PRINCIPLES OF ELECTRICAL ENGINEERING

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PRINCIPLES
OF
ELECTRICAL ENGINEERING

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PREFACE

This text is the outgrowth of experience in teaching the principles of electrical engineering to students of electrical engineering at the Massachusetts Institute of Technology. It aims to provide a substantial first course in the subject by presenting rigorously, and at the same time in understandable form, the really basic principles upon which modern electrical engineering rests. In furtherance of this purpose many problems and examples from current engineering practice are introduced. The book is not, however, to be mistaken for a complete condensed treatise on the entire subject. It is strictly a first course on the principles, and its study should be followed by detailed courses in direct-current and alternating-current machinery. Where-ever applications of the principles are introduced, they are for the purpose of illustrating these principles and rendering them real and alive to the student.

The book has the following special features, which we believe to be desirable:

1. The subject of the magnetic circuit has been stressed. It has been the common experience of teachers of electrical engineering that students beginning the subject find this a stumbling block. Much more space than is usual has, therefore, been devoted to this matter.

2. As a basis for explanation, the modern electron theory has been freely used. It has been found that this affords the most rational means of tying together the otherwise widely divergent principles with which the electrical engineer deals.

3. The subjects of thermionic emission, conduction through gases, electrolytic conduction and certain high-frequency phenomena have been included. A knowledge

of these matters is becoming more and more essential to the electrical engineer in any field.

4. The subject of the behavior of dielectrics has been approached from a standpoint which departs from the historical method of attack in order to gain clearness and unity of treatment.

5. About five hundred live problems are included for illustration, for practice in applying the principles and for the purpose of bringing before the student useful and interesting engineering data. Some of these are purposely made of such calibre as to merit the attention of the most able students.

The book is written primarily for students of college grade and presumes a knowledge of calculus and physics. The terminology and symbols employed are those recommended by the American Institute of Electrical Engineers.

Grateful acknowledgment is extended to Professor F. S. Dellenbaugh, Jr., for oscillographs of transients, and to Mr. E. L. Bowles, Mr. L. F. Woodruff and Mr. E. L. Rose for diagrams, proof-reading and the checking of problems.

W. H. T.
V. B.

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CHAPTER I

THE ELECTRICAL ENGINEER

A nation's standing in the scale of modern civilization can fairly be measured by the extent to which she is utilizing her natural sources of power. The truth of this is evident in the case of manufacturing nations, but nowadays it is also true of even agricultural communities since they depend for much of their prosperity upon the artificial fertilizer produced electrically by plants utilizing natural water power that formerly went to waste.

This development of the world's sources of power is the work of the engineer. Whether the task is the designing of some delicate mechanism to operate on the minute quantity of power available in a telephone circuit, or the harnessing of the floods of the Mississippi to supply whole cities with almost unlimited power, it is the task of the engineer. Wherever a project involves the generation, transmission or utilization of electric power, electrical engineers are needed.

1. Electricity not a Natural Source of Power. Electricity as it occurs in nature, however, is not one of the natural sources of power, although fully one-half the world's power supply is converted into the electrical form before it is finally utilized because that is often the cheapest and most convenient form in which to transmit and utilize power.

The chief natural sources of power are:

(1) Water flowing in natural or artificial waterways,
(2) Coal which is mined from the earth,
(3) Oil and gas which flow from wells.

In order to generate any large amount of power by means of water, either there must be an enormous flow of water at a low head or a smaller flow at a very high head. Thus in the plant at Swan Falls, Idaho, which is shown in Fig. 1, the head averages only 19 feet and the necessary flow per horse power is over 120,000 pounds per hour, while in the San Franciscito plant in California, shown in Fig. 2, the flow per horse power is less than 2600 pounds per hour but the head is 938 feet. The design of the water wheels and electric generators for operating under such diverse conditions must show a wide variance. In a problem of this kind, the electrical engineer's knowledge of physics, particularly mechanics, is called most prominently into play. Where coal or oil is used to produce power, a much smaller quantity per minute is required. For instance, in place of either of the two projects mentioned above, we could produce a horse power by burning about 1.5 pounds of coal or 0.9 pound of oil per hour. Thus by burning a small quantity of oil or coal, an immense amount of energy can be released. This is one of the reasons why the larger percentage of the power used in the world comes from coal and oil, in spite of the fact that the sources of these materials are limited in extent.

2. Why We have Central Power Plants. Great as the amount of power is that can be obtained from small amounts of oil and coal, it is, however, rarely ever convenient or economical to set up an engine at just the place where the power is to be used. When we wish to light a room by incandescent electric lamps which require about $\frac{1}{15}$ horse power each, we do not set up a small gasoline motor and generator or a steam engine and generator near each lamp. Neither do we set up in a shop a small engine near each

FIG. 1. Idaho Power Company's Plant at Swan Falls, Idaho, houses two 850-kva. and four 1,560-kva. generators operating under a head of 17 to 21 feet. All current is generated at 2,300 volts, and is stepped up to three high-tension voltages, connecting with three sub-divisions of the transmission system, one of 22,000 volts, one of 44,000 volts and one of 66,000 volts. *Westinghouse Elec. & Mfg. Co.*

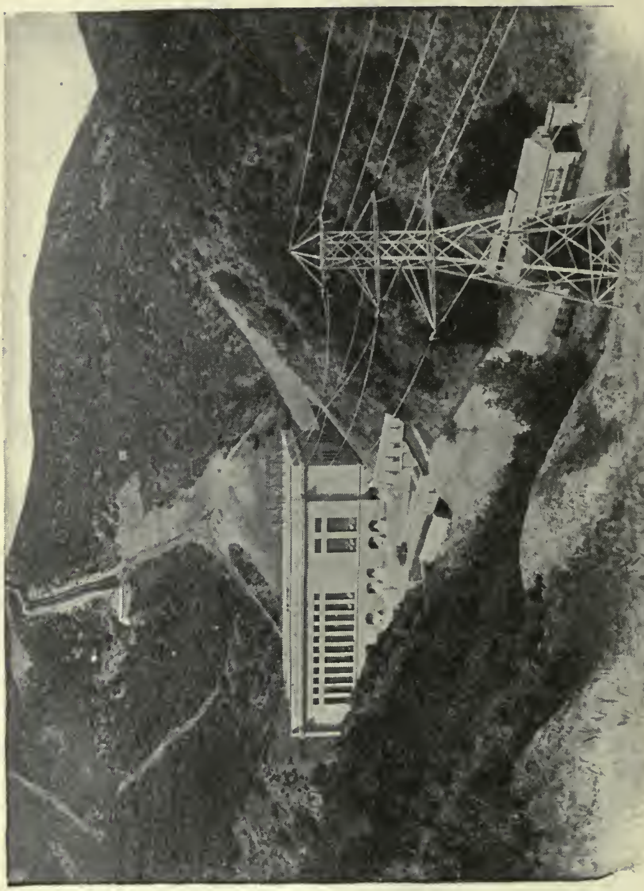

Fig. 2. Utilizing the power available from a large city's water supply. Three 9,375-kva. generators operating under a head of 938.2 feet at San Franciscito Plant, Los Angeles, Cal. Generator voltage is 6,600. Power is transmitted to Los Angeles at 100,000 volts.

machine to be driven. In fact, in a large manufacturing plant, we rarely have separate engines for separate buildings. One central plant supplies several buildings. A further development of this same scheme is to supply power to a whole city or community from a single power plant. Recently power plants have been developed which supply many cities at once. The modern tendency is to include larger and larger areas within one system, some engineers even prophesying that all the power stations in the whole country will eventually be connected into one vast superpower system.

3. Why Central Power Plants are Electrical. When it is desired to transmit power over great distances, as in the case of the systems mentioned above, practically the only way in which it can be done effectively is by means of electricity. In fact, when power is to be transmitted more than a few feet, electricity is usually the best form. Consider what a single room of an old-fashioned shop looked like with its forest of belts and shafts and pulleys. Then call to mind the picture that would be presented by a whole building or a group of buildings connected by rapidly moving belts and ropes. Try to imagine what even a small town would look like if we tried to transmit power from a central plant to the various manufacturing plants by means of belts and ropes, and it will be evident why central power stations are electrical.*

The advantages may be summed up as follows:

1. Electrical energy is transmitted by wires which are small and cheap, which do not move, which can be bent into

* It must not be inferred from this that all other means of power transmission could or should be superseded by electrical devices. Where distances are short, belts and shafting are often cheapest and most efficient. Where installations are temporary, distances not too long and only small quantities of power are needed, as for rock drills, hoisting engines and pumps, steam under pressure may often be used to advantage. Compressed air is also often preferable under some conditions.

almost any shape to pass around curves and obstacles and which are comparatively safe, neat and of small upkeep expense when once installed.

2. Electrical energy can be converted easily into nearly any other form at the spot where it is needed. It lights lamps, drives motors, heats stoves and furnaces, refines metals and electroplates.

3. Very little energy is lost in the wires and other devices during transmission over even great distances.

4. Electrical devices are easily stopped, started and controlled by simple compact devices which are rapid and accurate in their operation and durable in their construction.

4. Locations of Power Plants. For the purpose of economical distribution, a power plant should be located as near as possible to the center of the district it supplies. In the case of plants operated by water, such a location is generally impossible. Such large quantities of water under great pressure are needed for most plants that it is more practicable to locate the plant near the water way and transmit the electrical power to the district to be supplied than to bring so much water to a centrally located plant.

The problem of central locations of plants operated by gas, oil or coal is much simpler. The quantity of energy contained in a small weight of gas, oil or coal, the ease with which gas and oil can be piped over great distances and the numerous facilities for transporting coal make it practicable to locate the power plant at the center of the district to be served.

In the future, however, as the efficiency of transmitting electrical energy becomes greater and the cost of transporting gas, oil and coal becomes higher, it may be more economical to locate even the plants using these fuels in the oil and coal fields and transmit power by electricity to still greater distances than at present.*

* See "Electrical Transmission versus Coal Transportation", by Harold W. Smith, Electrical Journal, September, 1921.

5. Superpower System. The great saving in cost of energy and in conservation of coal and oil made possible

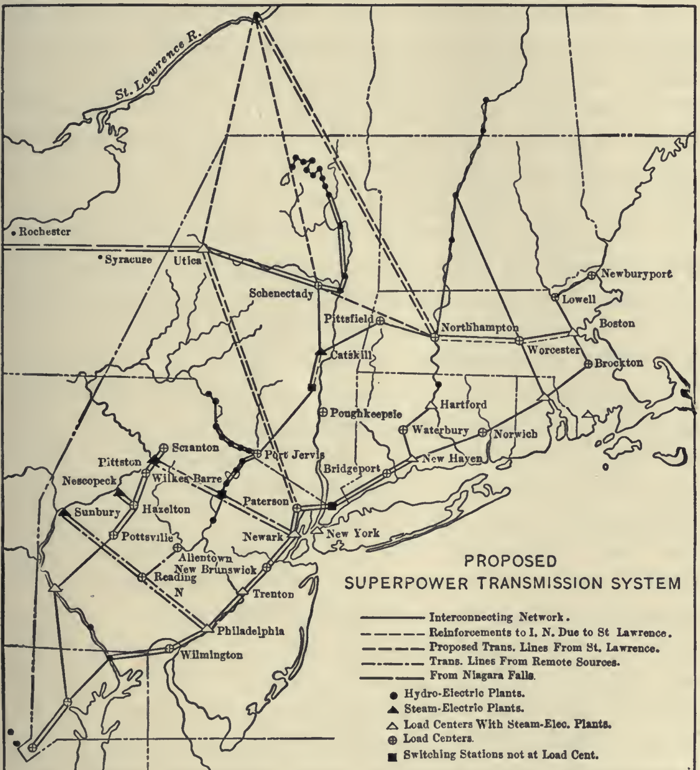

FIG. 3. It is proposed to develop all the water power and interconnect all the power plants within this section, which already carries on forty per cent of the manufacturing of the United States.

by developing a superpower system are strikingly illustrated in the Report of the Superpower Survey.*

* Professional Paper No. 123, Department of Interior, U. S. Geological Survey, entitled "A Superpower System for the region between Boston and Washington," by W. S. Murray and others.

It will be noted from Fig. 3 that the Superpower Zone extends from Washington, D. C., to Portland, Me., and from the Atlantic seaboard to Harrisburg, Pa., and Utica, N. Y. Within this zone reside twenty-five millions of the nation's population and here forty percent of the manufacturing of the nation is carried on. There are already in this area 315 electric public utilities operating about 1200 miles of line at 33,000 volts or higher, to which the superpower system will add about 4700 at 110,000 volts and 4000 at 220,000 volts. While most of the power is to be developed from the coal, oil and water power within this territory, it is also proposed to draw huge quantities from an enlarged power development at Niagara Falls and a new plant to be established on the St. Lawrence River. All of the available water power within the area will be put to use, the Potomac, Susquehanna and Delaware Rivers furnishing power to the central and southern portions, the Hudson and Connecticut Rivers to the northern portion. However, these water-power developments even when supplemented by those of Niagara and the St. Lawrence will take care of but little more than twenty percent of the requirements of the territory under consideration. For about eighty percent of the power in this vast system, coal and oil must be depended upon, and it is in the economical location and operation of these steam-electric stations that the great saving is to be accomplished. Note that these are to be located in the coal fields at Sunbury, Nescopee and Pittston, Pa., and at cities on the tidewater to which transportation is easy and cheap.

Most of this development bids fair to take place within ten years. One of the problems involves the designing of apparatus with an insulation capable of withstanding about a half-million volts pressure. A knowledge of the actions of different materials when subjected to this pressure is necessary, and for this reason the subject of electrostatics must be mastered by an engineer entering this part of the electrical

field. To this must be added a knowledge of the laws of economics, for the relative costs of different materials, methods and systems are always a vital factor in all great engineering projects.

6. Convenience of Electric Power. It is not only in the economical generation, transmission and utilization of huge quantities of power that electricity excels. It is also in the number of conveniences made possible by its use. Chief among these is the telephone. Each instrument requires only a minute quantity of power but makes possible communication with any of thirteen million stations in the United States. It has long been possible to carry on a conversation between New York and San Francisco over metallic conductors. Recently speech was transmitted from Havana to Key West by **submarine** cable, across the continent to San Francisco by **aerial** wire and **underground** cable, and over a thirty-mile strip of ocean to Catalina Island by **radio.** This accomplishment of combining wireless telephony with the present wire systems, with their twenty-five million miles of conductors, gives promise of far-reaching development in the near future.

The cross-continent transmission was made possible by vacuum-tube repeaters inserted in the line. The wireless transmission was accomplished through vacuum tubes used as rectifiers and amplifiers. In fact, the vacuum tube has recently come to play a prominent part in so many electrical developments that a knowledge of vacuum apparatus and the conduction of electricity through gases is almost a prime requisite for a modern electrical engineer in any field.

7. The Electrical Engineer. The two examples just given of progress in electrical engineering, the superpower project and long-distance communication, were chosen because of the great difference in their respective fields. One deals with vast quantities of power at high voltages, capable of being applied to thousands of different uses. The other employs insignificant amounts of power at the lowest volt-

ages and its use is restricted to the carrying on of conversation over distances. Yet both of these enterprises require the services of exactly the same types of electrical engineers with the same knowledge of fundamental laws of electricity and the same training in applying them.

Electrical engineers may be divided into three more or less clearly defined classes or types, depending upon their duties.

First type, — the electrical engineer who applies the laws of science to the development of electrical equipment. To this class belongs the research engineer, who pushes outward the boundaries of our knowledge of natural laws for the purpose of employing them more fully for mankind, the designing engineer, who plans the machines which make use of these laws, and the production engineer, who manufactures them. Each must have a thorough, up-to-date knowledge of the main branches of physics and chemistry as well as an intimate knowledge of the latest theories and discoveries in electrical science. The last two must also be familiar with the best shop methods especially as related to quantity production. To this class of engineers all electrical projects must look to supply the necessary electrical machinery and appliances.

Second type, — the electrical engineer who applies the electrical-engineering equipment to the use of man. He plans, constructs and operates power-transmission systems, telephone, telegraph and electric-railway systems. To this class belongs the engineer who has the responsibility of selecting the proper types and sizes of electrical equipment. He must be familiar with the latest designs and must have all the fundamentals of the science so well in hand as to be able to judge fairly the claims of various manufacturers concerning their outputs. Closely connected with this task is the writing and interpreting of specifications. Such an engineer must not only have a knowledge of the standard tests and behavior of electrical machinery under given con-

ditions but must also be able to describe these phenomena in clear, brief English which will never allow misunderstandings to arise as to his meaning. The economics of any project form a large factor, and so this engineer's decisions must be backed by the facts and theories of economics.

Third type,— the electrical engineer who acts as liason engineer between electrical engineering and other fields. Here belongs the consulting electrical engineer who is retained by any company to advise on electrical matters. Here also is the promoting engineer, who sees the problem in the large and realizes the possibilities of harnessing some waterfall, building a manufacturing plant or organizing a communication company. He must know the needs of the communities to be served by a project, the various uses to which the power can be put and the probable market for the company's commodities. Not only must he know these things well but he must also be able to explain his ideas in clear English to prospective investors and to prove to them the real value and soundness of the project. Such an engineer must have a broad knowledge of civil, mechanical and electrical engineering. As much of his responsibility is of a financial nature, he must be well grounded in economics and versed in business law and procedure. He does not need as intimate knowledge of the details of electrical machinery or of the minor points in electrical theory as the first two types of engineers. He should, however, be just as sure of his fundamentals, just as keen in his analysis and just as rigorous in his thinking.

It will be noted that all three engineers described above have certain common requirements in their education. They must all be familiar with the fundamental laws of physics, chemistry and mathematics, and must know thoroughly the general principles of electrical theory and practice which are the basis of all electrical engineering.

This text assumes a knowledge on the part of the student of the common facts and principles of physics, chemistry

and mathematics, and endeavors to present the general principles of electrical theory in such a manner as to afford a foundation upon which the student of electrical engineering, regardless of which branch he may select, can build his higher technical courses.

SUMMARY OF CHAPTER I

THE MODERN TENDENCY is toward greater and more efficient utilization of natural sources of power. This tendency is taking the direction of larger and larger central electric power stations, the application of electric power to more diverse uses and greater refinement of electric appliances. The field of the electrical engineer is thus gaining in extent and diversity.

THREE TYPES OF ELECTRICAL ENGINEERS are needed in this development:

- First, engineers to design and manufacture electric apparatus or machinery,
- Second, engineers to put this machinery to use,
- Third, engineers to connect electrical engineering and other fields.

CERTAIN FUNDAMENTAL EDUCATIONAL REQUIREMENTS are common to all three types, chief among which are the basic laws of mathematics, physics and chemistry and the general principles of electrical engineering.

PROBLEMS ON CHAPTER I

Prob. 1–1. If it requires 2.2 pounds of coal per hour to produce one horse power in a good modern steam power plant, how many tons of coal per day of 15 hours are used by a power plant delivering 25,000 horse power?

Prob. 2–1. A certain oil well flows 2000 barrels of oil per day. If this is burned under a boiler, how many horse power will it develop continuously? Assume that 1 pound of oil contains 18,000 B.t.u. of which 15 percent is available by this method of using the oil.

Prob. 3–1. If it requires 9 barrels of crude oil per day of 10 hours to run a 250-kilowatt plant at rated load, what percent of the energy in the fuel is available by the method used?

Prob. 4–1. The Big Creek reservoir of the Pacific Light and Power Company is 4.5 miles long, $\frac{1}{2}$ mile wide and has an average depth of 34 feet. The effective height of the reservoir above the water wheel is 1900 feet. How many foot-pounds of energy are stored in this reservoir?

Prob. 5–1. To how many tons of coal averaging 14,000 B.t.u. per pound is the water in the reservoir of Prob. 4–1 equivalent from the energy standpoint?

Prob. 6–1. The power plants in connection with the reservoir of Prob. 4–1 contain 6 water wheels of 20,000 horse power each. How many days would the water in the reservoir alone operate these wheels, assuming that the average load is one-half the capacity of the plants and that the efficiency at this load is 80 per cent?

Prob. 7–1. The highest recorded efficiency for water turbines was attained by the four 6000-horse-power wheels at New River, Va. Under a head of 49 feet, an efficiency of 93.7 percent was secured. What flow of water was necessary under these conditions?

Prob. 8–1. In the Mississippi River hydro-electric development at Keokuk, Iowa, there are 15 turbines each having a

normal rating of 10,000 horse power based on a head of 32 feet. Under these conditions they operate at an efficiency of about 88 percent. What is the flow of water through them?

Prob. 9–1. At full load the generators attached to the turbines in Prob. 8–1 have a guaranteed efficiency of 96.3 percent. How many kilowatts can each generator deliver under these conditions? Data from General Electric Review.

Prob. 10–1. In the Gatun hydro-electric development there are 3 Pelton-Francis turbines, each having a capacity of 3600 horse power when operating under an effective head of 75 feet. The total flow of water through the pen-stocks is 90,000 cubic feet per minute. What is the efficiency of the turbines under these conditions?

Prob. 11–1. Each generator attached to the turbines in the Gatun plant has a guaranteed efficiency of 95.1 percent when delivering 2000 kw. What horse power must each turbine develop under these conditions? Data from General Electric Review.

Prob. 12–1. Assuming an efficiency of 83 percent for the turbines in Prob. 11–1, how much water per minute must be supplied to each machine at an effective head of 75 feet?

Prob. 13–1. Assume the following conditions in a good gas-producing plant:

The producer delivers 75 percent of the energy in the coal to the gas engine.

The gas engine converts 35 percent of this energy into mechanical energy of the piston.

The piston delivers 90 percent of this energy to the shaft.

What is the over-all efficiency of the gas-producing plant?

Prob. 14–1. At $2.50 per ton for coal averaging 14,000 B.t.u., what will it cost for fuel per year of 3000 hr. to operate a 100-kilowatt electric generator with a gas-producing engine? The generator has an efficiency of 90 percent and the producer plant data as in Prob. 13–1.

Prob. 15–1. A 70-horse-power Diesel engine showed on test that it delivered 41.7 percent of the energy in the oil to the piston. The efficiency of the engine from the piston to the pulley was 90 percent. At 2 cents per gallon (7.6 pounds) for oil, how much more or less expensive per year of 3000 hours would it be to use a 70-horse-power electric motor of 80 per

cent efficiency instead of the Diesel engine? Electricity costs 4 cents per hour for each kilowatt. (This is not fair to the motor, as the oil engine would probably require much more attention.) 1 pound of oil = 14,500 B.t.u.

Prob. 16–1. A test was made on the shafts and belting of a certain machine shop eight stories high. A jack shaft on each floor was connected by belts to the engine shaft. When the shop was running at full load, the sum of the power being delivered by the several jack shafts was 196.7 horse power. The engine was delivering 257.2 horse power.

(*a*) What horse power was lost in the jack shafts and belting?

(*b*) What was the efficiency of the jack shafts and belting?

CHAPTER II

ELECTRIC UNITS AND ELECTRIC CIRCUITS

The electrical engineer deals primarily with the generation, transmission and utilization of electrical power. Therefore, while occasionally he has to deal with electricity at rest (static electricity), his usual concern is with electricity in motion, that is, with electric currents.

8. The Electron Theory. In every substance there are a large number of small particles of electricity. These are all of the same size and they are extremely small. We call these particles electrons.* The theory of their behavior under various conditions is called the electron theory and is in general use today.

The effects produced by electrons at rest are called static effects. More important are the effects produced when large numbers of electrons are in motion.

In many substances, such as glass for instance, the electrons are attached securely to the atoms of the material and can be broken loose only with great difficulty. These substances are insulators. In metals there are large numbers of free electrons, or electrons which are not attached and which can move through the metal. These metals are conductors. When the electrons move we say that the metal conducts a flow of electricity or a current of electricity.

A metal wire full of free electrons is similar to a pipe packed full of sand and then filled with water. The electrons may be likened to the water molecules and the atoms of the material to the grains of sand. The electrons may

* See "The Electron" by R. A. Millikan or "Within the Atom" by John Mills.

be made to filter through the metal just as the water may be forced through the sand. The stream of electrons is an electric current just as the movement of the water molecules is a stream of water.

Water is considered an incompressible fluid in studying flow in pipes. There are really large spaces between the water molecules, but it is difficult to actually crowd them together or to compress the water. Anyone who ever had an automobile cylinder full of water is familiar with this fact. We may similarly treat the electrons in a metal as an incompressible fluid. There are large spaces between them and yet on account of their great repulsion for one another, they are hard to compress.

Water can be slightly compressed but only at such high pressures that we rarely deal with them. Similarly electricity can be compressed but only at very high electrical pressures. To small pressures it is practically incompressible.

9. The Ampere — A Unit of Current. Just as the flow of water in a pipe is measured by the number of gallons which pass a given point per second, so the flow of electricity along a conductor is measured by the number of coulombs of electricity which pass a given point per second. The coulomb is just as definite a quantity of electricity as the gallon is a definite quantity of water. In fact, a coulomb is a certain number of electrons, just as a gallon is a certain number of molecules of water. There are about 6.3×10^{18} electrons in a coulomb. This number written out in full is 6,300,000,000,000,000,000. It will readily be seen why a large unit was chosen for ordinary computations to avoid the use of such large figures.

An electric current is hence usually measured in " coulombs per second " rather than " electrons per second." Even the expression " coulombs per second " was considered too cumbersome, so the rate of flow of " one coulomb per second " is designated as an **ampere** rate of flow. Thus to

designate a current of 10 coulombs per second, we have merely to express it as 10 amperes.

In defining the ampere, advantage was taken of the fact that if an electric current is passed through a solution containing a salt of a metal, it will deposit the metal on the negative plate. The ampere was accordingly legally defined by an Act of Congress, 1894, as follows:

" The unit of current shall be what is known as the **international ampere,** which is one-tenth of the unit of current of the centimeter-gram-second system of electromagnetic units and is the practical equivalent of the unvarying current, which when passed through a solution of nitrate of silver in water according to the standard specifications, deposits silver at the rate of one thousand, one hundred and eighteen millionths 0.00111800 of a gram per second."

The ampere, then, is the common unit for measuring rate of flow; thus a 50-watt, 110-volt tungsten lamp is said to take a current of nearly one-half ampere — that is, a current of about one-half ampere is flowing through it all the time it is glowing. An electric flat iron usually takes about five amperes, a one-horse-power 110-volt motor about ten amperes, when running under full load.

When a piston-type water pump is working, the flow of water in the pipe varies or pulsates. Similarly currents in wires are often variable. In fact, they may alternate, or change direction rapidly, just as water in a pipe would alternate in direction of flow if the pump were a simple piston without valves.

The American Institute of Electrical Engineers defines five types of currents thus:

Direct Current. A unidirectional current. In other words a current which never reverses in direction, although it may vary in amount.

Pulsating Current. A direct current which pulsates regularly in magnitude.

Continuous Current. A practically non-pulsating direct current. Usually, unless otherwise specified, when we speak

of a direct current we mean this particular kind of a direct current.

Alternating Current. A current which alternates regularly in direction. Unless otherwise distinctly specified, a periodic current in which the changes in magnitude and direction are regularly repeated and in which the net flow in one direction along the wire equals the net flow in the other direction.

With an alternating current, the electrons move slightly back and forth past a fixed point in the wire, without any gradual progression along the wire. In most alternating-current circuits the current changes direction one hundred and twenty times a second. This is called a sixty-cycle current. The frequency of the current is sixty cycles per second.

Oscillating Current. A periodic current whose frequency is determined by the circuit constants. The current that flows in the antenna of a radio-telegraph transmitter is usually an oscillating current. With this type of current the successive waves are often not of the same magnitude.

Several types of currents may flow in the same wire simultaneously.

The laws which govern the flow of alternating currents are very similar to those which govern the flow of continuous currents. In this text we will study the laws of the electric circuit when continuous currents flow, and then show some of the additional effects which must be taken into account when the current varies. Unless otherwise stated, an electric current in this text should therefore be taken to mean what is usually known as a direct current, although more exactly defined above as a continuous current.

10. The Volt — A Unit of Pressure. Just as it requires pressure to cause a flow of water, so it requires pressure to cause a flow or current of electricity. Just as we usually measure water pressure in pounds per square inch, so we measure electrical pressure or electromotive force in volts. When a lamp is rated as a 110-volt lamp, it means that it requires a pressure of 110 volts across the terminals of that lamp to force the proper amount of current through it. More pressure, that is, a higher voltage, would force through more current and probably damage the lamp.

There are several ways in which we can produce a pressure tending to force the electrons along a wire. Small pressures, or voltages, can be obtained chemically from batteries, either primary cells or storage batteries. Their action is exactly the same but the storage battery can be recharged when exhausted; that is, its chemicals can be restored to their original condition by forcing a current through in the reverse direction.

There are many other ways in which electrons may be caused to move along a wire. The most important of these is used in the electric generator, where the wire is moved rapidly through a magnetic field.

However the pressure may be produced, it is measured in volts. One volt as defined by Act of Congress is the 1/1.0183th part of the pressure delivered at 20° C. by a standard chemical cell, called the Weston cell, consisting of plates of cadmium and mercury in an electrolyte of mercurous sulphate and cadmium sulphate.

11. The Ohm — A Unit of Resistance. It was proved by Georg Simon Ohm in a paper published in 1826* that for a given metallic circuit a definite ratio existed between the pressure and the current — if the pressure was doubled, the current would also be doubled, etc. This ratio of pressure to current is called the **resistance** of the circuit and is measured in **ohms.** Thus a circuit is said to have five ohms resistance when the ratio of the pressure to the current is five. That is, five volts will force one ampere through it, ten volts will force two amperes, etc.

$$\frac{\text{Pressure}}{\text{Current}} = \frac{\text{Volts}}{\text{Amperes}} = \text{Ohms}.$$

The resistance of a wire to a flow of current is similar to the frictional resistance of a pipe to the flow of water. The flow of electricity can, however, be measured and calculated today much more accurately than can the flow of water.

* See Friedrich Mann's, "Georg Simon Ohm."

12. Ohm's Law. The law that the ratio of the pressure to the current in a given circuit is constant is called Ohm's Law, and is the fundamental law of the flow of electric currents. In its simplest form this may be written as follows

$$\text{Resistance} = \frac{\text{Pressure}}{\text{Current}}.$$

In practical units

$$\text{Ohms} = \frac{\text{Volts}}{\text{Amperes}}.$$

In symbols

$$R = \frac{E}{I}, \tag{1}$$

where R = resistance in ohms,

E = electromotive force (or pressure) in volts,

I = current in amperes.

The symbol I is used for current under an international agreement which insures having the same symbols used in electrical work the world over. This is very convenient indeed in studying books written in foreign languages. The letter I is taken from the French word for current, "intensité."

Example 1. When 220 volts are applied to the field coils of a certain motor it is found that a current of 1.8 amperes flows through the coils. What is the resistance of the coils?

$$R = \frac{E}{I} = \frac{220}{1.8} = 122 \text{ ohms.}$$

13. Absolute System of Electrical Units. The Abvolt, Abampere and Abohm. In electrical work just as in mechanics, we have two common systems of units. The first or practical system is generally used by engineers, and is based upon the ampere, volt and ohm. In addition there is the c. g. s. or absolute system. The units in this system have very simple relations to the fundamental units, the centimeter, gram and second. In much theoretical work it is more convenient to use the absolute system.

The unit of current in this system is the **abampere** and equals 10 amperes.

The unit of pressure is the **abvolt** and equals 10^{-8} volt.

The unit of resistance is the **abohm** and equals 10^{-9} ohm, that is, it is the resistance through which a pressure of one abvolt will force a current of one abampere.

Prob. 1–2. A dry cell in good condition has an internal resistance of approximately 0.08 ohm and an electromotive force of 1.4 volts. If a wire of negligible resistance is put across the terminals of a dry cell of 0.08 ohm resistance and 1.41 volts e.m.f., what current would momentarily flow in the wire?

Prob. 2–2. State the resistance, current and pressure of Prob. 1–2 in abohms, abamperes and abvolts.

14. General Application of Ohm's Law. Ohm's law may be applied to an entire electric circuit or to any part of a circuit.

When it is applied to the **entire** circuit, care must be taken to make sure that **all** the electromotive force of the circuit is used for the value of E, that **all** the current of the circuit is used for the value of I and **all** the resistance of the circuit for the value of R.

Similarly in applying the law to only a **part** of the circuit, care must be taken that the values for E, I and R include only the voltage, current and resistance of **that particular part** of the circuit under consideration.

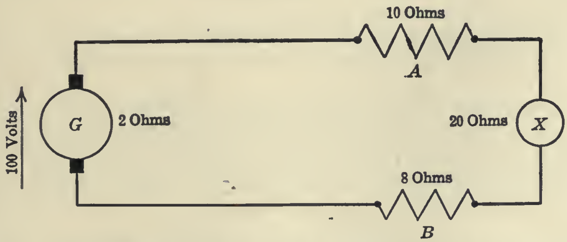

FIG. 4. The pressure of 100 volts is applied to the whole circuit.

Example 2. In Fig. 4 the generator G generates an electromotive force of 100 volts.

(*a*) How much current flows in the circuit?

(*b*) What is the voltage across the 10-ohm resistance?

(*a*) To find the current of the whole circuit, use the voltage of the whole circuit (the 100 volts e.m.f.) and the resistance of the whole circuit ($2 + 10 + 20 + 8 = 40$ ohms).

Thus $$I = \frac{E}{R} = \frac{100}{40} = 2.5 \text{ amperes.}$$

(*b*) To find the voltage across the 10-ohm resistor, use the current through the resistor and the resistance of the resistor. It is known from experiment that the same current flows in every part of such a circuit as that of Fig. 4. Thus there will be 2.5 amperes flowing through the 10-ohm resistor. The voltage across the 10-ohm resistor must equal the product of the current through the resistor and the resistance of the resistor or

$$E_R = I_R R_R = 2.5 \times 10 = 25 \text{ volts.}$$

Ohm's law should be very familiar to every engineer in all three of its possible forms.

$$I = \frac{E}{R}.$$

$$E = IR.$$

$$R = \frac{E}{I}.$$

Prob. 3–2. Assume that resistance X, Fig. 4, is not 20 ohms as marked but has an unknown value. The voltage across the resistor A is measured and found to be 15 volts. The rest of the data is as in Fig. 4. Compute:

(*a*) The voltage across B,

(*b*) The voltage across X,

(*c*) The resistance of X.

Prob. 4–2. A series tungsten lamp is rated to take 6.6 amperes when a voltage of 48 volts is applied to its terminals. It is desired to use two such lamps in series on a 115-volt system. What resistance must be connected in series with the lamps to limit the current to its rated value?

Prob. 5–2. The voltage at the generator end of a two-wire transmission line is maintained at 115 volts. When the line is carrying 42 amperes the voltage at the receiving end is 112 volts.

(*a*) What is the resistance of the line?

(*b*) What is the voltage at the receiving end when the line is carrying 84 amperes?

15. Kirchhoff's Laws. First Law. Two fundamental simple laws of the electric circuit have received the name of Kirchhoff's laws. They are fundamental natural laws applied to the conditions for the flow of electric currents in a circuit or network of conductors.

First Law. Experiments show that with direct current the amount of current flowing away from a point in a circuit is equal to the amount flowing to that point.

This is simply an expression for the law for the conservation of matter. When electric currents flow in a circuit, no electrons are lost in the process. At any point in the circuit just as many electrons arrive per second as leave. Of course if the insulation is poor and current leaks off the wire, this leakage must also be considered. With moderate voltages the amount leaking in this way is usually entirely negligible.

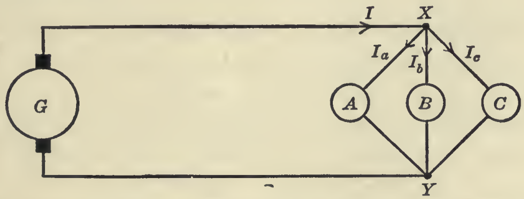

FIG. 5. The current I flowing to the point X must equal the sum of the currents I_a, I_b and I_c flowing away from X.

Thus in Fig. 5, the current I flowing to the point X must exactly equal the sum of the currents I_a, I_b and I_c which are flowing away from the point X. We express this by the equation

$$I = I_a + I_b + I_c.$$

In other words, electricity does not pile up or accumulate at any point in an electric circuit, at least when a direct current is flowing.

When we come to the study of varying currents and capacitances, we shall see how to extend this law to cover alternating or oscillating currents also.

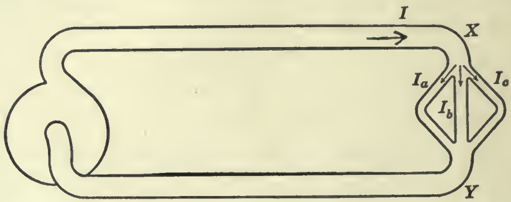

Fig. 6. The water current I must equal the sum of the currents I_a, I_b and I_c.

This law is exactly in accord with the hydraulic analogy of Fig. 6. The current of water I flowing to the point X in the pipe OX must exactly equal the sum of the currents flowing away from the point X in the three pipes A, B, and C, or

$$I = I_a + I_b + I_c.$$

16. Kirchhoff's Laws. Second Law. The difference of the electrical potential between any two points is the same regardless of the path along which it is measured. In other words, if there is an electric pressure acting between two points in a circuit it acts with equal force on all paths connecting the two points. By difference of electrical potential is meant the electrical pressure or the voltage between the points.

The engineer uses many terms for electrical pressure such as difference of potential, drop in potential, electromotive force, voltage and so on. These all mean electric pressures, but we usually speak of the electromotive force as the voltage supplied to the circuit by a battery or generator, and the pressure existing between the ends of a resistor is usually spoken of as a drop in potential, or simply as an IR drop.

Example 3. In Fig. 7 there are three paths from R to S:

(1) Through the resistor L,
(2) Through the resistor M,
(3) Through the generator G.

Consider the voltage between the points R and S,

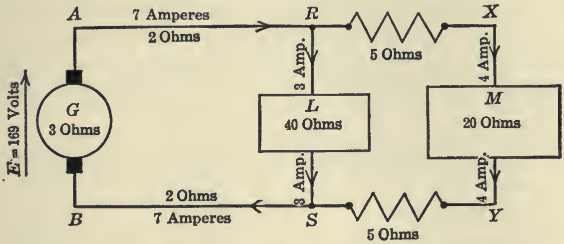

FIG. 7. The voltage drop from R to S is the same whether computed along the path $RABS$, RLS or $RXYS$.

1st, As computed from the current and resistance through the 40-ohm resistor (L);

2nd, As computed from resistance of path through the 20-ohm resistor (M).

(1) Voltage between R and S = Current through L times Resistance of L, $= I_L R_L$,

$$= 3 \times 40 = 120 \text{ volts.}$$

(2) Voltage between R and S = Current through path of M times resistance of path of M,

$$= 4(20 + 5 + 5) = 120 \text{ volts.}$$

Thus the voltage, or potential difference, between the points R and S is 120 volts, whether computed along the path of M or of L.

It remains to compute the potential difference between R and S along the path through the generator G.

17. Electromotive Force and *IR* Drop. The product of the current times the resistance is called the IR drop in potential. Since the electromotive force is the force which sends the current around the circuit, the IR drop may be thought of as the force which opposes the electromotive force in sending a current in this direction. When a steady

current is flowing, there is a condition of equilibrium and the opposing forces must be equal. Therefore the algebraic sum of the IR drops in any circuit must equal the algebraic sum of the electromotive forces.

In Fig. 7, if we represent the electromotive force of 169 volts as a positive force in the direction of current flowing around the circuit $ARSB$, we must represent the IR drops as positive forces in the direction opposite to current flow and the sum of the IR drops must equal the sum of the electromotive forces or 169 volts.

The IR drop through the generator	$= 3 \times 7$	$= 21$ volts.
The IR drop from A to R	$= 2 \times 7$	$= 14$ volts.
The IR drop from S to B	$= 2 \times 7$	$= 14$ volts.
The IR drop from R to S		$= x$ volts.
Total IR drop around circuit $ARSB$		$= (49 + x)$ volts

The algebraic sum of IR drops around a circuit must equal the algebraic sum of the electromotive forces.

Therefore

$$
\begin{aligned}
(49 + x) &= 169 \\
x &= 169 - 49 \\
&= 120 \text{ volts.}
\end{aligned}
$$

There is the same potential difference between the points R and S as found by computing it along the other two paths.

Kirchhoff's second law is very often stated in terms of the e.m.f. and the IR drop as follows:

$$\Sigma E = \Sigma IR$$

or

$$\Sigma E - \Sigma IR = O,$$

where ΣE is the algebraic sum of the electromotive forces in a circuit and ΣIR is the algebraic sum of the IR drops in the same circuit.

The reason that the algebraic sum must be used is because we may have electromotive forces opposing each other, as when a generator charges a storage battery. Also the currents in different parts of a particular loop in a network may be in different directions. In such a case the electromotive

force of the main source is called positive. Any electromotive forces opposing it are negative. *IR* drops opposing the main electromotive force are called positive and those aiding it, negative.

In applying the law in this form, great care must be exercised to see that when one sign has been given to the electromotive force in the direction of current flow, the opposite sign is given to the *IR* drop in the direction of current flow. These laws are of the greatest assistance in the determination of the current, voltage and resistance relations in circuits of the form of complicated networks.

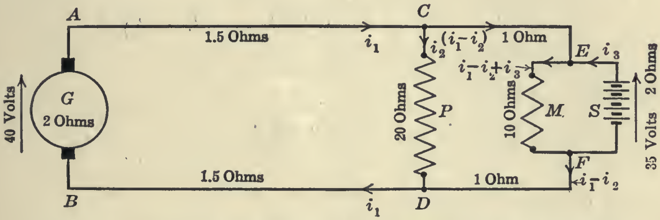

FIG. 8. Compute the current in the several parts of the line.

Example 4. In a circuit arranged as in Fig. 8, find the currents delivered by the generator G and storage battery S respectively.

We will assume the current to flow as follows, having care to apply Kirchhoff's first law of current flow that the sum of the currents flowing to any point must be equal to the sum of the currents flowing away from that point.

i_1	to flow from	A to C,
i_2	"	C to D,
$i_1 - i_2$	"	C to E,
$(i_1 - i_2 + i_3)$	"	E through the 10-ohm resistor to F,
i_3	"	F through the storage battery to E,
$i_1 - i_2$	"	F to D,
i_1	"	D to B and from B to A through generator G.

The currents may not in every case be in the direction in which we have assumed them to flow. If this is true, the results will show a minus sign for those branches in which we have assumed the wrong direction of flow. The numerical results will be correct, however, and a negative sign merely means that the current is actually flowing in the direction opposite to that marked in the diagram.

As we have three unknowns, we must write three equations.

(1) Consider the circuit *ACDB*:

$$\begin{aligned} 40 &= 1.5\, i_1 + 20\, i_2 + 1.5\, i_1 + 2\, i_1 \\ &= 5\, i_1 + 20\, i_2. \\ i_1 &= 8 - 4\, i_2. \end{aligned}$$

(2) In the circuit *CESFD*:

$$\begin{aligned} 35 &= 2\, i_3 - 1(i_1 - i_2) + 20\, i_2 - 1(i_1 - i_2) \\ &= 2\, i_3 - 2\, i_1 + 22\, i_2. \end{aligned}$$

Substituting value of i_1 from (1):

$$35 = 2\, i_3 - 2(8 - 4\, i_2) + 22\, i_2.$$

$$(3) \quad 51 = 2\, i_3 + 30\, i_2.$$

(4) In circuit *EMFS*:

$$\begin{aligned} 35 &= 2\, i_3 + 10\,(i_1 - i_2 + i_3) \\ &= 12\, i_3 + 10\, i_1 - 10\, i_2. \end{aligned}$$

Substituting value of i_1 from (1):

$$35 = 12\, i_3 + 10\,(8 - 4\, i_2) - 10\, i_2.$$

$$(5) \quad -45 = 12\, i_3 - 50\, i_2.$$

Combining (3) and (5):

$$\begin{aligned} 306 &= 12\, i_3 + 180\, i_2 \\ -45 &= 12\, i_3 - 50\, i_2 \\ \hline 230\, i_2 &= 351 \\ i_2 &= \frac{351}{230} = 1.526 \text{ amperes.} \end{aligned}$$

From (1) $i_1 = 8 - 4 \times 1.526$

$i_1 = 1.896$ amperes.

From (3) $i_3 = \dfrac{51 - 30\, i_2}{2} = \dfrac{51 - 30 \times 1.526}{2} = 2.610$ amperes.

Thus the generator *G* is delivering 1.896 amperes and the battery *S*, 2.610 amperes.

The answers may be checked by the use of Kirchhoff's second law in the form, "The difference of potential between any two points is the same along whatever path it is computed." For instance, there are four paths between the points C and D, and the potential difference computed along all four paths should be the same.

(1) Along CPD:

The difference of potential along this path is merely the IR drop through the 20-ohm resistance at P when a current of $i_2 = 1.526$ amperes is flowing through it.

$$V_{CD} = i_1 R = 1.526 \times 20 = 30.52 \text{ volts.}$$

(2) Along the path $CAGBD$:

$$\begin{aligned} V_{CD} &= E_G - i_1 (2 + 1.5 + 1.5) \\ &= 40 - 5\, i_1 \\ &= 40 - 9.48 = 30.52 \text{ volts (checks with (1)).} \end{aligned}$$

(3) Along the path $CEMFD$:

$$\begin{aligned} V_{CD} &= 1(i_1 - i_2) + 10(i_1 - i_2 + i_3) + 1(i_1 - i_2) \\ &= 12(i_1 - i_2) + 10\, i_3 \\ &= 12(1.896 - 1.526) + 10 \times 2.61 \\ &= 30.54 \text{ volts (checks with (1) to engineering accuracy).} \end{aligned}$$

(4) Along the path $CESFD$:

$$\begin{aligned} V_{CD} &= 35 - 2\, i_3 + 2(i - i_2) \\ &= 35 - 2 \times 2.61 + 2(1.896 - 1.526) \\ &= 30.52 \text{ volts (checks with (1)).} \end{aligned}$$

The use of Kirchhoff's laws will enable the engineer to solve any direct-current network whatever, no matter how complicated, if properly used. In fact by extending the meaning of these laws they can be made to solve alternating- or even oscillating-current networks.

In applying the laws, however, a great difference in the amount of labor involved in computation will result from different methods of going about the work. Certain general rules may be given for the methods of solving an electrical network, and if followed they will help in attaining speed and accuracy in such work.

In order to solve a network, first assign letters to the unknown quantities involved; second, form as many equations as there are unknowns; third, solve these equations simultaneously.

To avoid loss of time, proceed as follows:

1. Make a **good** diagram of the circuit.

Write on this diagram every constant of the circuit which is known, resistances, voltages or currents. Indicate the direction of the voltages or currents by arrows.

2. Assign letters to the unknowns.

Call currents i, voltages e, and resistances r with subscripts. Put an arrow on each e or i to indicate the direction in which it is **assumed.**

Use enough unknowns to completely "fix" the circuit, and no more. This means use as few unknowns as possible and still be enabled to write sufficient equations to solve the circuit.

In assigning letters to unknowns, apply Kirchhoff's laws as you go along, in order to keep down the number of unknowns. Thus if three wires meet at a point, and you have already called the current in two of them i_1 and i_2 respectively, call the third current $i_1 + i_2$ instead of i_3.

3. Apply Ohm's or Kirchhoff's laws to parts of the circuit to obtain equations between the unknowns.

Write as many equations as there are unknowns. There will always be the possibility of writing more than this. One of the possible equations not used at this time may be saved for a check.

Use Kirchhoff's first law where it is possible as it is very simple. In using the second law remember that the sum of voltages and drops may be taken about **any** closed loop in the network. Choose the simplest ones. Of course loops involving unknowns must be used. Be careful of signs.

4. Solve the equations algebraically for all the unknowns.

5. Check. This is important. Usually a loop not already utilized will give an excellent check from the second law.

Of course the solution of simple networks is easy. The networks actually met with in practice by the engineer are likely to be complicated. A study of the problems of this chapter will be only a beginning on actual cases of practice.

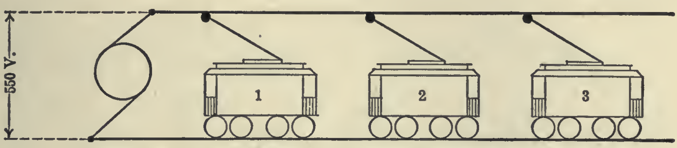

FIG. 9. What voltage exists across each car?

Prob. 6-2. In Fig. 9,

Car No. 1 is 1 mile from station and is taking 40 amperes.
Car No. 2 is 3 miles from station and is taking 20 amperes.
Car No. 3 is 4 miles from station and is taking 25 amperes.
Trolley wire has a resistance of 0.42 ohm per mile. Track resistance, 0.03 ohm per mile.

Find:

(*a*) Voltage across each car,
(*b*) Total voltage loss in line.

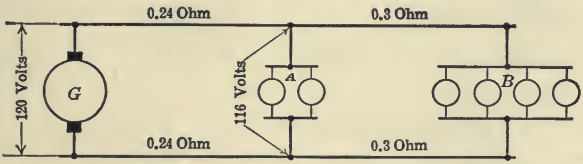

FIG. 10. The generator *G* supplies both group *A* and group *B* with power.

Prob. 7-2. In Fig. 10, the voltage across Group A is 116 volts; the resistance of each lamp in Group A is 100 ohms.

Find:

(*a*) Voltage across Group B,
(*b*) Average resistance per lamp in Group B,
(*c*) Line drop between Generator and Group A,
(*d*) Line drop between Group A and Group B,
(*e*) Current per lamp in A and in B.

Prob. 8–2. Assume trolley line, Fig. 41a, to be fed by two generators, G_1 of 560 volts and G_2 of 555 volts. Two cars are on the line, Car 1 taking 300 amperes, Car 2, 200 amperes. The resistances of the trolley wire and track are as marked.

Find the voltage across each car.

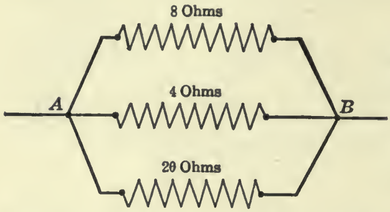

FIG. 11. The resistance of the parallel combination of resistors can be found by the application of Ohm's and Kirchhoff's Laws.

18. Solution of Series and Parallel Combinations of Resistors. Conductance. No special rules are needed for finding the resistance of any combination of series and parallel arrangements of resistors. It is merely necessary to apply Ohm's and Kirchhoff's laws.

Example 5. Three resistors are of 8, 4 and 20 ohms respectively and are arranged in parallel as in Fig. 11. What is the resistance of the combination?

The current per volt through the 20-ohm resistor	$=\frac{1}{20}=$	0.05 ampere.
The current per volt through the 4-ohm resistor	$=\frac{1}{4}=$	0.25 ampere.
The current per volt through the 8-ohm resistor	$=\frac{1}{8}=$	0.125 ampere.
The current per volt through the combination	$=$	0.425 ampere.

This amounts to assuming a difference of potential of one volt between points A and B. Thus, knowing the voltage across the combination (1 volt) and the current through the combination (0.425 ampere) we can find the resistance of the combination by Ohm's law:

$$R = \frac{E}{I} = \frac{1}{0.425} = 2.35 \text{ ohms.}$$

Note that the resistance of a parallel combination must be lower than the resistance of any separate branch of the combination. Regardless of how small is the resistance of that branch having the smallest resistance, the addition of the other branches between the same two points makes it easier for currents to flow between the two points and thus must lower the resistance between the two points.

The expression "current per volt" is often called the **conductance** of that part of the circuit to which it is applied. Note that the **conductance,** being the current per volt, equals $\frac{I}{E}$, which is the reciprocal of the expression for the **resistance,** $\frac{E}{I}$. For this reason the conductance is often measured in **mhos.** The conductance of an electrical appliance measured in **mhos** is the reciprocal of its resistance measured in **ohms.** Thus a lamp with a resistance of 10 **ohms** would have a conductance of $\frac{1}{10}$ **mho.**

Prob. 9–2. The numbers on the lines of Fig. 12 indicate resistance in ohms. Compute the resistance between

(*a*) *A* and *B*,
(*b*) *B* and *C*,
(*c*) *A* and *C*.

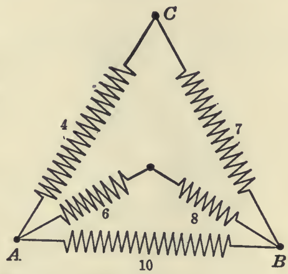

FIG. 12. Compute the resistances between two points of the triangle.

19. Special Networks, the Delta and the Star. A network arranged as in Fig. 13 is called a **Delta** or **Triangle.** The arrangement of Fig. 14 is called a **Star** or **Y.** These arrangements are often combined in a single network, as in Fig. 15, and present special difficulties in solution. The application of Kirchhoff's laws will solve such a network, but to find the equivalent resistance the simplest expedient consists of reducing star combinations to delta and vice versa, as the occasion requires.

To solve for the resistance between points A and B, in Fig. 15, for instance, the star (R', R'', R''') can be replaced by an equivalent delta (R_1, R_2, R_3), and the arrangement of Fig. 15 becomes that of Fig. 16, which can easily

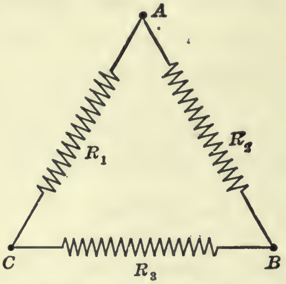

FIG. 13. A Delta or Triangle of resistors.

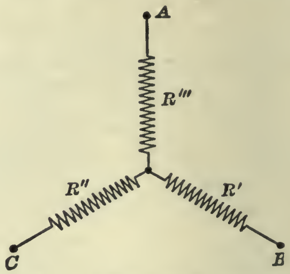

FIG. 14. A Star or Y arrangement of resistors.

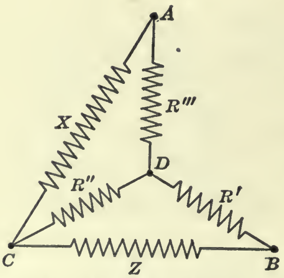

FIG. 15. A combination of Star and Delta arrangements of resistors.

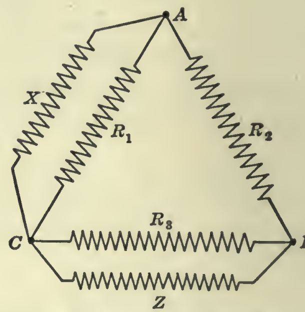

FIG. 16. This arrangement of resistors is delta equivalent of the arrangement of Fig. 15.

be solved for the resistance between A and B. It can be proved that star (R', R'', R''') is the equivalent of delta (R_1, R_2, R_3) if R_1 is made equal to

$$\frac{R'R'' + R''R''' + R'R'''}{R'}$$

and R_2 to

$$\frac{R'R'' + R''R''' + R'R'''}{R''}$$

and R_3 to

$$\frac{R'R'' + R''R''' + R'R'''}{R'''}.$$

Knowing R', R'' and R''', the values of the resistances of the delta R_1, R_2 and R_3 can thus be determined. Of course X and Z in Fig. 16 must also be known if the resistance between A and B is to be found.

Note: Special care must be taken in the notation. In the equivalent delta arrangement, R_1, R_2 and R_3 must be opposite in position with respect to R', R'' and R''' of the star.

Prob. 10–2. Prove that the delta arrangement of Fig. 13 is equivalent to the star arrangement of Fig. 14 if

$$R_1 = \frac{R'R'' + R''R''' + R'R'''}{R'},$$

$$R_2 = \frac{R'R'' + R''R''' + R'R'''}{R''}$$

and

$$R_3 = \frac{R'R'' + R''R''' + R'R'''}{R'''}.$$

Prob. 11–2. Prove that the star arrangement of Fig. 14 is equivalent to the delta arrangement of Fig. 13 if

$$R' = \frac{R_2R_3}{R_1 + R_2 + R_3},$$

$$R'' = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

and

$$R''' = \frac{R_1R_2}{R_1 + R_2 + R_3}.$$

Hint: Use equations of conductances.

Prob. 12–2. Compute the resistance between the points A and B, Fig. 15, where

$$\begin{aligned} R' &= 5 \text{ ohms}, \\ R'' &= 7 \text{ ohms}, \\ R''' &= 9 \text{ ohms}, \\ X &= 4 \text{ ohms}, \\ Z &= 8 \text{ ohms}. \end{aligned}$$

20. Measurement of Current, Voltage and Resistance.* The commercial method of measuring current is by means

* For details of construction, calibration and use of electrical instruments, see Laws' "Electrical Measurements."

of an ammeter inserted in the circuit at the point where it is desired to know the current.

In Fig. 17, ammeter A measures the current in the line AB; ammeter A_1 measures the current in branch BE or the current taken by the lamp L_1. Since the same current must flow away from L_1 as flows to it, the ammeter could just as properly be placed between L_1 and the point E.

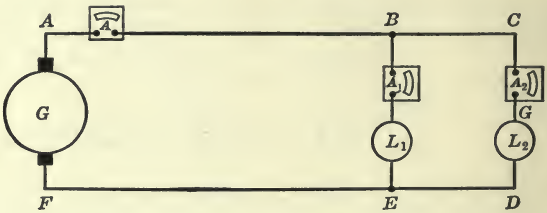

FIG. 17. The reading of the ammeter A is the sum of the readings of ammeters A_1 and A_2.

Ammeter A_2 measures the current taken by lamp L_2. Ammeter A_2 could be placed anywhere in the line $BCDE$ to measure this current. The sum of the readings of A_1 and A_2 should equal that of A.

An ammeter must have a very low resistance in order that it may not appreciably increase the resistance of the line in which it is inserted. The potential difference across the terminals of a well-known make of these instruments is about 0.05 volt for full-scale reading. Thus a 10-ampere ammeter would have $\frac{0.050}{10}$ or 0.005 ohm resistance, while a 100-ampere instrument would have $\frac{0.050}{100} = 0.0005$ ohm.

The **potential difference** between two points is commercially measured by means of a voltmeter, one terminal of which is attached to one point and the other terminal to the other point. Thus in Fig. 17*a*, voltmeter V_1 measures the difference of potential or voltage between the terminals of the lamp L_1. Voltmeter V_2 measures the potential

difference or voltage between the terminals of lamp L_2. A voltmeter V_3 attached to A and B would measure the potential difference between the points A and B. This value might differ slightly from the voltage across the lamp L_1, because the voltage across AB equals the voltage across the lamp L_1 plus the drop in that amount of wire which is used to connect the lamp to the points A and B.

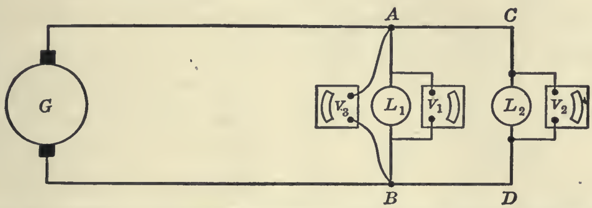

FIG. 17a. The voltmeter V_1 measures the voltage drop across the lamp L_1. Voltmeter V_2 measures the drop across the lamp L_2.

A voltmeter should have a very high resistance in order that it may not draw an appreciable current from the circuit to which it is connected. The resistance of a 150-volt instrument is in the neighborhood of 20,000 ohms or somewhat more than 100 ohms per volt.

For accurate work we must remember that when ammeters and voltmeters are connected into a circuit they alter the circuit conditions by adding new resistances and branches.

Thus a voltmeter connected between C and D of Fig. 17 will measure the voltage drop in the lamp plus the voltage drop in the ammeter. To obtain the actual voltage across the lamp we should subtract the computed drop in the ammeter from the voltmeter reading.

If the voltmeter is connected between G and D it will then read the correct voltage across the lamp, but in this case the ammeter will read not only the lamp current but also the voltmeter current. The latter may be computed and subtracted to obtain the actual lamp current.

In measurements taken on a circuit carrying fairly large currents at moderate voltages, the drops in ammeters and the current taken by voltmeters are usually negligible. If we were commercially testing a 20-horse-power, 550-volt motor, for instance, we should not ordinarily make allowance for the effect of putting in the meters.

21. The Wheatstone Bridge. Resistances may be measured roughly by using an ammeter and a voltmeter. The resistance to be measured is connected to a circuit which will pass through it a current it can safely carry. This current is measured with an ammeter and the drop due to it with a voltmeter. Corrections as outlined in the last section are made to these readings. The quotient then gives the desired value of resistance.

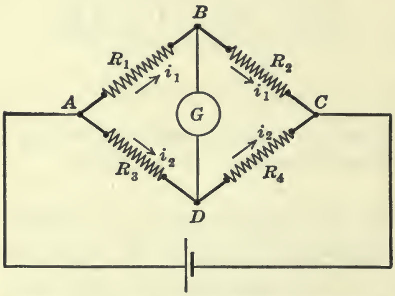

FIG. 18. The conventional diagram of Wheatstone's bridge.

A more accurate method of measuring resistance is by some form of the Wheatstone bridge. This bridge consists of an arrangement of four resistors, a steady source of electromotive force (such as a battery cell) and a galvanometer.

The four resistors are arranged in two parallel paths as shown in Fig. 18, where the resistors R_1 and R_2 form path ABC, and resistors R_3 and R_4 form path ADC. The battery is attached to the terminals A and C of these parallel paths and the galvanometer (G) is bridged across the intermediate points B and D. When the resistances of the resistors have been so adjusted that no current flows through the galvanometer, the points B and D must be at the same electrical potential.

If points B and D are at the same potential, the potential drop from A to D must equal the potential drop from A

to B. Similarly, the drop from D to C must equal the drop from B to C.

Let the current in branch ABC be i_1 and the current in branch ADC be i_2.

$$\text{Drop } A \text{ to } B = i_1R_1,$$
$$\text{Drop } A \text{ to } D = i_2R_3,$$
$$i_1R_1 = i_2R_3. \tag{1}$$
$$\text{Drop } B \text{ to } C = i_1R_2,$$
$$\text{Drop } D \text{ to } C = i_2R_4,$$
$$i_1R_2 = i_2R_4. \tag{2}$$

Divide (1) by (2) $$\frac{i_1R_1}{i_1R_2} = \frac{i_2R_3}{i_2R_4}.$$

Therefore $$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \tag{3}$$

If any three of these resistances are known in amount, the fourth can be found from equation (3).

In practice two of the resistors are usually constructed so that they may be made 10, 100 or 1000 ohms in value. A third resistance is variable between wide ranges, usually from several thousand ohms to a tenth or a hundredth of an ohm. The fourth is the resistor of which it is desired to measure the resistance.

Thus in Fig. 18, the resistors R_1 and R_2 might well have values of 10, 100 or 1000 ohms, while R_4 was the resistor with the wide-range adjustment. R_3 would be the same fraction or multiple of R_4 that R_1 was of R_2.

Example 6. Assume that in Fig. 18, R_1 was set at 10 ohms and R_2 at 1000, and that R_4 had to be set at 4124.6 in order to balance the bridge, that is, to obtain a condition in which there was no deflection of the galvanometer needle. What would be the value of R_3?

$$R_3 = \frac{R_1}{R_2} R_4$$

$$= \frac{10}{1000} \text{ of } 4124.6$$

$$= 41.246 \text{ ohms.}$$

Prob. 13–2. Fig. 19 represents a Wheatstone bridge in which the two resistors R_1 and R_2 of Fig. 10 are replaced by a single wire BD of high resistance and uniform cross-section.

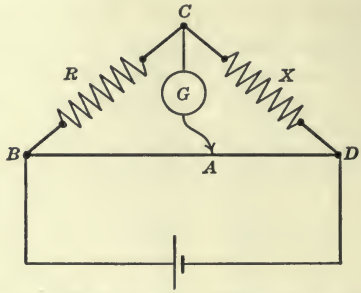

FIG. 19. The slidewire Wheatstone bridge.

A balance is obtained by sliding the contact point A along the wire. Write the equation for this form of bridge.

Prob. 14–2. Fig. 20 represents a Wheatstone bridge set up for a "Varley Loop" test for the location of a ground in a

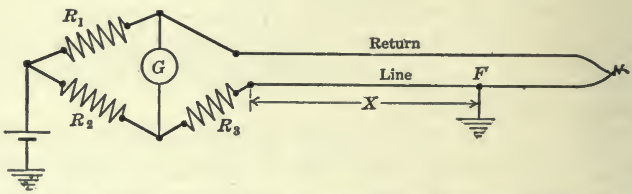

FIG. 20. A Wheatstone bridge arranged for a "Varley Loop" test, to locate the place F, where the line is grounded.

cable. The "return" wire need not have the same resistance per foot as the line. Derive the equation for the distance (X) out on the line to the grounded point "F." Resistance per foot of "line" is known.

SUMMARY OF CHAPTER II

THE VOLT (= 10^8 abvolts) is the practical unit of pressure.
THE AMPERE (= 10^{-1} abampere) is the practical unit of current.
THE OHM (= 10^9 abohms) is the practical unit of resistance.
OHM'S LAW:

$$\text{Amperes} = \frac{\text{Volts}}{\text{Ohms}} \qquad I = \frac{E}{R}$$

$$\text{Volts} = \text{Amperes} \times \text{Ohms} \qquad E = IR$$

$$\text{Ohms} = \frac{\text{Volts}}{\text{Amperes}} \qquad R = \frac{E}{I}$$

KIRCHHOFF'S LAWS; used in the solution of networks:

First Law: The sum of all the currents flowing to any point equals the sum of all the currents flowing from that point.

Second Law: The algebraic sum of the electromotive forces in any circuit equals the sum of the resistance drops.

CONDUCTANCE is the reciprocal of resistance.

CURRENT is usually measured by an ammeter.

VOLTAGE is usually measured by a voltmeter.

RESISTANCE may be measured by applying Ohm's law to values of the current and voltage or by the use of a Wheatstone bridge.

PROBLEMS ON CHAPTER II

Prob. 15–2. In Fig. 21,

Resistance of A = 100 ohms,
Resistance of B = 120 ohms,
Resistance of C = 160 ohms.

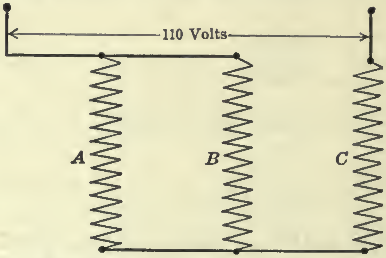

FIG. 21. A combination of series and parallel arrangements of resistors.

Find:

(*a*) Current through each resistance,
(*b*) Resistance of parallel combination (A and B),
(*c*) Combined resistance of system,
(*d*) Voltage across each resistance.

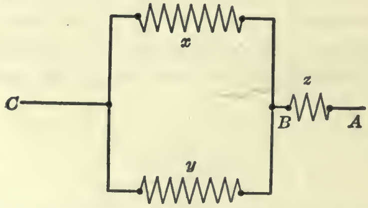

FIG- 22. A combination of series and parallel arrangements of resistors.

Prob. 16–2. In Fig. 22,

Voltage from A to B = 40 volts,
Current through resistance x is 2.5 amperes,
Resistance y = 5 ohms,
Resistance z = 4 ohms.

Find:

Current through y, Resistance of x,
Current through z, Voltage B to C.

Prob. 17–2. In Fig. 23, each lamp takes 12 amperes. Resistance of $AB = BC = 0.04$ ohm. Resistance of $CD = MK = 0.03$ ohm. Resistance of $EF = KF = 0.02$ ohm. What is the voltage across Group I and Group II, if the terminal voltage of $G_1 = 120$ volts and of $G_2 = 125$ volts?

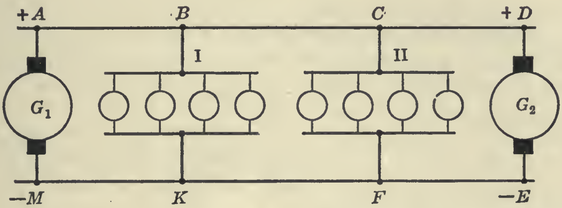

Fig. 23. Two lamp loads fed by two generators, one at each end of the line.

Prob. 18–2. A storage battery of 240 cells in series is "floated" at the end of a 4-mile trolley line, the resistance of which (line and return) is 0.08 ohm per mile. The generator voltage is 550 volts; each cell has an e.m.f. of 2.1 volts and an internal resistance of 0.001 ohm. (*a*) What current will the cells supply to the line, when there are 5 cars at the battery end of the line, each taking 65 amperes? (*b*) What will be the terminal voltage of the set of battery cells?

Prob. 19–2. What current will the cells in Prob. 18–2 take when there is no load on the line?

Prob. 20–2. What current will the generator be delivering when the 5 cars of Prob. 18–2 are half way between station and battery? Cars are carrying the same current as in Prob. 18-2.

Prob. 21–2. What current will the battery be delivering to or receiving from the line in Prob. 20–2?

Prob. 22–2. If there are only two cars on the line in Prob. 20–2, each taking 75 amperes, (*a*) what current is the generator delivering? (*b*) What current is the battery receiving or delivering?

Prob. 23–2. Six storage cells arranged as in Fig. 24 in two parallel sets of three cells in series are discharging through a

resistor of 1.2 ohms. Each cell normally has an e.m.f. of 2.1 volts and internal resistance of 0.02 ohm, but one cell in set *A* has " gone bad " and has an e.m.f. of 1.8 volts and an internal resistance of 0.14 ohm. What current is supplied to *R* by each set of cells?

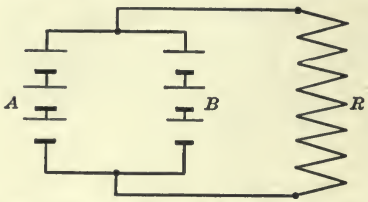

Fig. 24. Batteries *A* and *B* connected in parallel are supplying current to the resistor *R*.

Prob. 24–2. If the resistance of *R*, Fig. 24, is reduced to 0.02 ohm, and the other data left as in Prob. 23–2, what current will flow through each set of cells and in what direction?

Prob. 25–2. If the resistance of *R*, Fig. 24, is increased to 2.4 ohms and the other data left as in Prob. 23–2, what current will flow through each set of cells and in what direction?

Prob. 26–2. A building is supplied by a two-wire system, the wiring diagram of which is as per Fig. 25. Values on lines represent the resistances of the various sections of line wire.

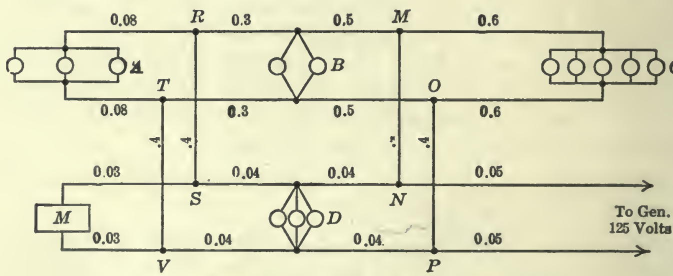

Fig. 25. The wiring plan of a two-story factory.

Motor *M* takes 40 amperes. Each lamp takes 4 amperes and the resistance of each is constant. (*a*) Find the voltage across each set of lamps. (*b*) Draw a diagram indicating the amount and direction of the current in each section of line.

Prob. 27–2. If the motor in Prob. 26–2 is not running, what will be the voltage across each group of lamps in Fig. 25? Each lamp is assumed to take 4 amperes.

Prob. 28–2. If the jumpers MN and OP are left out of Fig. 25, what will be the voltage across each set of lamps? Assume each lamp to take 4 amperes and the motor 40 amperes.

Prob. 29–2. An electric railroad is supplied with power by substations 10 miles apart. The trolley wire is connected at frequent intervals with a heavy copper conductor called a "feeder." The two may be considered jointly as equivalent to a single conductor whose resistance is 0.18 ohm per mile. The return path from the cars to the sub-stations consists of the track which is well bonded at the joints and has a resistance of 0.0305 ohm per mile. At a certain time two cars are running in the section between these sub-stations, one taking 200 amperes at a distance of 4.5 miles from one end, the other taking 240 amperes at a distance of 3 miles from the other end. The voltage at the busbars of each power station is 700. What is the voltage between trolley and track at each car?

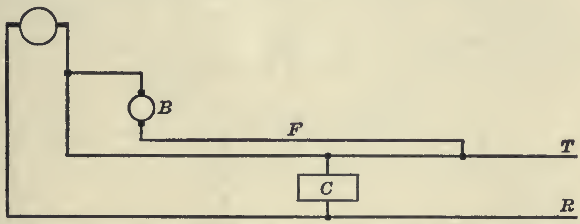

FIG. 26. The voltage of car C is "boosted" by means of the "booster" feeder B and F.

Prob. 30–2. Fig. 26 represents a method of raising line voltage. T is the trolley wire and R the rails. F is a feeder which is connected to the trolley wire at a distance of 4.2 miles from the power station. B is a booster, *i.e.*, a generator used to increase the voltage applied to the feeder. The following conditions exist. The terminal voltage of the generator is 600. The trolley wire has a resistance of 0.064 ohm per 1000 feet. The rails form a return conductor whose resistance is 0.0061 ohm per 1000 feet. The booster armature has a resistance of 0.024 ohm and generates an e.m.f. of 100 volts. The total resistance of the feeder is 0.82 ohm. C is a train requiring a current of 500 amperes. What is the lowest voltage which the train can receive in the region between the power station and the point of connection between the feeder and trolley?

Suggestion: Let x be the distance from the power station to the train. The minimum voltage occurs when dv/dx is zero.

Prob. 31–2. A storage battery and a motor armature are connected through leads of 0.5 ohm resistance to form a closed circuit. The chemical e.m.f. of the battery and its internal resistance are 120 volts and 0.12 ohm respectively. If the motor armature has a resistance of 0.45 ohm, what e.m.f. must it generate by rotation in order that the battery may discharge at its normal rate of 10 amperes?

Prob. 32–2. Two storage batteries are connected in parallel to be charged from 110-volt mains. As 110 volts is not sufficient to charge the batteries at the desired rate, a booster is placed in series with one of the mains, connected so that its voltage adds to the line voltage. The e.m.f. induced in the booster armature is 6 volts and the armature resistance is 0.06 ohm. The internal resistances of the batteries are 0.20 and 0.33 ohm respectively. The e.m.f.'s induced by chemical action in the batteries are 108 volts and 110 volts respectively. Compute the current in each part of the circuit.

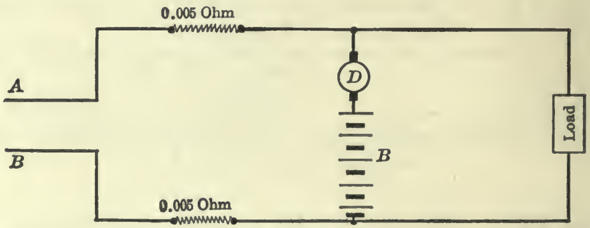

Fig. 27. The voltage of the "load" is made more nearly constant by means of the storage battery B and the booster D.

Prob. 33–2. The power received at AB, Fig. 27, is used to operate a group of elevators and fluctuates greatly. A storage battery (B) is used to equalize the load, in conjunction with a small dynamo (D) called a booster. The field coils of the booster are not shown and need not be considered. The booster is regulated in such a manner as to assist the battery to charge when the elevators are idle and to assist it to discharge when their demand is high.

The resistance of the booster armature is 0.012 ohm. The internal resistance of the battery is 0.015 ohm. In its average

state of charge the battery sets up an e.m.f. of 225 volts by chemical action. The voltage between mains at the primary source of power AB is 230 volts.

(*a*) What must be the value and what the direction of the e.m.f. induced in the booster armature in order that the battery may be charged by a current of 240 amperes when the elevators are idle?

(*b*) What value and direction must the booster's e.m.f. have in order that the battery and primary source may each supply half the current when the elevators require 450 amperes?

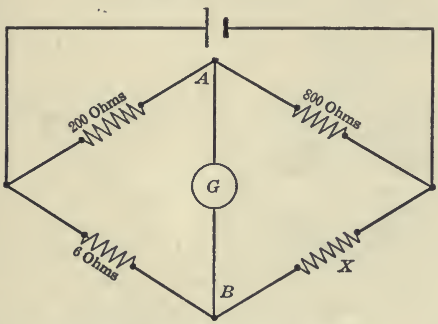

FIG. 28. The Wheatstone Bridge.

Prob. 34–2. A schematic diagram of a Wheatstone bridge is shown in Fig. 28. If the galvanometer G, whose resistance is 18 ohms, registers no current, what is the resistance of X?

Prob. 35–2. In Fig. 28, the battery has an e.m.f. of 2 volts and an internal resistance of 0.1 ohm; lead and contact resistance is negligible. (*a*) What would be the percentage error introduced in determining the resistance of X if the galvanometer failed to deflect when a current of 10^{-5} ampere was passing through it from A to B? (*b*) Would the true value of X be higher or lower than the value obtained by this measurement?

Prob. 36–2. Two wires each having a resistance of 0.32 ohm connect the terminals of a battery to two points a and b, between which are three branches whose resistances are respectively 4, 8.25 and 12.5 ohms. The battery generates by chemical action an e.m.f. of 124 volts and has an internal resistance of 0.16 ohm. (*a*) What current flows in each branch

from a to b? (b) What is the voltage across the battery's terminals?

Prob. 37–2. An electric circuit consists of the armature of a separately excited generator, two connecting wires and a storage battery, all in series. The armature has a resistance of 0.25 ohm and generates an e.m.f. of 115 volts by its rotation. Each of the connecting wires has a resistance of 0.2 ohm. The battery has a resistance of 0.12 ohm and generates by chemical action an e.m.f. of 108 volts opposed in direction to the e.m.f. of the generator. (a) Find the terminal voltage of the generator. (b) Find the terminal voltage of the battery.

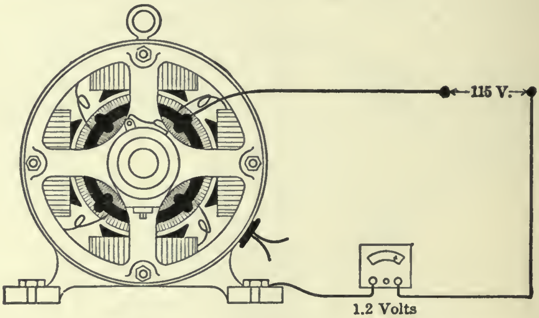

FIG. 29. An insulation test of a motor.

Prob. 38–2. The leakage conductance of the insulation between a pair of wires in a certain telephone cable is γ mhos per mile. The resistance of the conductor is r ohms per mile. Considering differential elements of insulation as connected in parallel by the conductors, compute the current input to the power end of the line when a potential of V_o volts is applied to this end. Assume that the distant end of the line is open. Compute the home-end resistance of the line by dividing the input voltage by the input current. If

$$\gamma = 0.0008 \text{ mho per mile},$$
$$r = 53 \text{ ohms per mile},$$
$$\text{length of cable} = 10 \text{ miles},$$
$$V_o = 100 \text{ volts},$$

compare the home-end resistance obtained by the above method with the leakage resistance obtained by dividing the leakage resistance per mile by the length of the cable. This problem requires knowledge of hyperbolic functions.

Prob. 39–2. In Fig. 29, one brush of the motor is attached to one side of a 115-volt circuit. The other side of the circuit is attached to the frame of the motor through a voltmeter which reads 1.2 volts. The resistance of the voltmeter is 18,650 ohms. What is the insulation resistance between the armature and the frame of the motor?

Prob. 40–2. Find the resistance between the points A and B in Fig. 30. The numbers on the figure indicate resistances.

Prob. 41–2. Compute the resistances in Fig. 30 between the points

(1) A and D,
(2) B and D,
(3) C and D.

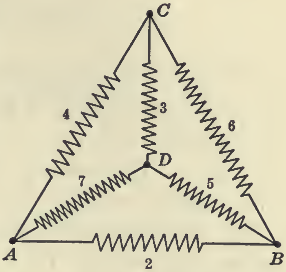

FIG. 30. A Star-Delta arrangement of resistors.

Prob. 42–2. A single-track railroad line is 5 miles long and is supplied with power from power stations at its two ends. The positive conductor consists of a third rail reinforced by a copper cable in parallel, the combination having a resistance of 0.0493 ohm per mile. The negative conductor consists of a pair of bonded track rails in parallel, the combination having a resistance of 0.0305 ohm per mile. The station at one end maintains a potential difference of 600 volts and that at the other end 575 volts between positive and negative conductors.

At what point along the line would an electric locomotive requiring 1200 amperes receive a minimum voltage between third rail and track? What would this voltage be, and what current would each station supply when the locomotive was at the point in question?

CHAPTER III

ELECTRIC POWER AND ENERGY

A current of electricity flowing along a conductor has been likened to a current of water flowing in a pipe. The analogy is so close that we may use the same method to compute the power required to maintain either a current of water or a current of electricity.

22. The Power Equation. A pump which is forcing a steady current of I pounds of water per second against a head or pressure of E feet is doing work at the rate of $I \times E$ foot-pounds per second. An electric generator which is causing a steady electric current of I amperes to flow under a pressure of E volts is doing work at the rate of $I \times E$ volt-amperes.

Since power is the time rate of doing work, the pump is said to have a power of $I \times E$ foot-pounds per second and the generator a power of $I \times E$ volt-amperes or watts. In direct-current circuits

$$1 \text{ volt-ampere} = 1 \text{ watt}.$$

In the form of an equation this is usually expressed

$$P = I \times E,$$

in which

P = power in watts,
I = current in amperes,
E = pressure in volts.

For convenience 1000 watts is called a kilowatt.

The power equation, as well as Ohm's and Kirchhoff's laws, applies in simple form only to direct-current circuits. It can also be extended to cover the power involved in the flow of alternating currents.

Example 1. At what rate is a pump working which is delivering 3000 pounds of water per second against a head of 22 feet?

$$P = IE$$
$$= 3000 \times 22 \text{ ft. lb. per sec.}$$
$$\text{or} \qquad \frac{66{,}000}{550} = 120 \text{ h.p.}$$

Example 2. A generator is delivering 300 amperes at a pressure of 220 volts. What power is it delivering?

$$P = IE$$
$$= 300 \times 220$$
$$= 66{,}000 \text{ watts}$$
$$\text{or} \quad \frac{66{,}000}{1{,}000} = 66 \text{ kilowatts.}$$

The following relations exist between electric units of power and mechanical units of power:

1 watt = 0.737 foot-pound per second
1 kilowatt = 1.34 horse power
1 foot-pound = 1.356 watt-seconds
1 horse power = 746 watts
or $\frac{3}{4}$ kilowatt (practically).

23. Voltage Not a Force. In the case of the flow of a fluid such as water, the equation for the power is

$$\text{Power} = \text{Pressure} \times \text{Current}.$$

Therefore

$$\text{Pressure} = \frac{\text{Power}}{\text{Current}}.$$

The thing we call **pressure** is therefore merely the **power per unit current.** Since the same equations hold in electric power, the **voltage** also must be the **power per unit current** of electricity. Because of the similarity of the equations for water power and for electric power, the electric voltage is called the electric pressure. The similarity between electric voltage and water pressure does not extend much further, for while water pressure can be expressed in terms of mechanical force, electric voltage remains strictly the

power per unit current and is not a force in the usual sense of the word. There is such a quantity as an **electric force** which is explained in Chapter XIII.

24. Use of the Power Equation. The same precautions must be exercised in the use of the power equation ($P = IE$) that are exercised in the use of Ohm's law. The power equation like Ohm's law may be applied to the **whole** of an electric current or to **any part.** But also, as in the use of Ohm's law, when the power of the **whole** circuit is to be computed, the summation of the electromotive forces of the **entire** circuit must be used for E. Similarly when the power of only a **part** of a circuit is to be computed, the value used for E must be the potential difference across **that part** of the circuit **only,** and the value of the current used for I must be the current through the **same part** of the circuit **only.**

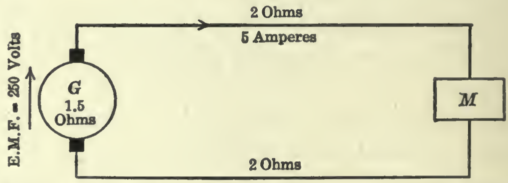

Fig. 31. The generator G supplies power to the line and the motor M.

Example 3. In Fig. 31, the generator generates an electromotive force of 250 volts. The generator has a resistance of 1.5 ohms and the line wires have resistances of 2 ohms each. What power is delivered to the motor M?

Power (to motor)	= Current (through motor) $\times$ voltage (across motor).
Voltage across motor	= electromotive force of generator $- IR$ drop in generator and line wires.
	$= 250 - (5 \times 1.5) - (5 \times 4)$
	$= 222.5$ volts.
Current through motor	= 5 amperes.
Power to motor	$= 222.5 \times 5$
	$= 1112.5$ watts
	$= 1.11$ kilowatts.

It will be noted that the result in the above has been rounded off to three figures. It would hardly be correct to express the result as 1.1125 kilowatts for we cannot measure as closely as this by any ordinary means. The number of figures given in a result should correspond to the accuracy which can be attained under the conditions of the problem. Ordinary voltages, currents and powers can be quickly measured with the usual instruments with the accuracy of about one-half of one per cent. By using care and special instruments they can be measured somewhat more accurately.

25. Power Consumed by Resistance. We have seen that the voltage required to force a current of I amperes through a resistance of R ohms is IR volts.

From this relation we can determine the power consumed in sending a current through a resistance.

$$P = IE$$

but

$$E = IR,$$

Therefore

$$P = I^2R$$

also

$$I = \frac{E}{R}.$$

Therefore

$$P = \frac{E^2}{R}.$$

We thus have three forms for the power equation

$$P = IE = I^2R = \frac{E^2}{R}.$$

The form $P = IE$ is universal, in that P is the power received or delivered when a current of I amperes flows under a pressure of E volts.

The forms $P = I^2R$ and $P = \frac{E^2}{R}$ apply only when E is the pressure required to send a current of I amperes through a resistance of R ohms.

Example 4. How much power is expended in overcoming the resistance of the line wires of Fig. 31?

$$P_R = I^2R = \frac{E^2{}_R}{R} = IE_R$$

Where

P_R = power consumed by resistance R,
E_R = voltage used in sending current through R
= $4 \times 5 = 20$ volts,
I = current through R,
$P_R = I^2R = 5 \times 5 \times 4 = 100$ watts

or

$$= \frac{E^2{}_R}{R} = \frac{20 \times 20}{4} = 100 \text{ watts}$$

or

$$= I\,E_R = 20 \times 5 = 100 \text{ watts.}$$

It may often be necessary to apply both forms of the equation to one part of a circuit in order to determine how the power is consumed or expended. This happens when power in a given part of a circuit is expended in more than one way.

Thus in the generator in Fig. 30, the power generated is expended in two ways, — first, in sending the 5-ampere current through the generator against the 1.5-ohm internal resistance, and second, in delivering current to the outside line.

Example 5. How much power is delivered to the line by the generator in Fig. 31?

Total power generated

$$\begin{aligned} P &= IE \\ &= 5 \times 250 \\ &= 1250 \text{ watts.} \end{aligned}$$

Power consumed by internal resistance

$$\begin{aligned} P_R &= I^2R \\ &= 5 \times 5 \times 1.5 \\ &= 37.5 \text{ watts.} \end{aligned}$$

Power delivered to line

$$\begin{aligned} &= P - P_R = 1250 - 37.5 \\ &= 1212.5 \text{ watts,} \end{aligned}$$

Or the power consumed by internal resistance may be found as follows:

Voltage (E_R) used to force current through current I through internal resistance R

$$= IR$$
$$= 5 \times 1.5$$
$$= 7.5 \text{ volts.}$$

Power consumed in R

$$= IE_R$$
$$= 5 \times 7.5$$
$$= 37.5 \text{ watts.}$$

In all of the above equations for computing the power used in the resistance R, note that the voltage used is E_R, the voltage required to force the current through the resistance R.

In the case of a motor, part of the electric energy delivered to it is used to overcome the resistance of the windings, the rest is transformed into mechanical power, some of which is in turn lost in friction and in other losses to be studied later.

Example 6. If the resistance of the motor in Fig. 31 is 3 ohms, how much mechanical power is developed?

In Example 3 we determined that 1110 watts were delivered to the motor.

Power consumed by resistance of motor

$$P_R = I^2 \times R_r$$
$$= 5 \times 5 \times 3$$
$$= 75 \text{ watts.}$$

Power transformed into mechanical power

$$P - P_R = 1110 - 75$$
$$= 1035 \text{ watts}$$
$$\text{or} \quad \frac{1035}{746} = 1.39 \text{ horse power.}$$

Some of this mechanical power would be used in overcoming the frictional resistance to motion, etc., so that the motor would probably deliver about one horse power which could be used in driving other machinery.

Prob. 1–3. A 110-volt motor having 0.625 ohm resistance receives 4.20 kilowatts at its terminals. What mechanical horse power is developed in the motor?

Prob. 2–3. Fig. 31*a* represents the electrical circuit of an arc lamp. The voltage across the terminals is 50 volts, and across the arc alone it is 40 volts. The arc takes 6.2 amperes. The resistance of the shunt coil *S* is 380 ohms. Find

(*a*) The power consumed by the lamp,
(*b*) The resistance of the ballasting resistance *R*,
(*c*) The power consumed by the resistances *S* and *R*.

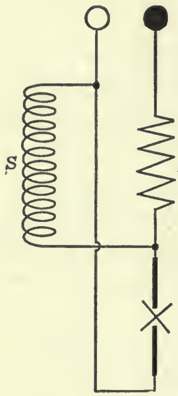

FIG. 31a. The circuits through an arc lamp.

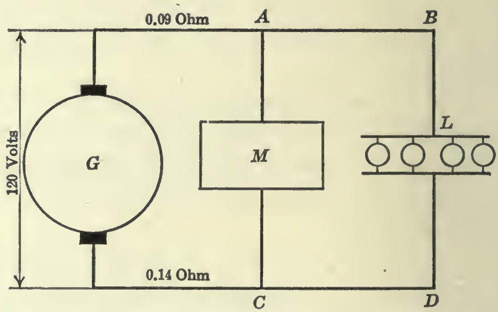

FIG. 32. The generator *G* supplies power to the motor *M* and the lamps *L*.

Prob. 3–3. The motor *M* in Fig. 32 has a resistance of 1.03 ohms and takes 20 amperes. Each lamp at *L* has a resistance of 40 ohms and takes 2.42 amperes.

(*a*) What power is taken by the lamp bank *L*?
(*b*) What power is taken by the motor?
(*c*) How much electrical power is converted into mechanical power in the motor?
(*d*) How much power is lost in the line wires?

26. Measurement of Electric Power. Since in direct-current measurements the power is the product of the volts and amperes, the combination of a voltmeter and an ammeter may be used to measure the power in any part of

a circuit. However, these two instruments have been combined in one instrument called a wattmeter.

The current is led through the field coils *FF* (Fig. 33) making a magnetic field proportional to the strength of the current. The voltage is impressed on the coil *P* which has a high resistance *R* in series with it. The turning tendency of the coil *P* is, therefore, proportional to the product of the current by the voltage, that is, to the power in a direct-current circuit.

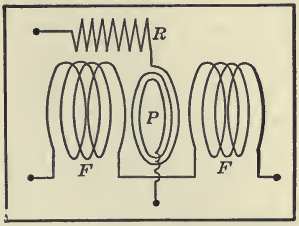

Fig. 33. The wiring diagram of a wattmeter.

An instrument of this type usually has four terminals, two for the current leads and two for the voltage leads. The greatest care must always be exercised not to confuse these two sets of terminals, for inasmuch as the current coils are of very low resistance, the instrument is ruined if the mistake is made of connecting the voltage leads to these coils.

Fig. 34 shows the proper method of connecting a wattmeter to indicate the power taken by the bank of lamps (*B*).

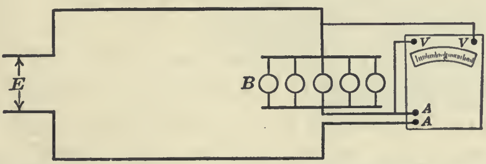

Fig. 34. The wattmeter measures the power taken by the lamp bank *B*.

The reason that a separate instrument is used for a wattmeter, instead of simply a voltmeter and an ammeter, is as follows. In a circuit in which the current is steady, that is, in a direct-current circuit, the power is equal to the product of *E* and *I*. When the current and voltage in the circuit are varying this is not true unless they are varying in ex-

actly similar manner. The relation is, however, true for the **instantaneous** voltage, current and power in any circuit. When used in a circuit in which the current and voltage are varying, the wattmeter gives a deflection corresponding to the average of these instantaneous values, and hence shows the actual average power in the part of the circuit to which it is connected. This cannot be done except where the wattmeter arrangement of coils is used. Hence while a wattmeter is not necessary in order to measure direct-current power, it is the only convenient way in which alternating-current power can be measured.

27. Electric Energy. Since power is the rate of doing work, the total amount of work done or energy expended in a given time is the product of the power and the time. Thus a 25-horse-power steam engine running at full load for 4 hours does 25×4 or 100 horse-power-hours of work. Similarly a 25-kilowatt generator running at full load for 4 hours delivers 25×4 or 100 kilowatt-hours of electric energy. A kilowatt-hour is the energy delivered or received by an appliance in one hour if the power is maintained at one kilowatt.

Similar units of electric energy are the watt-hour and the watt-second or joule. The values of these are evident from the names.

1 kilowatt-hour = 1.34 horse-power-hours.

1 watt-second (joule) = 0.737 foot-pound.

Prob. 4–3. If it costs a total of $0.0126 per kilowatt-hour to generate electric energy, at what price must it be sold per horse-power-year in order to realize a 10 % profit? Assume the power is to be used 24 hours per day and 365 days per year in a chemical plant.

Prob. 5–3. A generator supplies for 8 hours per day 40 kilowatts at 110 volts to a motor load situated 500 feet from the generator. The resistance of the line wires is 0.049 ohm per 1000 feet. It costs $0.056 per kilowatt-hour to generate electric energy. How much money would be saved in a year of 300 days by doubling the voltage of the motors and generator and supplying the motors with the same power?

Prob. 6–3. How many 25-watt lamps can be used (on the average) for $2\frac{1}{2}$ hours per night if the monthly bill is not to exceed $3.20? The price of electric energy is 10 cents per kilowatt-hour; a month is to be reckoned as having 30 days.

28. Heat Energy of Electricity. We have seen that when a current I is made to flow through a resistance R, energy is expended at the rate of I^2R watts. This energy is all transformed into heat just as all mechanical energy which a machine uses in overcoming friction is transformed into heat. This fact is strikingly called to mind in Fig. 35, which is a photograph of a large transformer. Note that a special arrangement of pipes is necessary so that the oil in the case can circulate and be cooled by the air passing over the pipes. Much of the heat thus dissipated originates in the coils and is due to the passage of the electric current through the resistance of the coils.

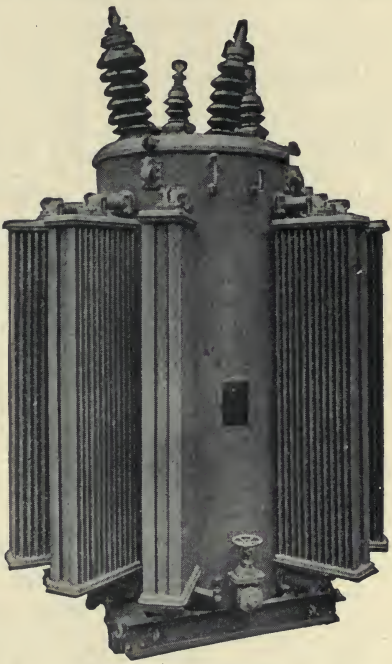

Fig. 35. A typical self-cooled radiator-type transformer of over 2000 kw. capacity. *Westinghouse Elec. & Mfg. Co.*

Thus the energy, represented by the expression (I^2Rt), always appears as heat and can be expressed equally well in heat units as calories or in electric units as watt-seconds. It is necessary to know merely the relation of a calorie to a watt-second. Careful measurements have determined that one watt-second equals 0.24 calorie.

Thus

$$H = 0.24\, I^2Rt,$$

where

$$\begin{aligned} H &= \text{heat generated in calories,} \\ I &= \text{current in amperes,} \\ R &= \text{resistance in ohms,} \\ t &= \text{time in seconds.} \end{aligned}$$

Of course the expression IE_Rt or $\frac{E^2{}_Rt}{R}$ can be used in the place of I^2Rt, providing E_R represents just the voltage required to force the current I through the resistance R.

Example 7. An electric water heater designed to operate on 110-volt lines has a resistance of 15 ohms. How long will it take to raise the temperature of a cup of water containing 250 grams from 10° C. to 90° C., assuming no loss of heat from the cup, and neglecting the specific heat of the cup itself?

Heat required to raise 250 grams of water 80° C.

$$\begin{aligned} H &= 80 \times 250 \\ &= 20{,}000 \text{ calories;} \end{aligned}$$

also

$$H = 0.24\, \frac{E_R^2 t}{R}.$$

Therefore

$$20{,}000 = \frac{110 \times 110 \times 0.24t}{15}$$

$$\begin{aligned} t &= \frac{20{,}000 \times 15}{110 \times 110 \times 0.24} \\ &= 103.3 \text{ seconds} \\ &= 1.72 \text{ minutes.} \end{aligned}$$

29. Efficiency of Transmission. Regulation. Since some energy is always transformed into heat whenever an electric current flows, no scheme of transmission can have an efficiency of 100 %. In other words, the I^2R loss is always present in the conductors of the transmission line whenever any electric energy is being transmitted.

The efficiency of transmission may be defined as the

ratio of the useful output to total input of the line. In the low voltage systems (under 5000 volts) generally used for short-distance transmission of direct currents, the I^2R loss is practically the only loss in the line. Computing the efficiency of transmission of such a line is therefore easily done.

Example 8. In the system shown in Fig. 36, the appliances at B draw 16 amperes and those at A, 20 amperes. The re-

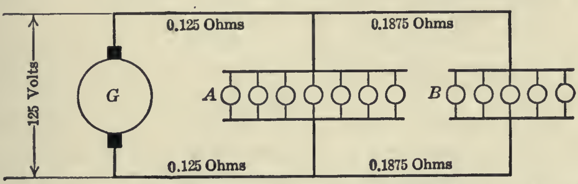

FIG. 36. A transmission system supplying power to two groups of lamps from one generator.

sistance of line and return between the generator G and group A is 0.25 ohm; between A and B 0.375 ohm. What is the efficiency of transmission?

$$
\begin{aligned}
\text{Total input into line} &= 125 \times (20 + 16) \\
&= 4500 \text{ watts.} \\
I^2R \text{ loss between } G \text{ and } A &= 36^2 \times 0.25 \\
&= 324 \text{ watts.} \\
I^2R \text{ loss between } A \text{ and } B &= 16^2 \times 0.375 \\
&= 96 \text{ watts.} \\
\text{Total line loss} &= 96 + 324 \\
&= 420 \text{ watts.} \\
\text{Useful output} &= 4500 - 420 \\
&= 4080 \text{ watts.}
\end{aligned}
$$

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{4080}{4500} = 90.7\%.$$

As a check we may compute the output more directly as follows:

$$
\begin{aligned}
\text{Line drop } G \text{ to } A &= IR \\
&= 36 \times 0.25 \\
&= 9 \text{ volts.} \\
\text{Voltage across } A &= 125 - 9 \\
&= 116 \text{ volts.}
\end{aligned}
$$

$$
\begin{aligned}
\text{Power used by } A &= 116 \times 20 \\
&= 2320 \text{ watts.} \\
\text{Line drop } A \text{ to } B &= IR \\
&= 16 \times 0.375 \\
&= 6 \text{ volts.} \\
\text{Voltage across } B &= 116 - 6 \\
&= 110 \text{ volts.} \\
\text{Power used at } B &= 110 \times 16 \\
&= 1760 \text{ watts.} \\
\text{Total power used by } A \text{ and } B &= 2320 + 1760 \\
&= 4080 \text{ watts.} \\
& \text{(check)}
\end{aligned}
$$

The **regulation of a transmission** line is a measure of the drop in voltage when the line is loaded. It is defined as the rise in terminal voltage when the load is changed from full load to no load, divided by the full-load voltage. The regulation of a generator or an alternating-current transformer is defined in exactly similar manner. The regulation of a transmission line should be low in order not to subject the load to undesirable fluctuations of voltage.

Example 8 (continued). Assuming the load above to be full load for the transmission line, we may compute the regulation as follows:

Voltage at B, full load = 110 volts.
Voltage at B, no load
(same as generator voltage) = 125 volts.
Change of voltage, no load to full load, = 125 − 110
= 15 volts.

$$\text{Regulation} = \frac{15}{110} = 0.136 = 13.6\%.$$

In making this computation we neglect any possible change of generator voltage with load, as we wish to compute the regulation of the transmission line only.

Prob. 7–3. An electric circuit has the form of a loop, *i.e.*, each conductor is connected to the power station busbar of like sign at both ends. The loop is 1.25 miles long and each conductor has a resistance of 0.26 ohm per mile. The two busbars at the power station have a potential difference of 575 volts. The loop circuit supplies two shop buildings, one at a

distance of 1800 feet from one end of the loop taking 400 amperes and the other at a distance of 2600 feet from the other end of the loop taking 560 amperes. What is the voltage between lines at each shop?

(*a*) What is the efficiency of transmission?

(*b*) Determine the voltage at each shop and the efficiency of transmission if that portion of the loop between the two shops were omitted.

Prob. 8–3. The field coils of a certain generator contain 20 pounds of copper (specific heat 0.095). The weight of the insulation is negligible. The resistance of the field coils is 250 ohms. How fast would the temperature rise if there were no cooling by convection or radiation and a current of 2.4 amperes were flowing in the coils?

Prob. 9–3. In Fig. 37, station I takes 50.0 amperes and station II takes 65.0 amperes. The resistance of *AB* and of *BC* each equals 0.0412 ohm. The resistance of *CD* and of

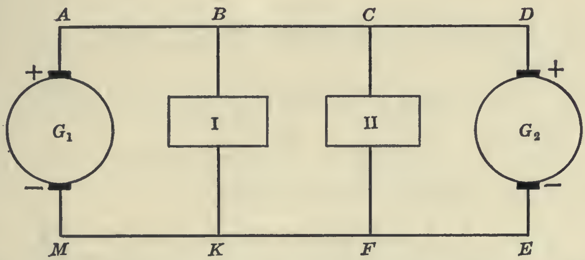

FIG. 37. A transmission system where two generators are used to supply power to two stations.

MK each equals 0.0330 ohm. The resistance of *EF* and of *KF* each equals 0.0214 ohm. The terminal voltage of G_1 is 121 volts and of G_2 is 125 volts.

(*a*) What is the voltage regulation at station I?

(*b*) What is the voltage regulation at station II?

(*c*) What is the efficiency of transmission?

Prob. 10–3. If generator G_2 were removed from the line in Fig. 37 and all other data remained as in Prob. 9–2, compute

(*a*) The voltage regulation of station I,

(*b*) The voltage regulation of station II,

(*c*) The efficiency of transmission.

30. The Three-wire System of Transmission. In order to transmit a given amount of power over a line with as small I^2R loss as practicable, it is necessary to make the current as small as possible. This can be done by increasing the voltage of transmission. Thus 10,000 watts may be transmitted in the form of 100 amperes at 100 volts pressure or of 50 amperes at 200 volts pressure, or in any other form the factors of which multiplied together equal 10,000 watts. But if transmitted as 50 amperes at 200 volts, the line loss would be only $\frac{50^2}{100^2}$ or $\frac{1}{4}$ as much as if it were transmitted as 100 amperes at 100 volts, the same line being used in both cases. Thus doubling the voltage has decreased the line loss to one-quarter as much.

For a given amount of power transmitted over a given line:

The line loss is proportional to the square of the line current.

The line current is inversely proportional to the line voltage.

Therefore the line loss is inversely proportional to the square of the voltage of transmission.

Prob. 11–3. At a certain place there are available 30 kw. of electric power. The line wires from this place to the load have a resistance of 1.5 ohms each. Plot a curve between efficiency of transmission and voltage at sending end for values of voltage varying from 100 volts to 10,000 volts.

In direct-current systems, only a limited advantage can be taken of the higher efficiency of high-voltage transmission. While motors can be constructed for operation at fairly high voltages, nearly all the other ordinary electrical appliances are operated most easily at about 110 volts.

This is particularly true of electric incandescent lamps. A lamp constructed to operate at 220 volts or 550 volts, for instance, would be expensive and of short life.

With alternating currents we are not subject to limitation

in this regard. Alternating-current power can be transformed from one voltage to another at will, by reliable stationary apparatus, and with little loss. The alternating-current transformer hence makes possible the transmission of large amounts of power over distances of even 200 miles and at voltages as high as 220,000 volts. At the point where it is utilized it may be transformed down to a safe convenient voltage around 110 volts. In fact, it is usually transformed several times between generator and user. The distance to which power can be transmitted economically depends upon the amount of power to be carried and the voltage used. Direct-current transmission is hence limited to comparatively short distances, although many miles of track are covered by a single station in direct-current railway practice where voltages of 3600 and even higher are used.

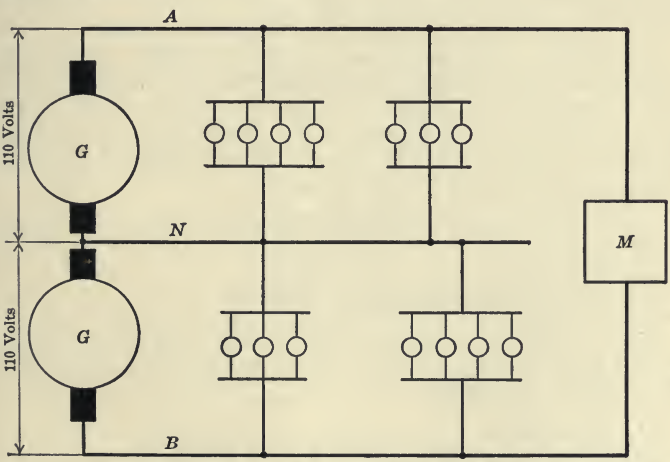

Fig. 38. Two generators in series used to supply a three-wire system to which are attached 110-volt lamps and a 220-volt motor.

Since there is no direct-current transformer, direct-current power must be transmitted at practically the same voltage at which it is used. Much advantage, however, is

gained by using three wires instead of two. Thus in Fig. 38, two 110-volt generators are shown connected in series. Therefore, across the outside conductors *A* and *B* there are impressed 220 volts, while only 110 volts exist between either of these conductors and the neutral wire *N*. Accordingly the lamps have practically 110 volts across them, while the motor has about 220 volts. This system therefore combines the advantage of affording both 110 volts and 220 volts, and yet having nearly the efficiency of a 220-volt line.

The greatest efficiency is secured when the loads on both sides of the neutral are "balanced" so that no current flows in the neutral. This is the condition which should be approximated.

Instead of connecting two generators in series, it is customary to use a special three-wire generator, or a large generator and a balancer set. These machines are fully described in texts on electrical machinery.

Prob. 12–3. Assuming that the cross-section of a conductor must be directly proportional to the current it is to carry, compare the weight of copper required by a three-wire system with the weight required by a two-wire system transmitting the same amount of power. All three wires in the three-wire system are to be the same size.

Prob. 13–3. Each lamp in Fig. 39 takes 1.2 amperes. Find:
(1) Line drop,
(2) Voltage across each set of lamps,
(3) Efficiency of transmission.

Prob. 14–3. If the lamp bank marked *S*, Fig. 39, is turned off, and the resistance of the other lamps remains the same as in Prob. 13–3, find:

(1) Line drop,
(2) Voltage across remaining sets of lamps,
(3) Efficiency of transmission.

Prob. 15–3. Assume that each lamp in Fig. 40 takes 2 amperes and that the resistance of the lamps remains constant. The brush potential of the generator is 220 volts. Each small machine maintains a terminal voltage of 110 volts. See Prob. 20–3. Find:

(a) Line drop in each section,
(b) Voltage across each set of lamps,
(c) Efficiency of transmission.

Prob. 16–3. If a break occurs in the neutral between *O* and *S*, Fig. 40, what would be the values of (a), (b) and (c), Prob. 15–3? Lamp resistance remains as in Prob. 15–3.

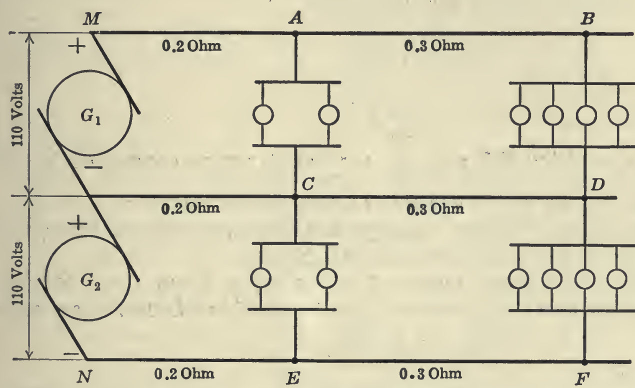

FIG. 39. Two generators feeding a balanced three-wire system.

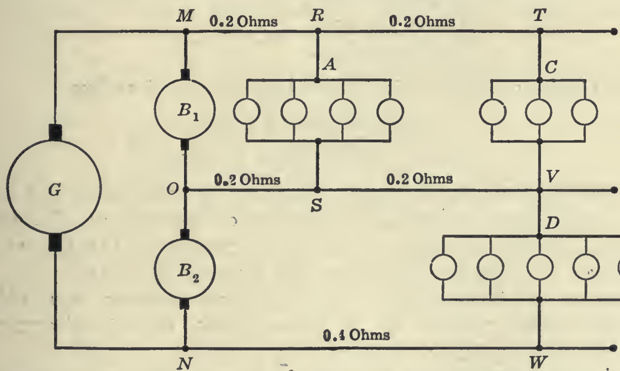

FIG. 40. A generator with a balancer set supplying power to a three-wire system.

Prob. 17–3. If a break occurs in the neutral between *S* and *V*, Fig. 40, what would be the values of (a), (b) and (c), Prob. 15–3?

SUMMARY OF CHAPTER III

Direct-current electric power is measured in watts and can be computed from the equations

$$\begin{aligned} P &= IE \\ &= I^2R \\ &= \frac{E^2}{R}. \end{aligned}$$

The quantities I^2R and $\frac{E^2}{R}$ represent power consumed by the resistance R, which appears as heat in the resistor.

The units of electric energy are the watt-second, commonly called the joule, and the kilowatt-hour.

The watt-second equals 0.24 calory. Thus the heat liberated by an electric current can be obtained from the equation

$$H = 0.24\, I^2Rt,$$

where

H is in calories,
I is in amperes,
R is in ohms,
t is in seconds.

The efficiency of electric transmission is the fraction

$$\frac{\text{power delivered to load}}{\text{power received from generator}}.$$

In the transmission of a given amount of power, the line loss is inversely proportional to the square of the voltage of transmission. Thus the higher the voltage, the higher the efficiency as far as it depends upon the I^2R loss of the line.

Because of the higher voltage of transmission available, the three-wire system is in general use for direct-current distribution.

The voltage regulation of a line is

$$\frac{\text{no-load voltage} - \text{full-load voltage}}{\text{full-load voltage}}.$$

These voltages are measured at the point where the load is attached to the line.

PROBLEMS ON CHAPTER III

Prob. 18–3. A shunt dynamo acts as a generator and supplies 4710 watts of power to a lighting system connected across its terminals. In addition the armature supplies the exciting current to the field coils which have a resistance of 136.4 ohms. The armature resistance is 0.12 ohm and the terminal voltage 124. (*a*) What is the armature current? (*b*) What e.m.f. is generated in the armature? (*c*) What power is lost as heat in the windings?

Prob. 19–3. A shunt generator supplies power to charge a storage battery. The armature of the generator has a resistance of 0.32 ohm. The shunt field winding of the generator has a resistance of 140 ohms. Each of the two conductors leading to the battery has a resistance of 0.2 ohm. The battery exerts by a chemical action an e.m.f. of 112 volts and has an internal resistance of 0.08 ohm. What e.m.f. must be set up in the armature of the generator to charge the battery at the rate of 24 amperes? If the rotational losses of the generator at this state are 160 watts, what mechanical power must be employed to drive the generator? What percentage of this mechanical energy is stored chemically in the battery?

Prob. 20–3. Fig. 41 represents the method of supplying a three-wire system from a two-wire source by the aid of a balancer set. At the original two-wire source a constant potential difference of 236 volts is maintained. The balancer set comprises twin dynamo machines whose armatures A and B are mechanically coupled and electrically in series between the outside wires. The middle wire is led out from the intermediate point. Each armature of the balancer set has a resistance of 0.028 ohm. The line resistances are as indicated. When the three-wire system is unloaded, the balancer set runs idle as two motors in series. When one side of the three-wire system is loaded more heavily than the other, the machine on the heavily loaded side operates as a generator and is driven by the other machine running as a motor. The field windings of the balancer set, not shown in the sketch, are to be so regulated that equal voltages are to exist between R and S and between S and T. The load on one side is 80 amperes and the load

on the other 10 amperes. The rotational losses of the balancer set amount to 472 watts and are supplied from the primary two-wire source.

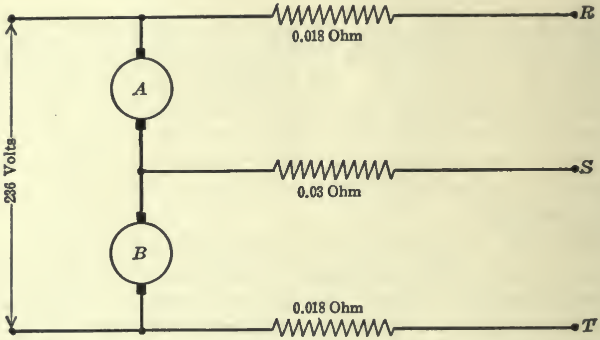

FIG. 41. A diagrammatic representation of a generator with a balancer set supplying power to a three-wire system.

(*a*) What e.m.f.'s must be induced in the two armatures under the circumstances above described?

(*b*) What are the voltages between *R* and *S*, and *S* and *T*?

(*c*) What are the voltages between *R* and *S*, and *S* and *T* when the load on each side is 80 amperes?

Prob. 21–3. If in Prob. 20–3 the load is removed between the lines *R* and *S*, what will be the voltages generated in the two armatures and what will be the voltage across line *ST* at a load of 80 amperes?

Prob. 22–3. A garage storage-battery set has been proposed having the following points: 1st, a d-c. generator having a 70-volt and a 105-volt adjustment, and an internal resistance under these two conditions of 0.15 ohm; 2nd, a smaller machine having two armature coils capable of generating 5 volts each. The internal resistance of each 5-volt coil is 0.04 ohm. These may be arranged so that they separately buck or boost the main generator voltage. In this way combinations can be obtained to give voltage from 60 to 115 volts in 5-volt steps with the exception of 85 volts and 90 volts. The rotational losses in the two machines total 400 watts. It is desired to charge 7 batteries of average back e.m.f. of 11.2 volts at a 40-ampere rate for 8 hours. Determine the arrangement of the

coils of this system so that the charging rate nearest to 40 amperes can be obtained. Under this condition find the saving over a system using a plain 115-volt generator with resistance in series with the batteries to control the current. Rotational loss, 300 watts. The cost of electric energy may be taken as three cents per kilowatt-hour. The average battery internal resistances is 0.03 ohm. (Electrical Review, Vol. 60.)

Prob. 23–3. In Prob. 22–3 what would be the overall charging efficiency for both systems and what would be the losses?

Prob. 24–3. A certain railroad electrification is five miles long. Power is sent out over a 1,000,000-circular-mil feeder whose resistance is 0.057 ohm per mile. The feeder is tied at both ends and every half mile to a # 0000 trolley wire whose resistance is 0.26 ohm per mile. The resistance of the return rail circuit is 0.02 ohm per mile. The generator, which is separately excited, has an armature resistance of 0.05 ohm and an induced electromotive force of 600 volts. At the distant end of the line is a storage battery whose resistance is 0.3 ohm. A train taking 700 amperes is three miles from the generator end of the line. If the generator is supplying 600 amperes, find:

(*a*) Voltage at generator end of the line,
(*b*) Voltage at car,
(*c*) Voltage at battery end of line,
(*d*) Electromotive force of battery,
(*e*) Number of coulombs delivered by the battery in ten seconds,
(*f*) Kilowatts input to car.

Prob. 25–3. If the controller on the train (Prob. 24–3) should be shut off so that no current was taken by the train, how much current (if any) would flow into the battery?

Prob. 26–3. Two railroad substations are 6 miles apart. The trolley wire is reinforced by a heavy copper feeder and the two in parallel are equivalent to a conductor whose resistance is 0.018 ohm per 1000 feet. The two rails serve in parallel as a return conductor and have a combined resistance of 0.006 ohm per 1000 feet. At a distance of 2.5 miles from one end of the line a train is running at a speed of 30 miles per hour against an opposing force of 7200 pounds. The electrical equipment of this train has an efficiency of 86 %. The voltage between busbars at the substations is 700. What is the voltage between trolley and track at the train? What current does the train receive from each substation?

Prob. 27–3. If a train moves from one end of the railroad referred to in Prob. 26–3 to the other end and requires at all times a current of 800 amperes, by what equation may the voltage at the train be expressed in terms of its distance, X, from the end of the line? Draw a curve to indicate approximately the general nature of the relation between the position of the train and the voltage acting on it.

Prob. 28–3. A railroad line is supplied with power from both ends. At one end a power station maintains a constant voltage between positive and negative busbars of 600. At the other end is a storage battery which sets up an e.m.f. of 540 volts by chemical action. The internal resistance of this battery is 0.08 ohm. The positive conductor is a trolley wire in parallel with a feeder, the two having a combined resistance of 0.098 ohm per mile. The negative conductor consists of the two track rails in parallel and has a resistance of 0.032 ohm per mile. The total length of the line is 7.2 miles. If only one train is running and is located 4 miles from the power station, what voltage is applied to it when it takes 200 kilowatts?

Prob. 29–3. A generator supplies power to a trolley load. Three miles from the generating station is a storage battery connected between trolley and rail. Two miles beyond the storage battery is a car taking 50 kilowatts. The trolley wire is # 0000 having a resistance of 0.26 ohm per mile. The resistance of the rail return (including both rails) is 0.04 ohm per mile. The e.m.f. and internal resistance of the storage battery are respectively 580 volts and 0.5 ohm. If the battery is discharging at the rate of 40 amperes, calculate the current taken by the car, the voltage at the car, the current delivered by the generator and the voltage at the generating station.

Prob. 30–3. How far from the generating station in Prob. 29–3 would the car be if the storage battery were just floating on the line (neither charging nor discharging)? The car still takes 50 kilowatts.

Prob. 31–3. How many kilowatts must be supplied to an electric steel furnace which is to deliver 1 ton of steel per hour? Consider 10 % of the heat to be lost in radiation.

Average specific heat of steel = 0.167

Average temperature of fusing point = 2400° F.

Average latent heat = 50 B.t.u. per lb.

Prob. 32–3. At 5 cents per kilowatt-hour, what would be the expense per day of 10 hours for heating a room containing 15 persons? Allow 30 cubic feet of air per minute per person. Air enters at 40° F. and is heated electrically to 70° F. Specific heat of air (constant pressure) is 0.237. Average weight of air is 0.08 pound per cubic foot. Allow 15 % loss in radiation, etc.

Prob. 33–3. A water rheostat is used as a load during a test on a 25-kilowatt generator. The water flows into the tank of the rheostat at a temperature of 54° F. and flows out at 170° F. How much water per hour is used if the generator is kept at full load? Neglect radiation and evaporation.

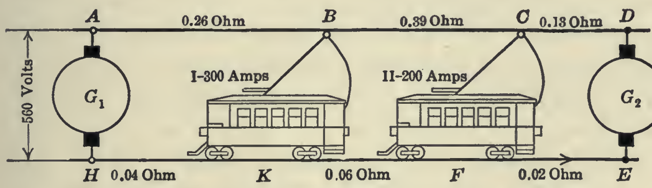

FIG. 41a. Trolley cars fed from both ends of the line.

Prob. 34–3 Generator G_2 in Fig. 41a maintains a terminal voltage of 555 volts. What is the efficiency of transmission when car No. I takes 300 amperes and car No. II, 200 amperes? The resistances of the different sections of the wire and track are as marked.

CHAPTER IV

THE COMPUTATION OF RESISTANCE

The resistance of a conductor is directly proportional to its length and inversely proportional to its cross-sectional area. If therefore the resistance of a conductor of given material of unit length and unit cross-section is known, it is possible to find the resistance of a conductor of the same material whatever its length or cross-section.

31. Resistivity. Expressed as an equation the above relation is

$$R = \rho \frac{l}{A}, \tag{1}$$

where

$R =$ the resistance of the conductor in ohms,
$l =$ the length of the conductor,
$A =$ the cross-section area of the conductor,
$\rho =$ the resistance of a conductor of unit length and unit cross-section, called the **resistivity** of the material.

The unit length chosen for international use is one centimeter and the unit area of a cross-section is one square centimeter. The resistance of such a centimeter cube is called the **resistivity** in ohm-centimeters. Of ordinary annealed-copper wire, at 20° C., determined from careful tests upon samples of the present-day grades in common use, the resistivity is 0.00000172 ohm-centimeter, usually stated as 1.72 microhm-centimeters.

Example 1. What is the resistance of an annealed-copper strap 45 centimeters long with a cross-section of 3 × 0.5 centimeters?

$$R = \rho \frac{l}{A}$$

$$= \frac{1.72 \times 45}{0.5 \times 3} = 51.6 \text{ microhms.}$$

Prob. 1–4. Prove by Ohm's and Kirchhoff's Laws that the resistance of a conductor is directly proportional to the length and inversely proportional to the cross-section area.

32. Resistance per Mil-Foot. Since, in America, the length of a conductor is usually measured in feet, it is customary to express l in equation (1) in feet. Similarly, since conductors are usually made circular in cross-section, it is customary to use circular rather than square measure in denoting the area of the cross-section. The unit of area chosen is the circular mil, which is the area of a circle one mil (one thousandth inch) in diameter. The area of a circle varies as the square of the diameter. Since the area of a circle one mil in diameter is one circular mil, the area of any circle in circular mils equals the square of its diameter in mils. A wire having a length of one foot and a cross-section area of one circular mil is called a **mil-foot wire.**

It is the resistance of this mil-foot wire expressed in ohms which is used for the value of ρ in the above equation when the length of the conductor is expressed in feet and the cross-section area in circular mils.

For this reason the equation is often written

$$R = \frac{\rho l}{d^2}, \tag{2}$$

where

$\rho =$ the resistance per mil-foot,
$l =$ the length in feet,
$d =$ the diameter in mils.

The resistance per mil-foot of annealed copper is 10.4 ohms at 20° C.

Example 2. What is the resistance per mile at 20° C. of an annealed-copper conductor $\frac{1}{8}$ inch in diameter?

$$\tfrac{1}{8} \text{ in.} = 0.125 \text{ in.} = 125 \text{ mils}$$

$$R = \frac{\rho l}{d^2}$$

$$= \frac{10.4 \times 5280}{125 \times 125}$$

$$= 3.51 \text{ ohms}$$

33. Resistivity of Metals used as Electrical Conductors. The three metals most used as electrical conductors are copper, aluminum and steel.

Soft-drawn copper has a resistivity of 1.724 microhm-centimeters at 20° C. and is the metal in greatest use as a conductor.

Hard-drawn copper is nearly 50% greater in tensile strength and has a resistivity of 1.772 microhm-centimeters, less than 3% greater than annealed copper, and is therefore extensively used for transmission lines.

Aluminum has a resistivity of 2.828 microhm-centimeters at 20° C., about 1.6 that of copper, but this disadvantage is often more than offset by its small weight per cubic centimeter, a conductor of aluminum having less resistance than a conductor of copper of the same length and weight.

The resistivity of steel depends largely upon its composition and treatment, but it is always much greater than that of copper or aluminum. It is approximately 21.6 microhm-centimeters for steel wire and varying from 13.8 to 21.6 for steel rails.

The resistivity of alloys is almost always higher than that of any one of the constituent metals. Some alloys of copper, nickel, zinc, manganese and chromium used as resistors have very high resistivities often approaching 100 microhm-centimeters.*

* See Appendix for Resistivity Tables.

Prob. 2–4. What line drop is there in a 2-mile hard-drawn copper trolley wire carrying 200 amperes if the wire is 0.625 inch in diameter?

Prob. 3–4. Each lamp in Fig. 42 takes 2 amperes at 112 volts. The lamps are 500 feet from the generator. The line wire is annealed copper $\frac{1}{8}$ inch in diameter. What is the voltage of the generator?

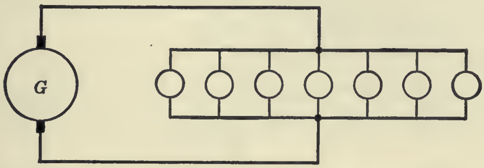

FIG. 42. A bank of lamps fed by the generator *G*.

Prob. 4–4. What will be the drop per mile in a line consisting of annealed-copper wire $\frac{1}{12}$ inch in diameter carrying 15 amperes?

Prob. 5–4. What will be the line drop in voltage and the loss in watts per mile in transmitting 12 kilowatts at 550 volts, if an annealed-copper wire is used having a diameter of 0.364 inch?

Prob. 6–4. A group of incandescent lamps takes 12 amperes. The line drop is not to exceed 2.6 volts. What must be the size of annealed-copper wire to be used if the lamps are 2500 feet from the generator?

34. Conductivity of Materials. In specifying the grade of material to be used for a conductor, it is customary to specify the conductivity instead of the resistivity. Just as conductance is the reciprocal of resistance, so conductivity is the reciprocal of resistivity.

$$\gamma = \frac{1}{\rho}, \tag{3}$$

in which

ρ = the resistivity in any system of units,
γ = the conductivity in the same system of units.

Thus the conductivity of soft-drawn copper at 20° C. is

$$\gamma = \frac{1}{0.000001724} = 580{,}000 \text{ mho-centimeters.}$$

The conductance of a wire may be computed from

$$G = \gamma \frac{A}{l}, \tag{4}$$

where G is the conductance in mhos and the other quantities have the same meanings as before.

Another term in common use is "percentage conductivity." By this is meant the percentage which the conductivity of a certain material is of the conductivity of Standard Annealed Copper.

Standard Annealed Copper at 20° C. has a resistivity of 1.7241 microhm-centimeters and a density of 8.89 grams per cubic centimeter. The conductivity of this standard is called 100%. The percentage conductivity of any material is rated as a certain percentage of this standard. Thus a rating of 95% conductivity for a material means that the material has a conductivity which is 95% of that of standard copper, or $0.95 \times 580{,}000$ mho-centimeters conductivity. The resistivity of such a material is 1.724/0.95 microhm-centimeters.

Ordinary copper runs between 98% and 100% conductivity, but copper may be obtained which is purer than the Standard and has more than 100% conductivity.

Aluminum averages 61% conductivity.

Prob. 7–4. What is the resistance per mil-foot of a lot of copper having 96% conductivity?

Prob. 8–4. What percent conductivity has a solid round aluminum wire 0.365 inch in diameter, one mile of which has a resistance of 0.672 ohm at 20° C.?

Prob. 9–4. What diameter must a copper wire of 96% conductivity have if a mile of it is to have the same resistance as the aluminum wire in Prob. 8–4?

35. Temperature Coefficient of Resistance. It will be noticed that when the resistance of a mil-foot of copper wire

was given as 10.4 ohms and that of aluminum as 17.1 ohms, the metal was assumed to be at a temperature of 20° C. The reason for stating the temperature is that experience shows that the resistance of any pure metal changes with the temperature. For each degree that the temperature of a copper wire rises above 20° C. (up to about 200° C.), the resistance **increases** 0.393 of 1 % of what it was at 20° C. Similarly for each degree that the temperature of a copper wire falls below 20° C. (down to about − 50° C.), the resistance **decreases** 0.393 of 1 % of what it was at 20° C. This percentage change in resistance is called the **Temperature Coefficient of Resistance.** For all pure metals this coefficient has nearly the same value.* Since the resistance per mil-foot of copper has been given as 10.4 ohms at 20° C., all computations of resistance of wires based on this value will give the resistances at 20° C. In order to find the resistance of a wire at any other temperature, it is necessary to find the increase or decrease in resistance and add it to or subtract it from the resistance at 20° C.

Example 3. The resistance of a coil of copper wire at 20° C. is 48 ohms. What will be the resistance of the coil at 50° C.?

The temperature rise = 50° − 20° = 30°.

For every degree rise, the resistance increases 0.393 %.

For 30° rise, the resistance of the coil increases 30 × 0.393 = 11.79 %.

The increase in resistance = 11.79 % of 48 ohms,
= 5.66 ohms.

The resistance at 50° = 48 + 5.66
= 53.66 ohms.

The process used in the above example may be expressed by an equation

$$R_2 = R_1 \,[1 + \alpha_1 (t_2 - t_1)], \qquad (5)$$

in which R_1 = the resistance at the temperature t_1,

R_2 = the resistance at temperature t_2,

α_1 = the temperature coefficient of resistance for the material at temperature t_1.

* See Appendix Table I

We have seen that α_1 for copper at 20° C. is 0.00393. For any other temperature, (t_2), the value (α_2) of the temperature coefficient may be found from the equation

$$\alpha_2 = \frac{\alpha_1}{1 + \alpha_1(T_2 - T_1)}. \tag{6}$$

Example 4. What is the temperature coefficient of resistance of annealed copper at 30° C.?

$$\begin{aligned} \alpha_2 &= \frac{\alpha_1}{1 + \alpha_1\,(T_2 - T_1)} \\ &= \frac{0.00393}{1 + 0.00393\,(30 - 20)} \\ &= 0.00378 \end{aligned}$$

Prob. 10–4. Prove that the above equation for α_2 is correct and explain how the coefficient changes with a change of the temperature used as a base, although the resistance change per degree is constant.

Prob. 11–4. The resistance of a copper wire is 4.90 ohms at 20° C. What is it at 80° C.?

Prob. 12–4. The resistance of the field coils of a generator is 220 ohms at 20° C. When the coils become heated to 75° C. what will the resistance be?

Prob. 13–4. What will the resistance of a coil of copper wire become at 7° C., if the resistance is 200 ohms at 20° C.?

Prob. 14–4. What will the resistance of a coil of copper wire become at 7° C., if the resistance is 200 ohms at 0° C.?

Prob. 15–4. The resistance of a field coil is 130 ohms at 12° C. What will it be at 180° C.?

Prob. 16–4. What will be the resistance of a copper wire at 10° C., if the resistance at 45° C. is 2.08 ohms?

36. Temperature Change Measured by Change in Resistance. Electrical machines are generally sold under a guarantee that the wire in the coils will not rise more than a given number of degrees when running under a specified load for a specified time.

By measuring the resistance of the coils when at room temperature (20° C. or 68° F.) and then again at the close of the run and applying the equation for temperature effect, the average temperature rise can easily be found.

Example 5. The primary coils of a transformer have a resistance of 5.48 ohms at 20° C. After a run of 2 hours, the resistance has risen to 6.32 ohms. What is the temperature rise of the coil?

The resistance increase $= 6.32 - 5.48 = 0.84$ ohm.

The percentage increase $= \dfrac{0.84}{5.48} = 15.3\%$.

The percentage increase for 1° rise above 20° C. $= 0.393\%$.
To produce 15.3% the temperature rise must be

$$\frac{15.3}{0.393} = 38.9°.$$

The average temperature rise in the coil $= 38.9°$.

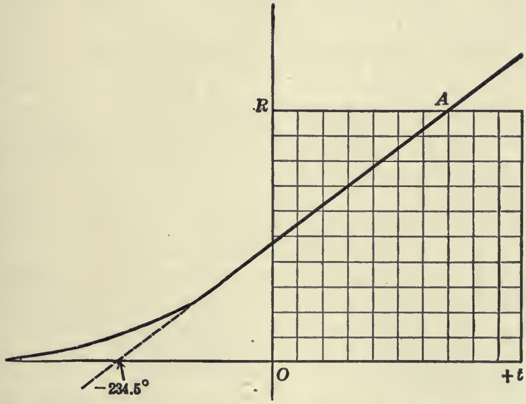

FIG. 43. A curve showing the relation between resistance and temperature of copper.

Instead of making the computations of the above example, it is generally more convenient to use the equations of the

graph showing the relation between temperature and resistance. Fig. 43 shows this graph for Standard Annealed Copper. For other pure metals the slopes would vary somewhat and the intercepts on the horizontal axis would differ slightly from $-234.5°$. The equation of this curve is

$$\frac{R_2}{R_1} = \frac{234.5 + t_2}{234.5 + t_1}. \tag{7}$$

Example 6. The above example would be worked out as follows by this equation.

$$\begin{aligned} t_2 &= \frac{R_2(234.5 + t_1)}{R_1} - 234.5 \\ &= \frac{6.32 \times 254.5}{5.48} - 234.5 \\ &= 59.1°. \\ t_2 - t_1 &= 59.1° - 20° \\ &= 39.1° \text{ C. rise.} \end{aligned}$$

Prob. 17–4. The cold (20° C.) resistance of an armature was 2.18 ohms. The hot resistance was 2.56 ohms. What was the temperature rise?

Prob. 18–4. It is generally specified that the temperature of the field coils of a dynamo must not rise more than 65° on full load. The resistance of a set of field coils before running was 80 ohms at 20° C. After run of 3 hours at rated load the resistance became 92.4 ohms. Did the machine meet the usual specifications?

Prob. 19–4. The resistance of certain coils in a machine was found to be 7.46 ohms at a temperature of 40°. It was specified that if the machine ran continuously under full load, the temperature of these coils should not exceed 105° C. (the usual limit). The resistance of the coils, measured after a long full-load run, was found to be 9.59 ohms. Did the machine meet this specification?

37. Temperature Coefficient of Alloys, etc. It has been stated that the temperature coefficients of resistance for all pure metals are nearly the same, that is, somewhere about 0.4%. Alloys, although in general of a much higher re-

sistance per mil-foot, have much lower coefficients, some having zero and even negative coefficients at certain temperatures.

"Manganin," an alloy consisting of copper, nickel, iron and manganese, for instance, has a resistance per mil-foot of from 250 to 450 ohms, according to the proportions of the different metals used, and a temperature coefficient so low as to be practically neg igible. Manganin wire is hence used to wind instruments where a fixed resistance is necessary, as in the resistance coils of a bridge or potentiometer. Before the war all manganin came from Germany. It is now made successfully in this country.

Certain substances, notably carbon, porcelain and glass, have large negative temperature coefficients of resistance and decrease in resistance rapidly when heated. The cold resistance of a carbon lamp filament is about twice as great as the hot resistance. The porcelain "glower" of a Nernst lamp when cold is a very poor conductor but when heated to incandescence it becomes a fairly good conductor. The filaments of tungsten lamps are pure metal and accordingly have a positive coefficient which is about 0.0051 at low temperatures.

38. Copper-Wire Tables. Tables have been prepared by the Bureau of Standards and adopted by the A. I. E. E. which give the resistance of 1000 feet of standard annealed-copper wire of different standard sizes and several temperatures. The sizes are designated by gauge numbers, the diameter in mils and the section area in circular mils, etc. There are several standard wire gauges. Brown & Sharpe (B. & S.) is in general use in America, and is commonly called the "American Wire Gauge" (A. W. G.). The Birmingham Wire Gauge (B. W. G.) is in general use in Great Britain. Table No. 1 gives the complete data for the American or Brown-&-Sharpe gauge numbers. By means of these tables it is easy to find the resistance of any length of wire of a given section area, etc.

Example 7. What copper wire (B. & S. gauge) should be used to transmit electric power 2 miles (out and back) if the resistance is not to exceed 2.7 ohms and the temperature to be assumed is 20° C.?

$$2 \text{ miles} = 2 \times 5280 = 10{,}560 \text{ feet.}$$

$$2.7 \text{ ohms for 2 miles} = \frac{2.7}{10.56} \text{ ohm per thousand feet}$$

$$= 0.256 \text{ ohm per thousand feet.}$$

From wire table:

No. 5 = 0.3133 ohm per thousand feet,
No. 4 = 0.2485 ohm per thousand feet.

No. 4 must be used in order not to exceed the limit of 0.256 ohm per thousand feet.

If the following simple facts concerning the above table are memorized, the gauge number and resistance and size of any wire can be roughly estimated without reference to the table.

No. 10 wire is practically $\frac{1}{10}$ inch (100 mils) in diameter, or 10,000 circular mils area and has practically 1 ohm resistance per 1000 feet.

As the wires grow smaller, every third gauge number **halves** the section area and **doubles** the resistance. For instance, No. 13 has about 5000 circular mils area and 2 ohms resistance per 1000 feet; No. 16 has 2500 circular mils area and 4 ohms per 1000 feet, etc. As the wires increase in size, every third gauge number doubles the circular mils area and halves the resistance; No. 7 for instance, has practically 20,000 circular mils and 0.5 ohm per 1000 feet, etc.

Another simple method for remembering the approximate resistances and weights of the different gauge sizes of copper wires is given in circular No. 31 of the Bureau of Standards.

Gauge Number	*Ohms per* 1000 *Feet*		
0	0.1		
1		0.125	
2			0.16
3	0.2		
4		0.25	
5			0.32
6	0.4		
7		0.50	
8			0.64
9	0.8		
10	1.		
11		1.25	
12			1.6
20	10.		
21		12.5	
22			16.

Note that the resistance of No. 0 is 0.1 ohm, No. 1 is 0.125 ohm and No. 2 is 0.16 ohm. The next number in each column is the third gauge number and the resistance is doubled in each case. Similarly for gauge Numbers 10, 11 and 12, the resistances are just 10 times those for No. 0, 1 and 2, respectively, etc. It is merely necessary to remember that for gauges No. 0, 1 and 2 the resistances are 0.1, 0.125 and 0.16 and that for every third number the resistance doubles.

The weight may be approximated from the fact that 1000 feet of No. 0 weigh approximately 320 pounds, and that the weight varies inversely with the resistance.

Example 8. If the line in Example 7 is to work at a temperature of 45° C., what number wire will be required?

Solution. The resistance is not given in the tables at 45° C., but at 20° C. We must then find out what resistance a wire will have at 20° C. if it has 2.7 ohms resistance at 45° C. The temperature rise above 20° is 45 − 20 or 25 degrees. For each degree rise the resistance has increased 0.393%. For 25 degrees rise the resistance has increased $25 \times 0.393 = 9.825\%$.

Thus at 45° the resistance is 109.83% of what it was at 20°. The resistance at 20° therefore equals

$$\frac{2.7}{109.83} \times 100$$

$$= 2.48 \text{ ohms for the 2 miles.}$$

$$\text{The resistance per 1000 feet} = \frac{2.48}{10.56}$$

$$= 0.235 \text{ ohm.}$$

The problem thus becomes: What size of wire has a resistance of 0.235 ohm per 1000 feet?

From table (No. 5),

No. 4 has a resistance of 0.2485 ohm per 1000 feet,
No. 3 has a resistance of 0.1970 ohm per 1000 feet.

No. 3, therefore, must be used in order not to exceed 0.235 ohm per 1000 feet.

Prob. 20–4. What size wire (B. & S.) will give a resistance of practically 1 ohm for the circuit of the example above at 35° C.?

Prob. 21–4. Thirty-five 50-watt, 110-volt lamps are to be used in a building so situated that it requires 200 feet of feeder wires (each way) from the generator to the distributing point. What size wire should be run in order that there shall not be more than a 3-volt drop in the feeders?

Prob. 22–4. How far can 20 amperes be transmitted through a No. 6 wire (B. &. S) with 4 volts line drop?

Prob. 23–4. A coil for an electromagnet has 800 turns of No. 23 (B. & S.) copper wire. The average length of a turn is 14 inches. What is the resistance of the coil?

Prob. 24–4. It is desired to construct a coil of not more than 290 ohms resistance. The coil must have 200 turns of about 16 inches average length. What size wire (B. & S.) should be used?

39. Stranded Wire. On account of their greater flexibility, stranded cables are often used instead of solid wires. A stranded cable is much easier to pull into a conduit and less likely to break when bent at sharp angles. When a size of wire larger than No. 0000 is required, it is prac-

tically always made in strands rather than solid but the smaller sizes are also very common in the stranded form.

For instance, instead of using a solid No. 4 wire, having a diameter of 204 mils and an area of 41,700 circular mils, it is much easier to use a cable made up of 7 wires each 0.077 inch in diameter. Each strand (wire) would then have an area of 77×77, or 5930 circular mils. Since the cable is made up of 7 of these strands, the area of the cable would be 7×5930 or 41,500 circular mils, which is practically the area of a No. 4 solid wire. The diameter of a stranded wire will always be slightly greater than that of a solid wire of an equivalent cross-section.

On the other hand, due to the spiral arrangement of the strands, the effective length as well as the mass is increased beyond what it would be for a rod of the same cross-section (number of strands times cross-section of each strand). The resistance is also increased. It is therefore customary to compute the resistance of a stranded cable by adding 2% to the resistance of a rod of the same cross-section. This 2% correction holds for only one value of the "lay" of the strands. For the method of computing the correction, see U. S. Bureau of Standards Circular, No. 31.

Prob. 25–4. To what size wire (B. & S.) is a stranded cable equivalent which is made up of 19 strands each 0.059 inch in diameter?

Prob. 26–4. It is desired to make a cable of 19 strands which shall be equivalent to a No. 0 (B. & S.) solid conductor. What size strands should be used?

Prob. 27–4. How many strands 0.061 inch in diameter will it take to make a cable equivalent to a No. 6 wire?

Prob. 28–4. It is desired to make a very flexible cable equivalent to No. 4 (B. & S.) wire. If strands of No. 22, (B. & S.) wire are used, how many will be required?

40. Aluminum. We have noted that although the resistance of aluminum wire is 17.1 ohms per mil-foot (practically 1.6 times that of copper) its weight is only 0.3 that

of copper. For this reason some transmission lines are strung with aluminum wire. While this necessitates a larger wire for the same resistance per 1000 feet, the weight of such a wire will be less than that of a copper conductor. However, such a line possesses a disadvantage over a copper line in that aluminum melts at a lower temperature than copper. A short circuit which would burn up but a few inches of a copper wire is likely to burn out long sections of an aluminum line. Likewise having a greater cross-section it collects a greater weight of ice per foot during a sleet storm. This is likely to weight the wire beyond its tensile strength. Aluminum is also more difficult to splice satisfactorily than copper, which adds to the cost of construction of a line with aluminum conductors.

Example 9. What size would an aluminum wire be which has the same resistance as a No. 4 copper wire?

Solution. No. 4 copper wire has a resistance per 1000 feet of 0.2485 ohm. The resistance of an aluminum wire is found from the equation

$$R = \frac{17.1}{d^2}.$$

Thus,

$$0.2485 = \frac{17.1 \times 1000}{d^2}.$$

$$d^2 = \frac{17.1 \times 1000}{0.2485}$$

$$= 68{,}800 \text{ circular mils.}$$

This would require an aluminum wire of gauge No. 1, of 83,690 circular mils. No. 2 has a cross-section of only 66,370 circular mils. An aluminum wire of this size would be called the equivalent of No. 4 copper wire.

Example 10. How would the weight per 1000 feet of the equivalent aluminum wire in the above example compare with the weight of the copper wire?

The weight of 1000 ft. of No. 4 copper wire = 126 lb.

The weight of 1000 ft. of 68,800 cir. mil copper wire = 208.2 lb.

The weight of 1000 ft. of 68,800 cir. mil aluminum wire = 0.30 × 208.2 = 62.5 lb.

Thus the weight of the aluminum equivalent is only

$$\frac{62.5}{126}$$

or 49.6% of that of the copper wire.

Prob. 29–4. What size aluminum wire will have the same resistance per mile as a No. 6 (B. & S.) copper wire?

Prob. 30–4. An aluminum cable is to be made up of 19 strands which will be equivalent to a No. 0 solid copper wire. What size must the strands be?

Prob. 31–4. If 83 No. 19 aluminum wires are used in making up a cable, to what size solid copper is such a cable equivalent?

41. Copper-Clad Steel Wire. On account of its lower cost and great tensile strength, copper-clad steel wire has lately come into use for trolley wires and transmission lines. This type of wire consists of a steel core to which has been welded a covering of copper. The resistance per mil-foot of such wire depends upon the relative sizes of the copper and steel cross-section areas. One company has put on the market two grades, one of so-called 30 % conductivity and the other of 40 % conductivity. This rating merely means that copper wires would have 30% and 40% respectively of the resistance of the copper-clad steel wire of the same size.

In the 30%-conductivity wire the area of the steel core is 79.5% of the entire cross-section while the copper has 20.5% area. In the 40%-conductivity wire, the steel makes up 68.2% and copper 31.8% of the total area.

Prob. 32–4. In a No. 00 copper-clad trolley wire of 30 % conductivity, how many circular mils would there be of steel and copper respectively?

Prob. 33–4. What would be the resistance of 1000 feet of the steel core of the trolley wire in Prob. 32–4? The mil-foot resistance of steel = 86.7 ohms.

Prob. 34–4. What would be the resistance of 1000 feet of the copper cover of the trolley wire of Prob. 32–4?

Prob. 35–4. From the resistance of the steel core and copper covering of 1000 feet of the trolley wire in Prob. 32–4, compute the resistance of the trolley wire. Would 1000 feet of a copper wire of the same size have 30% as much resistance?

42. Safe Carrying Capacity for Copper Wires. In installing wire in buildings, it is necessary to take into account another factor besides the voltage drop when determining the size to be used. An electric current heats any conductor through which it passes. If heat is generated in the wire faster than it can be dissipated from the surface of the wire, the temperature will continue to rise as long as this condition exists. It is necessary therefore to select a wire which will dissipate the heat generated by the current at such a rate that the temperature will never rise high enough to cause the insulation to deteriorate. The National Board of Fire Underwriters has therefore issued a table of the safe current capacity of copper wire of the sizes used in house wiring. Wherever local regulations do not specify otherwise, the currents carried by any interior wiring should not exceed the values given in this table. See Appendix.

Example 11. It is desired to install a conductor to carry 40 amperes. What size copper wire should be used?

From the table, No. 6 rubber-insulated wire will carry 46 amperes and is the size to be used.

If weather-proof wire can be used, No. 8 will do.

43. Determination of Right Sizes for Interior Wiring. In deciding upon the sizes which should be used in the different parts of any interior distributing system, it is necessary to take into consideration two factors, —

First: The size in each section must be such that the current in no wire exceeds the amount given in the Underwriters' Table of safe carrying capacities for wires. It is therefore necessary to determine accurately the current which each wire must carry and to make a tentative selection of size from the above table.

Second: The voltage drop throughout the system must then be computed in order to make certain that it does not exceed certain values, for if lamps are to be operated anywhere on the system a variation of more than 5% in the voltage at the lamps causes an unpleasant variation in the illumination. If the entire load consists of motors, heating appliances, etc., a drop of 10% is usually allowable. Any greater drop than this, however, would have a bad effect upon the speed of the motors.

Example 12. The panel board (P.B.) in Fig. 44 is situated 150 feet from the main switch. From the board run three

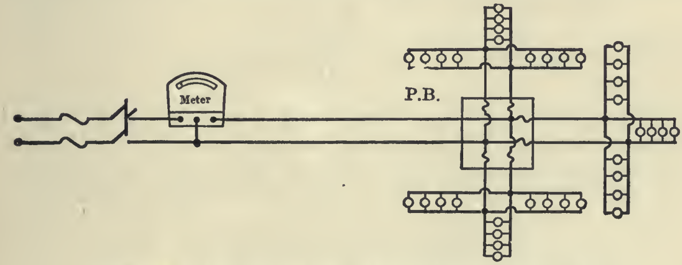

Fig. 44. A distribution diagram for house wiring.

branch lines. Each branch is supplied with 12 outlets for 50-watt, 110-volt lamps. What size must the mains be?

Solution. Each branch line carries 12 × 50 or 600 watts. This means $\frac{600}{110}$ or 5.45 amperes in each branch wire. Branch-wire sizes are determined according to Table III. Although from the table it is observed that No. 16 wire would carry this current safely, it will be noted that no size smaller than No. 14 can be installed in order that the wires may have sufficient mechanical strength.

Mains. Size according to Table III.

Each main must carry the current in all three branches or 3 × 5.45 = 16.35 amperes.

According to Table III, No. 12 wire must be used as the next size in common use, No. 14, can carry only 15 amperes.

Checks on above sizes for voltage drop.

The distance from the panel to the load center* of branches	= 50 ft.
The length of wire in each branch = 2 × 50	= 100 ft.
The resistance of 1000 ft. No. 14 (Table II)	= 2.525
The resistance of 100 ft. of No. 14 = $\frac{1}{10}$ of 2.525	= 0.253 ohm
The voltage drop in each branch = 5.45 × 0.253	= 1.38 volts
The length of the mains = 2 × 150 ft.	= 300 ft.
The resistance of 1000 ft. of No. 12 (Table II)	= 1.588 ohms
The resistance of 300 ft. of No. 12 = 0.3 of 1.588	= 0.476 ohm
The voltage drop in the mains = 16.35 × 0.476	= 7.78 volts
The total drop from the main switch to the lamps = 7.78 + 1.38	= 9.16 volts

$$\text{The percentage line drop} = \frac{9.16}{110} = 8.3\%$$

This is nearly twice as great a drop as is allowed in good practice because the brightness of the lamps would vary through wide ranges depending on how many were in use at one time. When only a few lamps were in use, the voltage of these lamps would be about the same as that at the main switch, 110 + 9.16, or about 119 volts. The voltage at this main switch would have to be 119 volts in order to maintain 110 volts at the lamps on full load. This would cause the lamps to glow far above their rated candle power and would either burn them out at once or shorten their life to a small percent of the normal rating. It would, therefore, be necessary to install larger than No. 12 mains. Let us try No. 10.

The resistance of 300 ft. of No. 10 = 0.3 of 0.9989 = 0.300 ohm.

The drop in the main = 0.300 × 16.35 = 4.91 volts.
The total drop = 4.91 + 1.38 = 6.29 volts.

$$\text{The percentage drop} = \frac{6.29}{110} = 5.72\%.$$

This is still somewhat too large a drop. It is, therefore, necessary to use No. 8 mains.

* *Note:* The load center is that point on the branch line at which, for convenience in calculation, all lamps may be considered to be concentrated.

Prob. 36–4. What size mains would be used in the above example if the panel board were situated 50 feet from the main switch and the same loads were on the branch lines?

Prob. 37–4. An installation requires seventy 50-watt lamps. The panel board is situated 80 feet from the main switch. What size main should be run? Note that the Underwriter Rules do not ordinarily allow more than twelve 50-watt lamps on a single branch line.

Prob. 38–4. If the panel board in the above problem could be placed 40 feet from the main switch, what size mains might be used?

Prob. 39–4. If the load on the installation of Prob. 37–4 consisted of motors instead of lamps but using the same total load, what size wire could be used for the mains?

44. Insulating Materials. Even the materials which we call insulators conduct electricity to a certain small extent. The resistance of such materials is very high compared with the resistance of metals. The resistivity of soft copper is 0.00000172 ohm-centimeter, which may be written more conveniently

$$1.72 \times 10^{-6} \text{ ohm-centimeter.}$$

Expressed in the same way the resistivity of glass under ordinary conditions is about

$$5 \times 10^{16} \text{ ohm-centimeters.}$$

This means that a cube of glass one centimeter on a side will offer a resistance of 50,000,000,000,000,000 ohms. Such an enormous resistance can be measured only with great difficulty. In fact, the value varies so much with different specimens and conditions of measurement that it is not known accurately.

Average values of the resistivities in ohm-centimeters for common insulating materials are about as follows:

Rubber	2×10^{15},
Impregnated paper	5×10^{14},
Varnished cambric	2×10^{14},
Glass	5×10^{16},
Fused Quartz	1×10^{18}.

Materials such as the above do not obey Ohm's law. When the voltage applied to a specimen is doubled, the current will be more than doubled. The resistivity of the material depends upon the voltage used in measuring it. More accurately, it depends upon the voltage gradient, or the voltage per inch of thickness, which is applied. The higher the voltage gradient, the lower will be the resistivity. The values given in the table above are for low potential gradients of about 1000 volts per inch.

Insulating materials have negative temperature coefficients of resistance which are usually large. As the temperature is increased, the resistance of a material rapidly diminishes. In fact, all materials which can stand the temperature are fairly good conductors at a red heat. The resistivity of glass at ordinary temperatures will vary as much as 10% for one degree centigrade change of temperature.

With insulating materials, the leakage over the surface is often greater than the conduction through the body of the material. With a porcelain transmission-line insulator, by far the greater part of the leakage current from the line flows over the surface. This is particularly true if the surface is dusty or wet. It is to lengthen these leakage paths and to keep part of them dry that an insulator is made with "petticoats." The surface resistivity is given as the number of ohms resistance between opposite edges of a piece of the surface one centimeter square. It is an extremely variable quantity, depending upon the humidity, the temperature and several other factors.

Example 13. A glass busbar insulator is a cylinder 10 centimeters high and of 4 centimeters diameter. A voltage of 30,000 is applied between the busbar on top and the support on the bottom. The temperature is 40° C. Take the volume resistivity at this temperature and gradient as 5×10^{11} ohm-centimeters and the surface resistivity as 2×10^{10} ohm-centimeters. How much current in microamperes leaves the busbar through the insulator?

The volume resistance is

$$R_V = \rho \frac{l}{A} = 5 \times 10^{11} \frac{10}{\pi(2)^2}$$
$$= 4 \times 10^{11} \text{ ohms.}$$

The current through the volume of the glass is

$$I_V = \frac{E}{R_V} = \frac{30000}{4 \times 10^{11}} = 7 \times 10^{-8} \text{ ampere}$$
$$= 0.07 \text{ microampere.}$$

The surface resistance is

$$R_S = \rho_S \frac{l}{w},$$

where l is the length and w the width of the surface path.

$$R_S = 2 \times 10^{10} \frac{10}{4\pi} = 1.6 \times 10^{10} \text{ ohms.}$$

The current over the surface of the glass is

$$I_S = \frac{E}{R_S} = \frac{30000}{1.6 \times 10^{10}} = 2 \times 10^{-6} \text{ ampere}$$
$$= 2 \text{ microamperes.}$$

The total current is hence practically 2 microamperes, that conducted through the volume of the glass being negligible.

It is, of course, useless to compute such a problem with great accuracy, for the constants of the material are never accurately known.

It will be noticed that the amount of current which leaks from the wire in the above example is small compared with the current probably carried by the wire itself. This is usually the case in ordinary wire circuits. Except in special cases the leakage can be entirely neglected.

45. Insulation Resistance of Cables. The insulation resistance of electrical machines, transformers or cables is sometimes measured and is used as a rough indication of the condition of the insulation. This is the resistance

measured between the copper wire and the frame or sheath. A moderate size of low-voltage motor, as usually insulated, when warm from carrying full load, should show an insulation resistance of about a megohm. When cold, the insulation resistance will have a value many times this amount.

The insulation resistance of lead-sheathed cables is often measured to determine whether the insulation is deteriorating. The insulating material of a cable, when considered as a conductor for leakage current, is not of uniform cross-section. The resistance must hence be found by an integration.

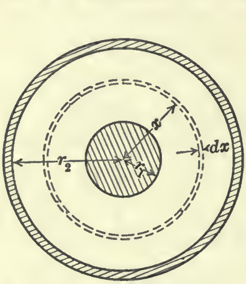

FIG. 45. A cross-section of a lead-sheathed cable.

Fig. 45 shows a cross-section of a single-conductor cable. The flow of leakage current is radial, out from the central copper conductor to the outside lead sheath. The cross-section of the path varies. We can consider it to be made up of a number of elementary paths in series.

Consider a centimeter length of the cable (perpendicular to the paper). Map out an element of the material of thickness dx and radius x. If ρ is the resistivity of the material, the resistance of this element will be

$$dR = \frac{\rho dx}{2\pi x}.$$

To obtain the total resistance we add the resistances of all such elements between $x = r_1$ and $x = r_2$: that is,

$$R = \int_{r_1}^{r_2} \frac{\rho dx}{2\pi x} = \frac{\rho}{2\pi} \log_\epsilon \frac{r_2}{r_1} \text{ ohms per centimeter of length.} \quad (8)$$

From this formula we can compute the insulation resistance of a cable.

Example 14. If $\rho = 2 \times 10^{14}$ ohm-centimeters is the resistivity of the insulation for a varnished-cambric insulated cable, where

$$r_1 = 1 \text{ centimeter}$$

and

$$r_2 = 4 \text{ centimeters,}$$

we have from the above formula

$$\begin{aligned} R &= \frac{2 \times 10^{14}}{2\pi} \log_\epsilon 4 \\ &= \frac{10^{14}}{\pi} 2.30 \times \log_{10} 4 \\ &= \frac{10^{14}}{\pi} 2.30 \times 0.60 \\ &= 4.4 \times 10^{13} \text{ ohms per centimeter length of cable.} \end{aligned}$$

The insulation resistance of a mile of this cable will equal this value divided by the number of centimeters in a mile (1.61×10^5). The cable will then have an insulation resistance of approximately

$$3 \times 10^8 \text{ ohms per mile,}$$

or

$$300 \text{ megohms per mile}$$

under the above assumption.

This formula must be used with considerable discretion, for insulation resistance is a variable matter depending upon very many conditions. The derivation of the formula will, however, show the manner in which the resistance of a non-uniform conductor can be computed.

SUMMARY OF CHAPTER IV

The RESISTANCE of a conductor can be computed from the formula

$$\mathrm{R} = \rho \frac{l}{\mathrm{A}},$$

where

R = the resistance in ohms,
l = the length of conductor,
A = the cross-section area,
ρ = the resistivity in units depending on the units of length and cross-section.

Resistivity is generally measured in OHM-CENTIMETERS, that is, the resistance between parallel faces of a centimeter cube, or in OHMS PER MIL-FOOT, that is, the resistance of a conductor one foot long and one circular mil in cross-section area.

THE MATERIALS MOST COMMONLY USED for conductors are copper for its low resistivity, aluminum for its light weight and steel for its strength.

CONDUCTIVITY is the reciprocal of resistivity. A certain grade of copper is taken as the standard and is said to have 100% conductivity and metals are rated as a certain percent conductivity of that standard.

The TEMPERATURE COEFFICIENT OF RESISTANCE is the change in resistance per degree per ohm at initial temperature. THIS COEFFICIENT IS ABOUT 0.004 for all pure metals. For alloys it is less and may be zero or even negative. For insulators it is always negative and of considerably higher order of magnitude than for conductors.

FOR COPPER, the effect of temperature change may be computed from the equation

$$\frac{\mathrm{R}_2}{\mathrm{R}_1} = \frac{234.5 + t_2}{234.5 + t_1},$$

where

t_1 = the initial temperature,
R_1 = the initial resistance,
t_2 = the final temperature,
R_2 = the final resistance.

In the United States the standard wire gauge is the "Brown & Sharpe" or "American Wire Gauge." Copper wire is commonly made in the sizes indicated in the Copper Wire Table and has the resistance and weight per 1000 feet indicated.

STRANDED WIRE has greater flexibility but for the same cross-section it has 2% more resistance per mile due to the spiral arrangement of the strands.

ALUMINUM WIRE has about 1.6 the resistivity of copper, but is only $\frac{3}{10}$ as heavy. It is hard to splice and is likely to suffer more from ice storms and short circuits.

COPPER-CLAD WIRE has a steel core which lowers the conductivity but increases the strength.

THE SAFE CURRENT-CARRYING CAPACITY OF WIRES is indicated in a table issued by the National Board of Fire Underwriters. A large current would sooner or later injure the insulation.

THE CORRECT SIZE OF WIRE FOR INTERIOR WORK is such that, first, no conductor will carry more current than that indicated in the Underwriters table, and, second, the voltage regulation of lamps shall not exceed 5% and that of motors 10%.

THE SURFACE LEAKAGE RESISTANCE OF INSULATORS is generally less than the volume resistance.

THE INSULATION RESISTANCE OF SHEATHED CABLES can be computed from the equation

$$R = \frac{\rho}{2\pi} \log_\epsilon \frac{r_2}{r_1},$$

where

R = the insulation resistance per centimeter length of cable,
ρ = the resistivity of insulation material,
r_2 = the outside radius of insulation,
r_1 = the inside radius of insulation.

The value of ρ in this equation depends upon many factors and is very uncertain.

PROBLEMS ON CHAPTER IV

Prob. 40–4. How far will a pair of copper line wires transmit 40 amperes with a line drop of 8 volts, if the wire is 0.262 inch in diameter?

Prob. 41–4. What size iron wire will have the same resistance per mile as a No. 4, B. & S. copper wire?

Prob. 42–4. To what size copper wire is a stranded aluminum cable equivalent which is made up of 19 strands each 0.059 inch in diameter?

Prob. 43–4. How many strands of aluminum wire 0.061 inch in diameter will it take to make a cable equivalent to a No. 6 copper wire?

Prob. 44–4. It is desired to make a flexible aluminum cable equivalent to No. 4 copper wire. If strands of No. 19, B. & S. wire are used how many will be required? Cables are usually made of 7, 19, 37 or 61 strands. Use one of these numbers.

Prob. 45–4. What are the cross-sectional dimensions of a round conductor with an iron core surrounded by copper having a mean conductivity of 40% at 0° C., and offering a resistance of 10 ohms per mile at 25° C? The copper used has a conductivity of 100%, the iron 16.8%, both at 0° C. The annealed copper standard of 10.4 ohms per mil-foot at 20° C. is to be taken.

Prob. 46–4. It is desired to determine the length of wire and the mass of copper on the field winding of a small dynamo. The percent conductivity of the copper is not known, but the specific gravity is taken to be 8.89 and the cross-section of the wire is found by measurement to be 1022 circular mils. Accurate measurements of the resistance of the winding at two temperatures are made with the following results:

Resistance at 20° C. = 126.5 ohms,
Resistance at 70° C. = 150.2 ohms.

From the above data, calculate the length of wire in feet, the mass of copper in pounds and the percent conductivity of the copper. With your answer, state the reference book and page from which you take any necessary additional data.

Prob. 47–4. A transmission line of 3 wires, each 12 miles long, is to be designed for a current of 110 amperes in each wire and a total heat loss of 150,000 watts at 20° C. The conductors are to be of copper of 97% conductivity. What is the least weight of copper which meets these conditions? What cross-section in circular mils would each such conductor have? What is the smallest A.W.G. size of wire which comes within the conditions of the problem? If this size of wire is used and has 97% conductivity, what is the actual heat loss?

Prob. 48–4. The field coils of a dynamo are composed of wire having a mean temperature coefficient of 0.00393 per degree centigrade, based on 20° C. The resistance of the coils is 116 ohms after they have long stood inert at a temperature of 26° C. Later when heated by the passage of the current, their resistance is found to be 136.5 ohms. What is their mean temperature in the latter state?

Prob. 49–4. Power is to be transmitted over a two-wire transmission line to a factory three miles from the generating station. The factory requires a current of 500 amperes at a potential of 600 volts, and it is specified that the losses in the transmission line shall not be more than 10% of the received energy. The resistivity of copper at 20° C. is 10.6 ohms per mil-foot and that of aluminum 17.1 ohms per mil-foot. Their temperature coefficients at 20° C. are 0.00384 and 0.0039 respectively. If the line is to operate in a climate whose average temperature is 50° *Fahrenheit*, find the smallest volumes of copper and aluminum it would be possible to use for the construction of the entire line.

Prob. 50–4. If copper costs 15 cents per pound and aluminum 32 cents per pound, which metal will be the cheaper to use in Prob. 49–4?

Prob. 51–4. A motor is connected to the 220-volt busbars of a power station by means of a one-mile line of aluminum wire (conductivity 60%). The cross-section of each wire is 33,100 circular mils. The motor field is wound with copper wire and its resistance at 20° C. is 150 ohms. The motor armature is also wound with copper and its total resistance at 20° C. is 0.5 ohm. Compute the resistances of all parts of the circuit when the temperature of the line wires is 30° C. and of the motor 70° C.

Prob. 52–4. Compute the weight of copper wire necessary to replace the aluminum line wires in Prob. 51–4 and to give the same resistance at 30° C.

Prob. 53–4. Two industrial plants, approximately 2000 feet apart, decide to combine their power plants into one situated at one of the plants. The generators are rated at 235 volts and the motors are 230 volts and 220 volts so divided in number that the 230-volt motors may be used at the plant near the power house and the 220-volt motors at the more distant plant. The total demand of the 220-volt motors is 400 amperes and of the 230-volt motors it is 700 amperes. Under these conditions the voltage drop through the feeders of the 700-ampere demand is 5 volts, of the line between the plant, 10 volts, and of the feeders to the other demands, 5 volts. The feeders in each plant are 500 feet long. Find the weight of copper necessary for this arrangement. The Electric Journal, February, 1915, page 7.

Prob. 54–4. In Prob. 53–4 find the losses in the line and feeders. If the resistance of the main switchboard connections and the armatures of the machines in parallel is 0.008, what is the generated e.m.f. of the machines and what is the loss in the switchboard and armature windings? Assume the generating units to be identical machines.

Prob. 55–4. If in Prob. 54–4 one-half the motors are taken out of service at the nearer plant, what would be the voltage at the switchboard, at the other motors of the other plant and at the motors of the nearer plant?

Assume the generated e.m.f. of the machines to be the same as in Prob. 53–4.

Prob. 56–4. If in Prob. 54–4 the entire motor load is taken off at the more distant plant, what should be the generated voltage of the generators in order to maintain 230 volts at the end of the feeder in the nearer plant? What would be the open-circuit voltage at the end of the feeder in the more distant plant?

Prob. 57–4. An aluminum bar has a uniform thickness of 0.30 inch. Its breadth varies uniformly from 9 inches at one end to 12 inches at the other. Its length is 18 feet. Its volume conductivity is 61%. What is its resistance at 20° C.? What is the percent error when its resistance is computed from the mean cross-section?

Prob. 58–4. A 110-volt system has an insulation resistance for each wire of 200 megohms per mile. What will the leakage be on a 5-mile line?

Prob. 59–4. Insulation resistance should be high enough so that not more than one-millionth of the rated current leaks through the insulation. On this basis, what should be the insulation resistance per mile of a 2-mile line, tramsmitting 120 kilowatts at 550 volts?

Prob. 60–4. The resistance of a car heater when cold (20° C.) is 120 ohms. If the temperature rises to 150° C. when hot, how much less current does it take when hot than when cold? The material of the heater is iron wire. The voltage is 550.

Prob. 61–4. A rough rule for the safe carrying capacity of copper is "1000 amperes per square inch cross-section." According to this rule what should be the diameter of a round wire capable of carrying 250 amperes?

Prob. 62–4. According to the rule in Prob. 61–4, what should be safe carrying capacity of No. 0000 (B. & S.)? Compare the value with that in Table **III.**

Prob. 63–4. Derive the equation for the resistance of a round rod having a uniform taper.

Prob. 64–4. It is desired to transmit 100 kilowatts of electrical power to a small town and to deliver it at 220 volts. The distance is 4 miles and it is thought that the line will pay if an amount of power equal to 10% of that delivered is allowed for line loss. (*a*) Find the size wire necessary and the voltage at which the power must be generated. (*b*) What size wire would be necessary if a three-wire (220-440 volts) system were used? (*c*) Compute the weight of copper saved. (Use the nearest sizes of wire, B. & S. gauge.) (*d*) Compare the copper for these two cases with that necessary where the power is transmitted at 4400 volts (alternating current), assuming that alternating-current power equals volts times amperes and that the voltage drop in the alternating-current line is all resistance drop.

Prob 65–4. A transformer is an electrical appliance consisting of a laminated iron core around which are wound the primary and the secondary windings. It is enclosed in a pressed steel tank which is filled with oil. The heat generated in the iron core and copper flows into the oil by conduction and causes the oil to circulate. The circulation of the oil causes a fairly uniform distribution of temperature throughout the transformer. Assume that the temperature is the same at all points within the transformer and that the watts radiated per unit area are proportional to the temperature difference. The results of a heat-run on a certain transformer show a temperature rise of

22° C. in 2 hours. Heat was generated within the transformer at the rate of 225 joules per second. The radiating surface of the transformer is 10 square feet. The heat radiated per square foot per degree of temperature difference equals 0.5 joule per second.

(*a*) With a room temperature of 25° C. what will be the temperature of the transformer at the end of one hour? What will be the ultimate temperature?

(*b*) If the heat is generated at twice the above rate, what will be the temperatures asked for in (*a*)?

(*c*) If this transformer is taken out of circuit when its temperature is 12° C. above room temperature, how long will it take for the temperature to drop 6 degrees? 10 degrees?

(*d*) Construct a temperature-rise-versus-time curve for a heat input of 200 watts.

(*e*) Construct a cooling curve (temperature-versus-time) starting with the transformer 10° C. above room temperature.

(*f*) What is the average specific heat of the transformer material?

Prob. 66–4. A Wheatstone bridge is constructed with a slide wire, as shown in Fig. 19. Determine the relations existing among the four resistances for maximum sensitivity to unbalance.

Prob. 67–4. One of the cables across the English Channel has a conducting core of #16 copper wire 64 mils in diameter (British Standard Gauge). It is fifty miles long, and may be assumed to follow a path which is approximately parabolic, reaching a maximum depth of one mile. The temperature of the water varies linearly with the depth, and may be taken as fifty degrees F. at the surface, and twenty-five degrees F. at the max. depth. Compute the total resistance of the cable.

CHAPTER V

ELECTROLYTIC CONDUCTION

In the preceding chapters we have considered conduction through metals. In all such cases the wire is left entirely unchanged chemically no matter how long the current is allowed to flow. We will now study a type of conduction in which the flow of current is accompanied by a chemical change in the conductor.

46. Electrolytes and Ionization. Pure water is a fair insulator. The addition of a minute amount of a soluble acid, base or salt will produce a conducting solution. Certain fused salts and non-aqueous solutions are also conducting. When a current is passed through a liquid of this sort, a chemical change appears in the neighborhood of the electrodes. Such liquids are called electrolytes.

Pure water is an insulator because there are no carriers of electricity present. When a salt is dissolved in the water, there still are no free electrons, but a new kind of carrier of electricity is present. This is the ion. When a quantity of a salt is dissolved, part of the molecules separate into component parts. These are not, however, the atoms of which the molecule is constructed. The parts are similar to atoms or groups of atoms except that some have extra attached electrons which do not normally belong to them and others have too few electrons to be normal. These parts are called ions and they are electrically charged. An ion having less than its normal number of electrons is positively charged and an ion having more than its normal number is negatively charged. When an electromotive force is applied to an electrolyte, the ions move through the liquid, those charged positively with a deficiency of

electrons moving in the direction of the applied electromotive force, and the negative ions charged with extra electrons, in the opposite direction. An ion is negatively charged when it has extra electrons attached because the electrons themselves are negative particles of electricity. A normal atom is made up of a so-called nucleus which is positively charged, and just enough negative electrons to exactly balance this positive charge. Therefore if there are more than this number of (negative) electrons attached, the atom is negatively charged and becomes a negative ion. Similarly an atom lacking in (negative) electrons would have a surplus of positive charge and would become a positive ion. Note that there are two currents flowing through the fluid, one made up of ions carrying an excess of small particles of electricity (the electrons), the other made up of ions which are deficient in these particles of electricity. The total electric current flowing is the sum of these two. It will be remembered that it is the flow of the "excess" electrons only which constitutes the current of electricity in a metallic conductor, the positive charges being bound. Thus, curiously enough and rather unfortunately, this flow of electrons is in the direction opposite to that which was chosen as the positive direction before the electron theory was known. Therefore when we speak of the flow of electricity as being in a certain direction, we may mean that there is a current of electrons flowing in the opposite direction. Of course our conventional meanings of "positive" and "negative" make a current of negative particles in one direction equivalent to a current of positive particles in the other direction: thus no real confusion should result.

This splitting a substance into positive and negative ions involves the separation of the parts of a chemical substance, and chemical effects are in evidence at the ends of the electrolytic conducting path. These chemical effects may involve the release of gas bubbles at the electrodes. They may produce electroplating or cause the metal of the elec-

trodes to go into solution or bring about changes in the chemical nature of the electrolyte near the electrodes.

Electrolytic conduction is made use of in primary batteries and in storage batteries, for the plating and refining of metals and for the production of various chemicals. On the other hand, under certain conditions it will produce harmful effects such as the destruction of pipes and concrete reinforcements by stray currents from trolley systems and other sources.

47. Electrolytes and Dissociation. A liquid conductor in which conduction takes place by reason of the presence of charged ions is called an **electrolyte.** The plate or **electrode** by which the current **enters** the solution is called the **anode** and that by which the current **leaves** is called the **cathode.** The anode is hence at a positive potential with respect to the cathode. Negatively charged ions move toward the anode and positively charged ions toward the cathode.

When an ion reaches an electrode it recovers its normal supply of electrons and becomes an atom. As a free, uncombined atom in the nascent state, it usually immediately combines chemically with neighboring atoms or molecules. This combination may involve atoms of its own kind, those of the solution or those of the electrodes. Positive ions arriving at the cathode take electrons from it. Negative ions arriving at the anode give up electrons to it. The electrolyte as a whole remains uncharged and hence just as many electrons are taken from the cathode as are passed to the anode. The electrons which pass through the external metallic circuit are hence passed through the solution by being carried along by ions.

Univalent atoms in solution become singly charged ions. Thus if common salt is dissociated into its ions, we have

$$NaCl \rightleftarrows Na^{+} + Cl^{-},^{*} \tag{1}$$

* The arrow indicates a chemical reaction. When two arrows are used, they indicate a reversible reaction, or one that can proceed in either direction until equilibrium is reached.

which means that a molecule of sodium chloride splits into a positively charged ion of sodium with one electron less than normal and a negatively charged ion of chlorine with one excess electron. Polyvalent atoms give rise to ions with a number of charges equal to the valency. Thus if calcium chloride is dissolved and dissociated, we have

$$CaCl_2 \rightleftarrows Ca^{++} + Cl^- + Cl^-. \tag{2}$$

One molecule of the salt thus produces a calcium ion with two extra positive charges and two singly charged negative chlorine ions.

Ions are not necessarily made up from single atoms. Chemical groups or radicals may produce single so-called complex ions. Thus when ammonium chloride is dissociated, we have

$$NH_4Cl \rightleftarrows (NH_4)^+ + Cl^-; \tag{3}$$

that is, the ammonium radical produces a single positive ion. On the final dissociation of sulphuric acid we obtain

$$H_2SO_4 \rightleftarrows H^+ + H^+ + (SO_4)^{--}; \tag{4}$$

that is, two positive hydrogen ions and a doubly charged negative ion formed from the (SO_4) group.

There may be several kinds of ions in a solution simultaneously. Thus if sodium chloride and potassium bromide are dissolved together, we shall have positive sodium and potassium ions and negative chlorine and bromine ions.

Not all of the dissolved substance is dissociated. A greater proportion is split into ions when the solution is dilute. The proportion also varies greatly with the substance used. Strong acids or bases are most highly dissociated. An acid may be defined as a substance which produces hydrogen ions in solution; a base as a substance which produces hydroxyl ions, that is, ions of the form $(OH)^-$. A strong acid such as nitric acid is largely dissociated, while a weak acid such as boric acid produces relatively few ions.

The conductivity of an electrolyte is equal to the current produced by unit potential gradient (volts per centimeter) in the solution. The conductivity is the sum of the conductivities produced by each sort of ion separately. These separate conductivities are proportional to the number of ions present and to their mobility, that is, the speed with which they move through the electrolyte under a given impressed voltage.

A strong solution of hydrochloric acid is highly conducting, while a strong solution of boric acid is of comparatively high resistance. The wide difference disappears for very dilute solutions, when both acids are largely dissociated.

A normal solution of hydrochloric acid has a resistivity of about 3 ohm-centimeters at 20° C. By a normal solution is meant one in which the number of grams of acid per liter of solution is equal to the molecular weight of the acid (in this case 36.5) divided by the number of replaced or replaceable hydrogen atoms.

Even pure water is not a complete insulator. Pure water dissociates very slightly into hydrogen and hydroxyl ions.

$$H_2O = H^+ + (OH)^-. \tag{5}$$

This dissociation is so slight that the resistance of a cubic centimeter of chemically pure water is approximately 0.5×10^6 ohms at 18° C.*

48. Electric Potential Series. If the electrodes are of different metals, the voltage between them will not be equal simply to the resistance drop in the electrolyte. In fact, if two plates of different metals are dipped in an electrolyte which is capable of combining chemically with one of them, a voltage will appear between the plates, even when there is no current flowing. This chemically produced voltage is the voltage produced and used in primary and in storage batteries.

* "Handbook of Chemistry and Physics," Chemical Rubber Co.

The potential produced on open circuit in this manner depends simply upon the metals employed and not upon the electrolyte so long as the latter is chemically active with respect to one electrode.

The elements may be arranged in a table called the potential series, which is shown below for some of the more common elements.

TABLE

Potential Series of Elements Compared with Hydrogen

Manganese	+ 1.07 volts
Zinc	+ 0.77
Cadmium	+ 0.42
Iron	+ 0.35
Cobalt	+ 0.23
Nickel	+ 0.22
Tin	+ 0.19
Lead	+ 0.15
Hydrogen	0
Copper	− 0.33
Mercury	− 0.75
Silver	− 0.77

If two substances are inserted in an electrolyte, the chemical potential between them may be found from the above table by taking the algebraic difference between the voltages set opposite the two elements in question. In any electrolytic cell, the voltage between the electrodes will then be equal to this voltage plus or minus the resistance drop in the electrolyte. The plus sign will be used if the current is caused to flow in a direction opposite to the chemical voltage and the minus sign if the current is in the reverse direction. The element highest up in the table will be positive and hence the chemical voltage will be in a direction toward this terminal.

A certain care must be used in employing this table to make sure that the element used is actually the one forming

the surface of the electrode. Suppose that two nickel electrodes are dipped in a solution of zinc sulphate and a current is passed through the combination. At the first instant there will be no chemical voltage, for the electrodes are both of nickel. After the current has flowed for a very short time, however, this will no longer be the case. In any solution of zinc sulphate there will be some positive ions $(Zn)^{++}$ and some negative ions $(SO_4)^{--}$.

The positive zinc ions arriving at the cathode will be discharged and become atoms of zinc which will adhere to the cathode. The cathode will soon become covered with a plated layer of zinc and we shall have the zinc-nickel chemical voltage of 0.55 volt opposing the flow of current.

In many cases, a gas such as hydrogen will collect at one of the electrodes. This gas in a thin layer completely covers the electrode below the surface of the liquid. We then have practically a hydrogen electrode as far as the chemical voltage is concerned.

Prob. 1–5. Describe how to make a normal solution of sulphuric acid: a one-tenth normal solution.

Prob. 2–5. A normal solution of H_2SO_4 has a conductivity of 0.198 mho-centimeter at 18° C. A $\frac{1}{10}$ normal solution has a conductivity of 0.0225, a $\frac{1}{100}$ normal solution 0.00308, a $\frac{1}{1000}$ normal solution 0.000361, all under the above conditions. Make a table and curve showing the relation between concentration (in percentage normal) of sulphuric acid and the "equivalent conductivity" — that is, the conductivity divided by the concentration.

Prob. 3–5. From the curve of Prob. 2–5, what statement can be made in regard to the dissociation of sulphuric acid in aqueous solution?

Prob. 4–5. By using the curve of Prob. 2–5, find the conductivity of a solution formed by adding 150 grams of sulphuric acid to one liter of water.

Prob. 5–5. If the solution of Prob. 4–5 were used between two storage-battery plates 6 inches square and separated

$\frac{1}{8}$ inch, what would be the resistance of the electrolyte between the plates?

49. Quantity Relations. Faraday discovered that when a current of electricity is passed through an electrolyte, the amount of chemical action produced is proportional to the quantity of electricity passed. That is, for example, if we are plating nickel, the amount of nickel deposited is proportional to Q, where

$$Q = IT, \tag{6}$$

provided an unvarying current I flows for a time T. If the current i is not constant, the quantity of nickel deposited is proportional to Q, where

$$Q = \int_o^T i\, dt. \tag{7}$$

This fact is made use of in the U. S. legal definition of an **ampere** current of electricity, see p. 19.

He also discovered that the **weight of metal deposited** or of gas liberated at an electrode by a given amount of electricity is **proportional to the atomic weight** of the substance deposited or liberated, provided the valence is the same in each case. The valence is the number of hydrogen atoms with which one atom of the substance in question forms a stable compound.

Thus an ampere flowing for one hour in a nickel plating bath will deposit 1.09 grams of nickel, while the same current in the same length of time will deposit 1.22 grams of zinc, the solution being of a bivalent salt in each case. The ratio 1.09 : 1.22 is the same as the ratio 58.7 : 65.4 of the atomic weights of the two metals considered. If the valence is different for the ions, the amount deposited is proportional to the atomic weight divided by the valence.

These laws may all be summed up in the single expression

$$m = \frac{w\,Q}{96{,}540\,n} \text{ grams,}$$

where

- m is the mass in grams deposited or liberated,
- Q is the quantity of electricity in coulombs passed through the solution,
- w is the atomic weight of the element under consideration,
- n is the valence of the element.

The constant

$$\frac{w}{96{,}540\, n} \tag{8}$$

is called the **electrochemical equivalent** of the given element. It is the mass of the element in grams deposited by one coulomb.

The constant 96,540 was determined experimentally. It is the number of coulombs necessary to deposit or liberate an amount of a univalent element in grams numerically equal to its atomic weight, that is, a "gram-atom" of the substance. Thus, for instance, it is the amount of electricity necessary to liberate one gram of hydrogen at the cathode in decomposing water. The same amount of electricity will, of course, liberate $\frac{16}{2}$ or 8 grams of oxygen at the anode, since the atomic weight of oxygen is 16 and the valency is 2. This large quantity of electricity is called a "faraday."

$$1 \text{ faraday} = 96{,}540 \text{ coulombs.}$$

It will be remembered that there are approximately

$$6.3 \times 10^{18}$$

electrons in a coulomb. Multiplying, we see that one faraday corresponds to

$$6.1 \times 10^{23} \text{ electrons}$$

moved along the circuit. If a univalent element is being dealt with, each electron moved through the circuit cor-

responds to one ion discharged and become an ordinary atom which may then enter into some chemical combination or become an atom of gas. The very large number of electrons given above is hence also equal to the number of atoms in a gram-molecule of a substance; for example, the number of hydrogen atoms in a gram of hydrogen gas. This number is therefore a very important physical constant.

The above rules may be applied also to combinations of atoms which appear as ions. Thus when sulphuric acid is dissolved in water, we have $(SO_4)^{--}$ ions. Their valency is 2, since each one combines with two hydrogen atoms. The molecular weight is equal to that of sulphur plus four times that of oxygen, or

$$32 + (4 \times 16) = 96.$$

The electrochemical equivalent is

$$\frac{96}{2 \times 96{,}540}$$

or approximately 0.0005. One coulomb of electricity hence discharges 0.0005 gram of (SO_4) ions at the anode when passed through such a solution. These discharged ions enter immediately into chemical combination, for SO_4 cannot exist uncombined.

In applying the above formula for quantity relations, great care must be used in several ways if correct results are to be obtained. Auxiliary reactions often take place and bring the results into error. The auxiliary reactions may be due to several causes. Several ions may be passing through the solution toward a single one of the electrodes at the same time. In this case the total number of ions passed will be governed by the above rule, but the rule does not indicate what will be the proportion of each kind. This is determined by the mobilities of the different ions. The rule must therefore be used with great care, and only

where there is a thorough knowledge of the chemistry involved.

Prob. 6–5. Zinc and copper strips are dipped in a solution of sulphuric acid to form a simple primary cell. What voltage will it give at first?

Prob. 7–5. After the current has flowed for a very short time in the above cell, one of the electrodes will become covered with a layer of hydrogen which will prevent the electrolyte from touching the metal underneath. What will then be the voltage given by this polarized cell?

Prob. 8–5. In a plant for refining copper an electric current is passed through a solution of copper sulphate for twenty-four hours. The amount of copper deposited in this time is 2.65 pounds. What average current was used?

Prob. 9–5. Two electro-plating vats are arranged in series; one is for nickel plating and the other for silver plating. If 3.26 ounces of silver are deposited in a given time, how much nickel is deposited in the same time?

Prob. 10–5. How many electrons per second are moved when one ampere current flows?

50. Primary Cells. If a strip of copper and a strip of zinc are dipped into a weak solution of sulphuric acid, we have a battery. The chemical voltage of the Cu-Zn combination of 1.10 volts, as taken from the table of Article 48, will be found to exist between the electrodes. The copper will be positive. If the two electrodes are connected by an external circuit, a current will flow in accordance with Ohm's law. The internal resistance of the cell will depend upon the size and position of the electrodes and the concentration of the electrolyte.

There are $(H)^+$ and $(SO_4)^{--}$ ions in the solution. When current flows, hydrogen will appear at the copper cathode in bubbles of gas in an amount determined by Faraday's laws. The $(SO_4)^{--}$ ions will arrive at the zinc anode, become discharged and immediately combine with atoms of zinc to form zinc sulphate, $ZnSO_4$, which will instantly dissolve in the solution. The amount of zinc dissolved will hence also be found by Faraday's law.

There are, however, several difficulties with this sort of cell. First, it polarizes. That is, after the current has been allowed to flow a short time, the copper becomes covered with a layer of hydrogen, and we have only the chemical voltage of an H–Zn combination, about 0.77 volt. A greater disadvantage is the fact that the layer of gas increases the internal resistance of the cell until it becomes very high.

Second, the zinc will be dissolved by what is called **local action,** which takes place in the following manner. Different parts of the zinc are of different purity. These form small **local** short-circuited electrolytic cells, which cause the zinc to be rapidly attacked. This is, in fact, the way in which metals are ordinarily **corroded.**

Third, the liquid is easily spilled.

The usual form of commercial primary battery is the so-called "dry cell." Polarization is prevented by including

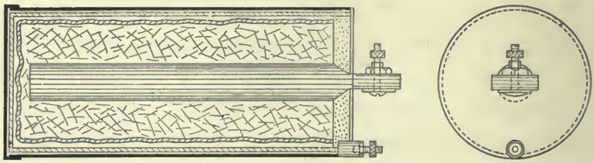

FIG. 46. The cross-section of a dry cell.

certain chemicals, such as manganese dioxide, which combine with or absorb the gases as fast as they are formed. Carbon and zinc electrodes are used, the zinc forming the container for the electrolyte. Local action is prevented by using certain zinc alloys which are very homogeneous. The electrolyte, usually ammonium chloride, is absorbed in blotting paper or plaster paris, and the whole is hermetically sealed with a tar compound to prevent spilling. A section of such a cell is shown in Fig. 46.

Such cells are convenient where small amounts of power are required in easily available form, as for door bells, flash lights, gas-engine ignition and so on. When any large

quantity of electric energy is needed, it is too expensive to obtain it in this manner. A short computation of the cost of zinc alone will show this.

Assume a dry cell to give 1.5 volts terminal voltage. To obtain one kilowatt-hour from dry cells of this sort connected in parallel would require

$$\frac{1000 \times 3600}{1.5} = 2.4 \times 10^6 \text{ coulombs.}$$

Such a quantity will, in accordance with Faraday's laws, require the consumption of a quantity of zinc in grams equal to

$$m = \frac{2.4 \times 10^6 \times 65.4}{96{,}540 \times 2} = 810 \text{ grams}$$

or

$$1.8 \text{ pounds.}$$

At 30¢ a pound for zinc, the energy thus obtained would cost 54¢ per kilowatt-hour for the zinc consumed alone. Other costs, such as that of fabrication, would be even larger.

The electrical energy obtained comes from the conversion of the chemical energy released in combining the zinc with sulphuric acid to form zinc sulphate. Energy obtained in this way from a metal is thus inherently expensive, for it costs more to form sulphates from metals than to form oxides from coal.

Prob. 11–5. The zinc plate of a certain battery cell weighs 3.52 ounces. How many hours will the battery cell last if it is required to deliver an average current of 2.14 amperes?

Prob. 12–5. A cell is desired which will deliver an average of 0.08 ampere for four months. What weight should the zinc plates have?

Prob. 13–5. (*a*) How many faradays of electricity are moved in the cell of Prob. 12–5? (*b*) How many electrons?

51. Storage Batteries. A storage cell is similar in action to a primary cell, except that the chemical actions involved

are completely reversible. After the cell is discharged, the passage of current in the reverse direction restores the electrodes and the electrolyte to their original conditions.

If there were no secondary chemical actions, the same number of ampere-hours would need to be passed through on charge as were taken out on discharge. Practically, a somewhat greater quantity of charge is necessary. An ampere-hour is 3600 ampere-seconds or 3600 coulombs. Storage batteries are usually rated in ampere-hours to give the quantity of electricity which can be taken from them without discharging them below a safe condition.

There are two commercial forms of storage batteries, the lead battery and the nickel-iron battery. Each has its particular field of usefulness. The lead battery is capable of passing heavier currents for a short period and is hence used for automobile starting. It is of lower internal resistance. At the present time the lead cell is cheaper in first cost and is extensively used for stationary power batteries, in direct-current power and lighting service, for absorbing the peak load and for short stand-by service in case of break-down of generators. The principal nickel-iron or alkaline battery is the Edison battery. Its uses have not as yet been completely developed. On account of lightness and the ability to withstand shocks, it is extensively used as a vehicle battery.

The lead battery illustrated in Fig. 47 has plates of lead peroxide, positive, and metallic lead, negative. The electrolyte is dilute sulphuric acid. It will be noted that neither of the plates is soluble in the electrolyte. The lead peroxide, PbO_2, is moulded into a grid of lead in the "pasted" type of cell, and is formed in place chemically in the "Planté" type. The negative plate is composed of lead in a spongy condition so that it offers a very large surface to the electrolyte.

On discharging the battery the materials of both plates are

converted to lead sulphate. The simplest chemical reaction is

$$PbO_2 + Pb + 2\,H_2SO_4 \rightleftarrows 2\,PbSO_4 + 2\,H_2O. \qquad (9)$$

This may be read either way, to the right for discharge and to the left for charge. As a matter of fact, this simple equation does not completely express all that takes place,

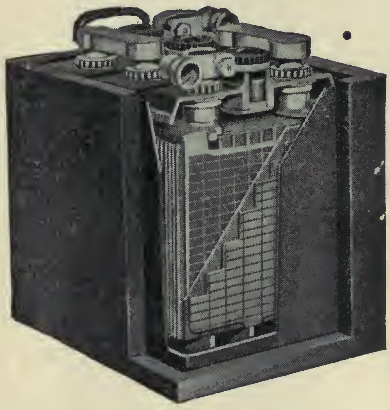

FIG. 47. Cut-away view showing the construction of a lead storage cell. *Electric Storage Battery Co.*

and the chemistry of the cell is quite complicated, especially in the presence of impurities.

The water formed on discharge further dilutes the electrolyte. For this reason the specific gravity of the electrolyte as measured by a hydrometer is an accurate measure of its condition, provided the concentration is properly adjusted when the battery is fully charged.

The voltage delivered by a lead battery varies with its charge. When fully charged each cell will deliver about 2.1 volts on normal load. When this has dropped to about 1.8 volts, the battery is considered fully discharged. Overdischarge beyond this point is likely to form sulphate in such form that it cannot be converted back to active ma-

terial on charge. This is called "sulphating" of the battery. On charge the voltage is higher than on discharge, partly on account of RI drop in the electrolyte and partly on account of secondary chemical reactions. It rises from about 2.1 to 2.6 volts per cell. Typical charge and discharge curves are shown in Fig. 48 for normal current in each case. Since the current is constant, the efficiency of

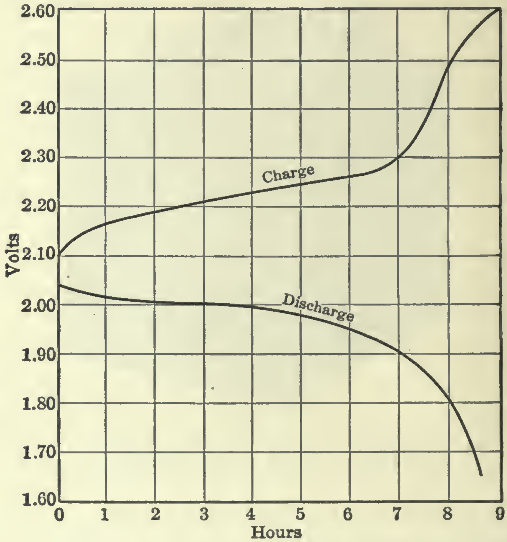

FIG. 48. Curves for a lead cell showing the relation between terminal voltage at normal current and time of charge and discharge.

the battery may be found by dividing the area under the discharge curve by the area under the charge curve.

Large plates placed very close together are used to keep down the internal resistance, which is remarkably low. On short circuit several hundred amperes may be drawn from a single cell of forty-ampere-hour size. An excessive current of this sort will, however, be likely to cause the plates to buckle or to sulphate on the surface. This last

effect is due to the fact that on heavy currents the electrolyte in the interior of the plates becomes very dilute, and does not have time to become the same by diffusion as the main body of the liquid.

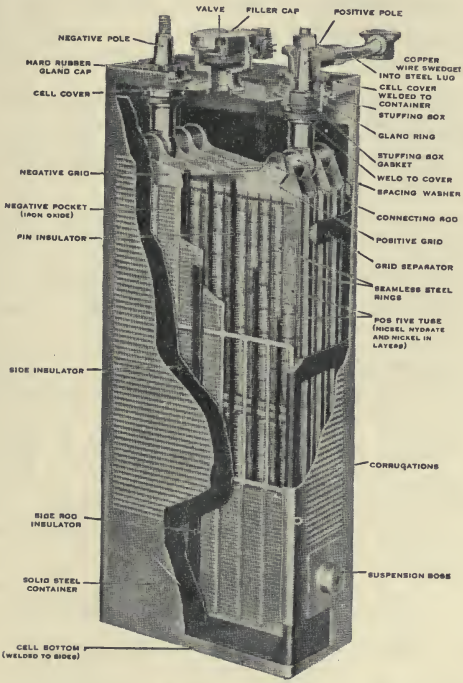

FIG. 49. Cut-away view of an Edison storage cell. *Edison Storage Battery Co.*

The weakness of lead structurally and its great weight gave considerable difficulty in the construction of plates that would not sag or buckle, and that would retain the active material in good electrical contact. Great ingenuity has been shown in the form of lead grid used to make plates.

The alkaline cell is principally the outcome of attempts to overcome the weight and mechanical weakness of the lead plate. The Edison cell illustrated in Fig. 49 uses elec-

trodes of nickel peroxide for the positive plate and finely divided iron for the negative. The electrolyte is a 21% solution of potassium hydroxide, with some lithium hydrate. The iron has a small amount of mercury added to it to render it more readily active, and there are flakes of nickel added to the nickel peroxide to increase the conductivity.

The chemistry of this cell is very complicated and not entirely understood. The products of discharge are iron oxide and a further oxide of nickel. The principal reaction is

$$5\,Fe + 5\,Ni_2O_3 \cdot 6\,H_2O + 9\,H_2O \rightleftarrows 10\,Ni(OH)_2 + 5\,Fe(OH)_2. \quad (10)$$

Read from left to right for discharge and from right to left for charge. The electrolyte is not changed in composition or in concentration by the action and hence the specific gravity is not an indication of the state of charge. The way in which the electrolyte enters into the action without becoming changed is as follows, expressed in simple form. The ions in the solution are K^+ and $(OH)^-$. The positive plate is that by which the current enters on charge and leaves on discharge. On discharge the K ions move toward the positive plate. Then

$$4\,K + 3\,NiO_2 \rightarrow Ni_3O_4 + 2\,K_2O. \quad (11)$$

At the other plate the (OH) ions on arriving give

$$8\,(OH) + 3\,Fe \rightarrow Fe_3O_4 + 4\,H_2O. \quad (12)$$

The K_2O is unstable and immediately after forming, decomposes water of the solution, thus:

$$4\,K_2O + 4\,H_2O \rightarrow 8\,KOH. \quad (13)$$

Thus the electrolyte is left unchanged. The charge and discharge curves and the voltages obtained are shown in Fig. 50.

One intermediate reaction is needed to explain several phenomena peculiar to this cell.

On charge, NiO_2 is first formed which gradually decomposes into the lower oxide Ni_2O_3 according to the equation

$$4\,NiO_2 = 2\,Ni_2O_3 + O_2 \qquad (14)$$

giving off the oxygen. This decomposition accounts for the continual gassing which takes place during charge and even continues for considerable time after charging is discontinued.

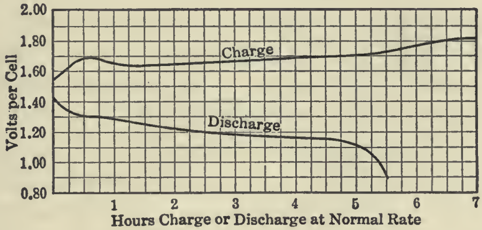

FIG. 50. Curves for an Edison cell showing the relation between terminal voltage at normal current and time of charge and discharge.

If a cell is allowed to stand idle after a charge, it will take one or two days for the higher oxide to completely decompose into Ni_2O_3. This reduction is much hastened by discharge, since any NiO_2 existing in the positive plate will be used up first. This rapid reduction of NiO_2 in discharge probably accounts for the marked drop in voltage which takes place at the beginning of the discharge of a freshly charged cell. It also accounts for the higher capacity obtained when a cell is discharged immediately after being charged.

Prob. 14–5. A lead storage cell has an internal resistance of 0.002 ohm. What will be the difference in its terminal voltage in charge and discharge at 40 amperes?

Prob. 15–5. A 280–ampere-hour lead storage vehicle battery has an average voltage of 32 volts. How much energy can be drawn from it?

Prob. 16–5. If the average voltage required during charge of the battery of Prob. 15–5 is 40 volts and the time of charge 10% longer than the time of discharge at normal rate, what is the efficiency of the battery?

Prob. 17–5. The vehicle of Prob. 15–5 requires six horse power output to drive it at 20 miles per hour and the motors are 60% efficient. (*a*) How far can it be driven at this speed on one charge? (*b*) What is the cost of energy per mile at 3¢ per kilowatt-hour?

52. Electrolytic Refining of Metals. If both electrodes are of the same metal, there is very little chemical voltage present. The only electrical energy used in passing a current through the combination is that lost in heating the electrolyte. There will always be a minute chemical voltage due to differences in purity of the metals and to differences in concentration in the parts of the solution.

In the electro-deposition of metals for plating articles, the cathode is always the article to be plated, the electrolyte a dissolved salt of the metal to be plated and the anode a piece of the same metal or an inert conductor. In the latter case there is a chemical voltage and the electrolyte must be renewed from time to time. There are many "kinks" to good plating technique. Voltages from one to six are usually used.

Copper, nickel and other metals are refined by plating them. Electrolytic copper thus prepared is very pure and is used for wires, sheets and many other purposes where even small impurities are undesirable.

In copper refining, the electrolyte is copper sulphate. The impure metal is used as the anode and a thin sheet of pure copper is used as a starter at the cathode. A small amount of common salt is used to precipitate any silver which dissolves from the anode and the silver thus recovered pays a large part of the costs of the process. Over a million tons of copper are annually refined in this manner. The impurities are insoluble in the solution or are made so by the addition of certain chemicals, and thus do not plate out but appear as a mud in the bottom of the tank. The copper obtained is about 99.95% pure, the chief impurity being hydrogen dissolved in the metal.

The electrolyte is used hot in large cells which are usually connected in series to give a convenient voltage to handle and generate. A current density of about 20 amperes per square foot of cathode is used. From Faraday's laws, about 0.0026 pound of copper is deposited per ampere-hour. Thus to deposit a ton of copper requires 770,000 ampere-hours, or for $\frac{1}{10}$ ton per cell per day, about 3200 amperes. If this were delivered at 100 volts for a long series of cells, a 320-kilowatt generator would be necessary. This would supply about 50 cells, giving 5 tons of metal per day. Several series may be used on one large generator. If the above electrical energy is estimated at two cents per kilowatt-hour, the cost per pound for energy alone is 0.15 cents per pound. A considerable fraction of the cost of electrolytic copper refining, strange to say, is the interest on the investment in copper tied up in the process.

Prob. 18–5. How long must a current of 500 amperes run in an electro-refining vat to deposit enough copper to make one mile of No. 00 wire?

Prob. 19–5. Zinc-coated iron is commercially known as "galvanized iron." How thick a plate of zinc will be put on a sheet of iron having 40 square feet (both sides) if a current of 12 amperes runs for 16 hours through the zinc-plating solution?

Prob. 20–5. If the electric energy of Prob. 18–5 costs 1.25¢ per kilowatt-hour and the refined copper is worth 18¢ per pound, at what voltage must the refining vat be run to make the cost of energy equal 5% the cost of copper refined?

53. Electrolysis. Electrolytic processes are useful in many ways. They enter into the experience of an electrical engineer in one way which is not desirable. This is in electrolysis.

The earth is a conductor. Moist earth with some dissolved salts present acts as an electrolyte. The resistivity of this electrolyte is high, but the earth is so large that the total resistance of the earth between two points may be low. In fact, the resistance between two

plates buried in the earth will be found to be due almost entirely to the current paths near the electrodes and practically independent of the distance between the plates. The resistance of the ground return of a telegraph circuit will be in the neighborhood of twenty ohms, with well constructed " grounds " or connections to the earth.

Where a trolley system is operated with a rail return for the current, large stray currents in the earth will always be found. These do no harm except where they take short cuts through sections of metal. They do no harm here where they enter the metal, but where the current leaves, the metal is dissolved or electrolysed. Water pipes are particularly subject to difficulty in this regard and will soon be eaten through and leak if large currents pass. Great care must be used also that there is not electrolysis of the reinforcing rods of concrete structures, for the failure of the structure may follow. The electrolysis of metal pipes buried in the earth has been the cause of much litigation.

Stray currents from trolley systems may be largely minimized by the use of negative feeders, that is, of large conductors connecting to the rail system to carry the current from points out on the line back to the power house. These negative feeders may also be connected to pipes to protect them, but care must be taken here that more harm is not done than good, due to the resistance of leaded or gasket joints of high resistance in the pipe line.

The amount of metal dissolved by electrolysis with a given current may be computed roughly by Faraday's laws. Secondary reactions prevent great accuracy.

There is electrolysis even when alternating current flows, but it is usually less than 1% of what would be caused by the same amount of direct current. With alternating current the metal is dissolved on one half cycle and plated back on the next. Not all that is dissolved will plate back, even when the two half cycles of current are identical, due to diffusion of the salt formed and to secondary reactions.

With alternating current the difficulty will occur both where the current leaves and where it enters the pipe, for these conditions are reversed when the direction of the current changes.

FIG. 51. Nine 2500-kw. converters used to supply current for the production of aluminum. There are thirty-six similar machines installed in the Marysville, Tenn. Plant of the Aluminum Co. of America. *Westinghouse Electric and Mfg. Co.*

54. Other Electrochemical Processes. Electrochemistry is a profession by itself. It is rapidly becoming of fundamental importance in our complex life. A large number of chemical processes are now conducted by the aid of electricity and can only be given passing mention here.

Fused salts in place of salts dissolved in water may be decomposed by electrolytic action. This often requires the high temperature obtained from the electric furnace. In other cases, moderate temperatures may be used.

Aluminum is manufactured in this manner by electrically decomposing an ore of aluminum such as bauxite, H_3AlO_3,

Aluminum requires the largest amount of electricity per pound of any metal obtained commercially by electrolysis. Fig. 51 shows part of a battery of thirty-six 2500-kilowatt converters used to supply current to one plant producing aluminum. The cost of aluminum is largely determined by the cost of electric energy. Metallic sodium and potassium are similarly produced, but by the use of much higher temperature.

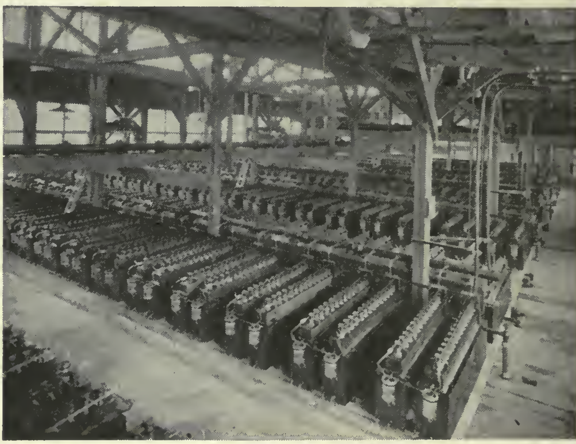

FIG. 52. Cell room showing six banks of 74 cells each. United States Government Chlorine-Caustic Soda Plant, Edgewood Arsenal, Md. *Hon. Benedict Crowell in "America's Munitions."*

Gaseous hydrogen and oxygen are commercially produced in one process by the electrolytic decomposition of water.

The fixation of atmospheric nitrogen for the production of fertilizers and explosives is of vast importance to the agricultural and military interests.

Carborundum, calcium carbide, fused quartz, silicon, graphite, phosphorus and steel alloys of various sorts are typical important electric-furnace products.

Immense quantities of caustic soda and chlorine are now

produced commercially from the electrolysis of sodium chloride solution in specially constructed cells. Fig. 52 shows one room of a chlorine plant at Edgewood, Md., which produced $12\frac{1}{2}$ tons of gaseous chlorine per day. It may safely be said that the electrochemical industries will expand rapidly within the next decade; in fact, their growth is limited only to the extent of future electric-power development.

SUMMARY OF CHAPTER V

AN ELECTRIC CURRENT FLOWS THROUGH A LIQUID because there are ions present in the liquid. Such a liquid is called an ELECTROLYTE.

A NEGATIVE ION is an atom with a greater number of electrons than normal attached.

A POSITIVE ION is one deficient in electrons.

AN ELECTRON is a particle of negative electricity, the smallest particle into which it is possible to divide electricity.

A CURRENT OF ELECTRICITY really consists of a flow of these negative particles. The direction of this flow is the reverse of that which we are accustomed to call the positive direction of current flow.

ACIDS AND SALTS IN SOLUTION break up into positive and negative ions. The positive ions move through the liquid to the cathode or plate at which the electric current leaves the solution. The negative ions move in the opposite direction.

THE CONDUCTIVITY OF A LIQUID is proportional to the number of ions present and their mobility.

A DIFFERENCE OF ELECTRICAL POTENTIAL is set up between two different metals when they are dipped into an electrolyte which is capable of combining chemically with one of them. The amount of this potential difference depends solely upon what metals are used.

THE ELECTRIC POTENTIAL SERIES OF METALS is an arrangement based on the voltages which exist between the various metals if one terminal is hydrogen.

FARADAY'S LAW states that the weight of an element liberated from a solution can be found by the equation

$$m = \frac{wQ}{96{,}540\,n}\text{ grams,}$$

where

m = the weight in grams liberated,
w = the atomic weight of the substance,
Q = the coulombs of electricity passed,
n = the valence of the element.

THE ELECTROCHEMICAL EQUIVALENT of an element is the constant

$$\frac{w}{96{,}540\ n}$$

and is the weight in grams deposited by one coulomb.

ONE FARADAY $= 96{,}540$ coulombs
$= 6.1 \times 10^{23}$ electrons,

which is also the number of atoms in a gram-atom of a substance.

A PRIMARY CELL consists of two different metals in contact with an electrolyte which combines chemically with one of them. The metal consumed is generally zinc.

A STORAGE CELL is similar to a primary cell except that the chemical actions are entirely reversible.

THE LEAD STORAGE CELL consists of lead-peroxide positive plates, metallic lead negative plates and dilute sulphuric acid electrolyte.

THE EDISON CELL has nickel-peroxide positive plates, iron negative plates and 21% solution of potassium hydroxide electrolyte.

ELECTROLYTIC REFINING OF METALS is carried out on a large scale to secure pure copper. The process consists of electro-plating from a solution of copper sulphate and depends upon electrochemical action.

ELECTROLYSIS DOES DAMAGE to water pipes and reinforcing rods in concrete where any stray currents in leaving the pipes or rods flow through some electrolyte surrounding the pipes or rods.

A LARGE NUMBER OF CHEMICAL PROCESSES are now conducted by the aid of electricity. Aluminum, oxygen, caustic soda and chlorine are a few of the more common substances commercially produced by electrolytic action.

PROBLEMS ON CHAPTER V

Prob. 21–5. If the internal resistance of a bivalent nickel plating bath is 0.5 ohm, and 1 volt is applied, what energy in kilowatt-hours is used in plating one pound of nickel?

Prob. 22–5. If the voltage in the above problem is increased to 2 volts, what will be the new amount of energy per pound?

Prob. 23–5. Ten thousand articles each with a surface area of 1 square foot are to be plated with 0.003-inch thick nickel coat. Take the density of nickel as 8.6 grams per cubic centimeter. If 1 volt is used in the solution of Prob. 21–5 and the energy cost is 20¢ per kilowatt-hour, what is the energy cost per article?

Prob. 24–5. If the labor cost for plating bath attendance is 4¢ per article under the conditions of Prob. 23–5, and this can be reduced to 3¢ if the time of plating is halved, will it pay to use a 2-volt supply?

Prob. 25–5. Hydrogen under standard conditions of temperature and pressure weighs 0.09 gram per cubic foot. How long will it take 1000 amperes to liberate enough hydrogen by electrolytic decomposition to fill a balloon which is spherical and 40 feet in diameter?

Prob. 26–5. At a voltage of 1.5 and at 10¢ per kilowatt-hour, what is the cost of energy for filling the balloon of Prob. 25–5?

Prob. 27–5. Compare the energy costs of plating nickel and silver (assuming the resistance of the plating bath to be the same in each case) for equal weights deposited. For equal thicknesses on an article. (Density of silver is 10.5 grams per cubic centimeter.)

Prob. 28–5. A dry cell delivering 1.5 volts will supply 1 ampere intermittently for a total of 50 hours. The cell costs 30¢ new. What is the cost of power obtained in this way?

Prob. 29–5. An average per plug of 0.04 joule output of electrical energy is generally used by the ignition system to produce a proper spark at the spark plug of a gasoline engine. With an 8-cylinder 4-cycle engine running 3200 r.p.m., what is

the total input in watts to produce the sparks, assuming an efficiency of the ignition system of 20%?

Prob. 30–5. If the engine of Prob. 29–5 is supplied by cells of the type of Prob. 28–5, how long will 6 such cells last on continuous running?

Prob. 31–5. An iron water pipe receives a stray current of 5 amperes from a trolley system. Where this current leaves the pipe, electrolysis occurs. Assuming Faraday's law, how long will it take to remove 10 pounds of the iron of the pipe?

Prob. 32–5. A copper plate is riveted to the iron hull of a ship. The salt sea water forms an active electrolyte. Before polarization what voltage acts and in what direction? After the cathode is covered with hydrogen, what voltage acts?

Prob. 33–5. The anode is attacked when the current flows in a case such as in Prob. 32–5. Will the presence of the copper plate cause damage to the iron plates of the hull? What if the copper plate is replaced by one of zinc?

Prob. 34–5. Iron is protected against corrosion in the presence of moisture which may contain salts which render it active as an electrolyte by covering with a coating of zinc. This is done in several ways, as in the galvanizing process. Tin is also used. Compare the two as regards corrosion of the iron if there are pinholes in the coating.

CHAPTER VI

THE MAGNETIC CIRCUIT

Practically all electric power machinery depends for its operation upon the inter-relation between electricity and magnetism. The generator, the motor, the transformer, contain not only electric circuits but also magnetic circuits, and depend for their operation upon the mutual effects produced between the two.

55. Relation Between Electricity and Magnetism. When an electric current flows along a wire, there are evidences of its presence not only within the wire itself but also in the space around it. Inside the conductor, the electric current produces heating and chemical effects, but it produces magnetic effects in the space outside of the wire as well as within it. As early as 1820, Oersted discovered that a compass needle in the vicinity of a wire carrying current was deflected, and Ampere, upon hearing of this result, soon worked out the law underlying this effect. This law bears his name. It is principally to Faraday, however, that we owe the complete examination of the quantitative relationship between electric current and magnetism, for it was due to his painstaking experimentation and clear logic that most of the laws underlying present-day electrical engineering were discovered and put into form.

Without the knowledge of these various relations between electric and magnetic circuits, there would be no electrical engineering to-day. The physicist would have at his command only voltaic cells and weak permanent magnets. They would be largely toys, and powerful electrical machinery could not exist. It is only by the use of coils carrying electric current that powerful magnetic fields can be established,

such for instance as in the large lifting magnets which raise twenty tons at a load. By the use of these powerful fields, we can in turn generate large voltages and currents, that is,

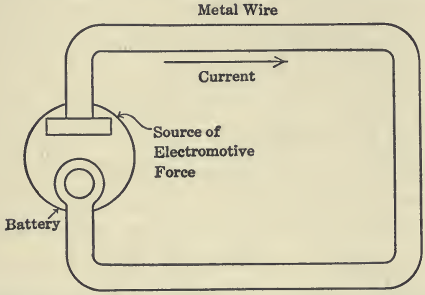

FIG. 53. The electromotive force of the battery causes a current to flow around the metallic circuit.

large amounts of electric power, making possible the 50,000-kilowatt turbo generator as we know it to-day. On the other hand, even such a delicate instrument as the telephone receiver depends for its action upon electromagnetic effects.

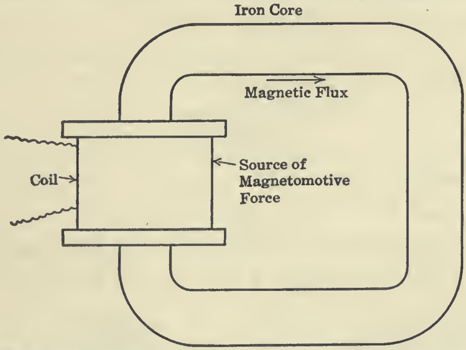

FIG. 54. The magnetomotive force of the coil sets up a magnetic flux in the iron circuit.

56. The Magnetic Circuit. Just as electricity or electric current is caused to flow in an electric circuit, so magnetism or magnetic flux can be set up in a magnetic circuit. In Fig. 53, the battery acts as a source of **electromotive force**

and forces an electric current through an electric circuit consisting of a metal wire. Similarly in Fig. 54, a coil carrying an electric current can be caused to act as a source of **magnetomotive force,** and to force magnetic flux through an iron core which constitutes a magnetic circuit.

It will be seen that in many ways these two circuits are similar and it is found that the laws governing them may be written in much the same form. There are, however, important differences, principal among which differences is the following. In the electric circuit, the current produces an effect in the wire such as heating, even when the current is absolutely steady. A magnetic flux, however, produces its principal effect when it is varying in the magnetic circuit, and in general does not make its presence felt when it is steady. A magnetic circuit may carry a steady magnetic flux indefinitely without producing any heating of the circuit, and consequently without any expenditure of energy, at least so far as the magnetic circuit itself is concerned. There are other important differences which will be brought out below.

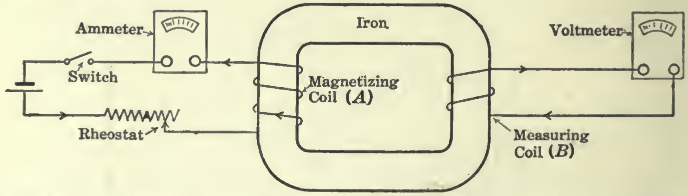

FIG. 55. A change of current strength in coil A will set up a voltage in coil B and cause the voltmeter to deflect.

57. Measurement of Flux. A magnetic circuit may be constructed by making a closed iron core and winding upon it a coil of wire. When a current is passed through this coil, as for instance by using a battery as shown in Fig. 55, magnetic flux will be set up in the iron. The presence of this flux may be made apparent in several ways, but it will be convenient for us to examine at this time one way

which is of principal importance. If we wind a second coil of wire on this same core and connect it to a voltmeter, with the zero position at the middle of the scale, the following effects may be observed. Although the two coils have no electrical connection whatever with each other, yet when the switch in coil A is closed, the voltmeter connected to coil B will suddenly deflect, thus showing that a voltage in a certain direction has been set up in the measuring coil. Since these two coils have no electrical connection, it is reasonable to suppose that the flux set up by the current in coil A has something to do with producing the voltage set up in coil B. When, however, a current is flowing steadily in the magnetizing coil A, the voltmeter in coil B will return to its zero position, and there will now be no indication of any effect in the core. If, however, the switch in the circuit of coil A is opened, the voltmeter will suddenly deflect in the opposite direction, thus showing voltage in the opposite direction from that of the first voltage. It thus appears that the flux produces an effect, namely, introduces a voltage into the measuring coil, **while it is in the process of changing,** — a voltage in one direction when flux is being set up and a voltage in the opposite direction when it is decreasing to zero.

We may observe these effects more exactly by keeping the battery switch closed and utilizing a rheostat and an ammeter in the battery circuit. We shall find that when the ammeter reads steadily at any position, the voltmeter will remain at the center of its scale. When the ammeter shows that the current is increasing, the voltmeter will deflect in one direction; and when the ammeter shows that the current is decreasing, the voltmeter will deflect in the opposite direction. By operating the rheostat at various rates, we can make the current change quickly or slowly. It will be found that the voltage produced will be exactly proportional to the rate at which the flux in the magnetic circuit changes. This is the same, except for certain effects

to be noted below, as the rate at which the current in the magnetizing coil changes. It is natural, therefore, for us to use the voltage thus produced in a measuring coil to measure the amount of magnetic flux produced in the iron magnetic circuit by the magnetizing coil around it.

Since it is hard to make accurate measurements of the rate at which the current is varying at given instants as outlined above, it is more convenient to use a ballistic galvanometer for measuring the amount of flux in the magnetic circuit. The ballistic galvanometer is simply a galvanometer with a very long period. If a voltage is applied to such an instrument for a short interval and then removed, the deflection of the galvanometer will be proportional to the product of the voltage applied and the length of time that it is connected. The reason for this effect will be more apparent when we have studied the theory underlying electrical instruments, but for the present we may simply assume that the ballistic galvanometer can be used to measure **in volt-seconds** the amount of a sudden impulse applied to it.

If the voltage applied to a ballistic galvanometer is varying during the short interval that it is applied, the deflection of the galvanometer will be proportional to the integral of the voltage taken over the interval of time; that is,

$$d = k_1 \int e \, dt, \tag{1}$$

where d = the deflection of the galvanometer, t = the time in seconds and k_1 = the constant of the galvanometer. We have seen above that the voltage produced by the changing flux in the magnetic circuit is proportional to the rate at which the flux changes; that is,

$$e \propto \frac{d\phi}{dt} \text{ or } e = k_2 \frac{d\phi}{dt}, \tag{2}$$

where ϕ = the magnetic flux in maxwells and k_2 = the proportionality factor. Then the deflection of a ballistic

galvanometer connected to a measuring coil about a magnetic circuit will be proportional to

$$d = \int k \frac{d\phi}{dt} dt, \tag{3}$$

or simply

$$d = k\phi, \tag{4}$$

where k is a constant combining k_1 and k_2. That is, the galvanometer deflection is a measure of the total amount by which the flux changes in the magnetic circuit. Thus, if we connect a ballistic galvanometer as shown in Fig. 56, putting the galvanometer in place of the voltmeter, and suddenly change the amount of flux in the magnetic circuit by varying the battery, the deflection of the galvanometer will be proportional to the total amount by which the flux is changed. The deflection which occurs when the flux in the circuit is first set up may be taken as the measurement of the amount of flux in the circuit.

As the c.g.s. unit of magnetic flux we take that amount of flux which, when established in the magnetic circuit, will produce **one abvolt-second** in a coil of one turn wound on the magnetic circuit, and we call this unit of flux a **maxwell.** The number of maxwells of flux in a magnetic circuit may thus be measured by measuring the abvolt-seconds indicated by a ballistic galvanometer connected to a measuring coil around a circuit when the flux is set up or decreased to zero, the measuring coil being of one turn. It is more convenient in practice to use a measuring coil of several turns (an equal voltage being set up in each turn, of course) and divide the resulting deflection of the galvanometer by the number of turns used. It is also better, for reasons explained below, to use an arrangement such as is shown in Fig. 56 and reverse the magnetizing current by means of a reversing switch. The deflection of the galvanometer when the reversing switch is thrown from one side to the other is thus due to a change of flux from a maxi-

mum in one direction to a maximum in the other direction. The amount of flux as computed from the galvanometer deflection should be divided by two, in order to give a cor-

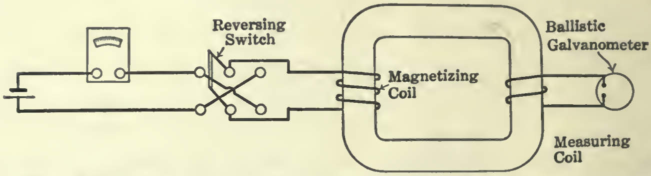

FIG. 56. Reversing the switch in the circuit of the magnetizing coil causes a voltage to be set up in the measuring coil.

rect result for the maximum amount of flux in the magnetic circuit.

Prob. 1–6. The flux through a coil of one turn is increased gradually from zero to 150,000 maxwells in two seconds. What is the average voltage produced in abvolts? In volts?

Prob. 2–6. If the coil of Prob. 1-6 has 200 turns, what average voltage will be produced?

Prob. 3–6. If the voltmeter in Fig. 55 shows a voltage of 2 millivolts, and the coil to which it is connected has 1000 turns, at what rate is the flux changing in the iron core?

58. Flux Lines. Euler, in his "Letters to a German Princess," written in 1761, describes a magnet and speaks of the "lines of flow of the magnetic fluid." It was from this sort of description of the magnetic effect in a magnetic circuit that we have obtained the terms "magnetic flux" and "magnetic flux lines."

Just as it is convenient for us to map out the manner in which water flows through a nozzle (Fig. 57) by drawing lines to indicate the direction which the water takes at every point, so it is convenient for us to represent the path which the magnetic flux takes in a magnetic circuit by means of lines showing at every point its direction, (Fig. 58). We may also show the direction which an electric current takes at every point in an electric circuit by means of lines similarly drawn.

Faraday was the first to utilize this manner of describing the path taken by magnetic flux in computations, and to show the great benefit of thus representing magnetic flux.

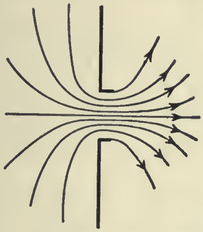

FIG. 57. The lines show the amount and direction of water flowing from an orifice.

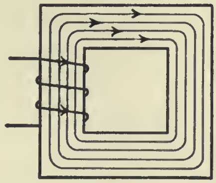

FIG. 58. The lines show the amount and direction of magnetic flux.

Suppose that in Fig. 57, in drawing the lines representing the flow of water through an orifice, we observe the following arrangement. Draw one line for each cubic inch per second of water flowing. That is, consider the stream of water to be made up of a large number of small streams flowing together and each carrying one cubic inch of water per second. Represent each of these unit streams by one line. The lines will then show not only the direction that the water takes, but also the amount of water flowing. The closer together these lines are situated, the more rapidly will the water be flowing at that point. The density of the lines, that is, the number of lines per square inch passing through a cross-section perpendicular to the stream, will, in fact, show the velocity of the water at any point in inches per second.

Similarly, in representing the magnetic flux in a magnetic circuit, we draw one line to represent a flux of one maxwell; in fact, we often abbreviate by saying so many **lines of flux** instead of saying so many **maxwells** of flux. The number of **lines per square inch** of cross-section, that is, the density of the lines, we take as a measurement of the **intensity** of the magnetic effect produced. A flux density of **one max-**

well (or one line) **per square centimeter** of cross-section is called a flux density of **one gauss.** In ordinary magnetic circuits constructed of iron, we employ flux densities of many thousands of gausses.

The lines thus used to describe the direction and intensity of magnetic flux do not have any physical existence, any more than do the stream lines describing the flow of water. They are simply a convenient means of describing the magnetic effect.

When electric current flows in an electric circuit, there is an actual transfer of electrons along the circuit. When magnetic flux flows in a magnetic circuit, on the other hand, we do not now believe that there is any actual flow of anything. This is indicated by the fact noted above, that although an electric current produces heating of the wire

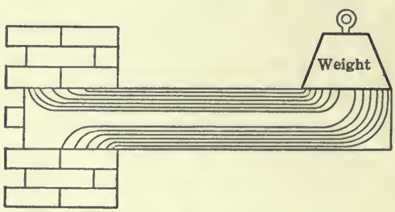

Fig. 59. The lines show the strains at various parts of the beam.

while it flows, magnetic flux, as long as it is steady, produces no heat or chemical effect in the iron through which it passes. **The magnetic flux** is more nearly in the nature of a **strain** set up in the material. The flux lines should be considered as being lines of strain due to stress. We might similarly indicate the direction and intensity of the strains in a beam under load as shown in Fig. 59, the horizontal lines representing the direction of the strains at the various points.

It is important to note that magnetic-flux lines are always

continuous. In other words, a magnetic-flux line can never have an end. This is true of the flow lines in a stream of water. For a flow line to end would mean that a certain quantity of water per second disappeared entirely at some point in the stream. In the same way, the magnetic-flux lines are always closed lines, and go completely around the magnetic circuit whatever its shape may be.

59. Magnetomotive Force. A voltaic battery introuced into an electric circuit provides an **electromotive force** which forces current around the circuit. Similarly a coil of wire carrying current produces a **magnetomotive force** which may force a magnetic flux around a magnetic circuit. We should note, however, one important difference. The voltaic battery is an integral part of the electric circuit, and produces its electromotive force right in the circuit itself by means of chemical action. On the other hand, a magnetomotive force in a magnetic circuit may be supplied by a coil of wire carrying current even though the coil does not physically touch the magnetic circuit at all. It is merely necessary that the coil of wire surround one part of the magnetic circuit, that is, that the coil and the magnetic circuit **interlink.**

The total magnetic flux, that is, the number of magnetic lines produced in a magnetic circuit, has been found experimentally to be proportional to the current in the magnetizing coil when the magnetic circuit does not contain iron or other magnetic material. The flux has also been found to be proportional to the number of turns of wire used in the magnetizing coil, that is, to the number of times the current in the coil links the magnetic circuit. The **magnetomotive force** of a magnetizing coil is hence proportional to the **product of the number of turns in the coil and the electric current flowing.** This assumes that the turns are all wound in the same direction. If a number of turns are wound on in the reverse direction, their effect will cancel the effect of an equal number of turns correctly wound.

The magnetomotive force is hence proportional to the **product of the electric current** and the number of **linkages** between the wire and the magnetic circuit.

60. Reluctance. The current flowing in an electric circuit is proportional to the electromotive force and inversely proportional to the resistance. This is Ohm's law. Similarly, the **amount of flux** in a magnetic circuit is **proportional** to the **magnetomotive force applied** and **inversely proportional** to what is called the **reluctance** of the circuit.

The resistance of a uniform electric circuit is found experimentally to be proportional to the length of the circuit and inversely proportional to the cross-section. The proportionality factor is called the resistivity of the material used, and its reciprocal the conductivity of the material. Similarly, the reluctance of a magnetic circuit is proportional to the length of the circuit and inversely proportional to its cross-section. Analogous to the **conductivity** of an electric circuit we have the **permeability** of a magnetic circuit. The conductivity depends upon the material used for making an electric circuit, its temperature and sometimes other effects. Similarly, the permeability of a magnetic circuit depends upon the material used and upon other very important factors to be outlined below. The conductivity of a given material is defined as the conductance (that is, the reciprocal of the resistance) of a unit cube of the material used in the electric circuit. In the same way, the permeability of a given material is the reciprocal of the reluctance of a unit cube of the material of a magnetic circuit.

The unit of resistance is the **ohm.** The unit of reluctance is the **oersted.**

61. Ohm's Law for Magnetic Circuits. For the electric circuit, we write

$$\text{current} = \frac{\text{electromotive force}}{\text{resistance}} \cdot$$

This may be abbreviated to

$$I = \frac{E}{R},$$

where, if we are using the c.g.s. system of units, I is the current in abamperes, E is the electromotive force in abvolts and R is the resistance in abohms. In the practical system of units, I is the current in amperes, E is the electromotive force in volts and R is the resistance in ohms. The resistance, we have seen, may be written

$$R = \frac{l}{\gamma A},$$

where l is the length and A is the cross-section of the magnetic circuit and γ is the conductivity of the material used.

Similarly, for the magnetic circuit, we may write

$$\text{flux} = \frac{\text{magnetomotive force}}{\text{reluctance}},$$

which may be abbreviated to

$$\phi = \frac{\mathcal{F}}{\mathcal{R}}, \tag{5}$$

where ϕ is the flux in **maxwells**, $\mathcal{F}$ is the magnetomotive force in **gilberts** and $\mathcal{R}$ is the reluctance in **oersteds**. These units are all in the c.g.s. system. In the same way, the reluctance may be written

$$\mathcal{R} = \frac{l}{\mu A}, \tag{6}$$

where l is the length and A the cross-section of the magnetic circuit and μ is the permeability of the material used.

We saw above that the magnetomotive force was proportional to the current and to the number of turns in the magnetizing coil.

Magnetomotive force, however, is not simply NI, where

N is the number of turns, but we define the magnetomotive force as

$$\mathcal{F} = 4\pi NI,$$

where $\mathcal{F}$ is the magnetomotive force in gilberts, N is the number of turns and I is the current in abamperes. If the current is expressed in amperes, the magnetomotive force in gilberts will be

$$\mathcal{F} = \frac{4}{10}\pi NI. \tag{7}$$

The proportionality factor, 4π, is introduced in this manner in order to make the values of permeability come out conveniently. We shall see below that if we define the magnetomotive force in the above manner, we can show that the permeability of air comes out unity. This is, of course, convenient in calculation. In selecting electrical units, we might have chosen a different-sized unit for the volt if we had wished; and if we had chosen correctly, the resistivity of copper might have been made to come out as unity. It would not have been worth while to do this with electrical units, for many different materials are used as conductors and their resistivities vary greatly. On the other hand, the permeabilities of almost all materials are practically identical with that of air, namely unity. There are only a few magnetic materials which have permeabilities differing greatly from unity; iron, the principal material used in practice, may have permeabilities of several thousand. It is convenient to have our unit of magnetomotive force, the gilbert, so chosen that the permeability of nearly all materials is almost exactly unity.

The flux flowing in a magnetic circuit which is interlinked by a magnetizing coil of a certain number of turns, N, and carrying a certain current, I abamperes, can accordingly be found from the formula

$$\phi = \frac{4\pi NI}{\dfrac{l}{\mu A}};$$

or if the current is expressed in amperes instead of abamperes, by

$$\phi = \frac{\dfrac{0.4\pi NI}{l}}{\mu A}. \tag{8}$$

The flux density, that is, the intensity of magnetization produced, can be found by dividing the total flux by the cross-section. This flux density is thus expressed in lines per square centimeter or lines per square inch. The density in lines per square centimeter is also called the density in **gausses.** The symbol B is always used to denote flux density. Thus

$$B = \frac{\phi}{A} = \frac{\dfrac{4\pi NI}{l}}{\mu} \text{ gausses.}$$

This last formula may be written in the form

$$B = \frac{4\pi NI}{l}\mu \text{ gausses.} \tag{9}$$

The quantity $$\frac{4\pi NI}{l} \text{ or } \frac{\mathcal{F}}{l}$$

is the magnetomotive force per unit of length of the circuit. It is thus expressed in **gilberts per centimeter.** It is analogous evidently to the potential gradient along an electric circuit. This quantity, called the **magnetizing force,** has received the letter H to designate it. We thus have

$$B = \mu H; \tag{10}$$

that is, the flux density is equal to the magnetizing force times the permeability. This is analogous to the expression for the electric circuit that the current density is equal to the conductivity times the potential gradient. We shall make much use of this expression later.

Prob. 4–6. The magnetizing coil in Fig. 55 has 40 turns and the ammeter shows a current of 2 amperes. What magnetomotive force is acting on the magnetic circuit?

Prob. 5–6. If the core in Prob. 4–6 is of annealed steel of permeability 2500, 4 square inches in cross-section, and of 20 inches total length, what is the reluctance of the magnetic circuit?

Prob. 6–6. What flux will exist in the core under the conditions of Prob. 5–6?

Prob. 7–6. What is the magnetizing force and what is the flux density in Prob. 6–6?

Prob. 8–6. A cast-iron ring of square cross-section, 3 inches inside diameter, 4 inches outside diameter, and 0.5 inch thick, is wound with a coil of 200 turns. When a current of 2.7 amperes is passed through this coil, the ring carries a total flux of 8000 maxwells. (*a*) What is the permeability of the cast iron? (*b*) What is the average magnetizing force?

62. Reluctances in Series and in Parallel. When a magnetic circuit is uniform, its reluctance may be immediately computed, provided its permeability is known, from the expression

$$\mathcal{R} = \frac{l}{\mu A} \text{ oersteds.} \tag{6}$$

If the circuit is not uniform, however, we may still compute its reluctance, provided we know the permeability of each part, by the same process as was used in computing the resistance of a non-uniform electric circuit.

If two reluctances are connected in series, the total reluctance is the sum of the individual reluctances. Thus, in Fig. 60, the total reluctance of the magnetic circuit shown is

$$\mathcal{R} = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} \cdot \tag{11}$$

The same method may be used for any other combination of series reluctances we may wish to consider.

For two reluctances in parallel, the total reluctance is,

similarly, the reciprocal of the sum of the reciprocals of the separate reluctances; that is,

$$\mathcal{R} = \frac{1}{\dfrac{1}{\mathcal{R}_1} + \dfrac{1}{\mathcal{R}_2}}. \tag{12}$$

For the magnetic circuit shown in Fig. 79*a*, constructed of two concentric iron rings of dimensions and permeabilities indicated, the reluctance of the combination is found thus:

For the inner ring

$$\mathcal{R}_1 = \frac{2\pi 4}{1000 \times 3 \times 1.4} = 0.00599 \text{ oersted};$$

For the outer ring

$$\mathcal{R}_2 = \frac{2\pi 5}{2000 \times 3 \times 0.6} = 0.00874 \text{ oersted};$$

For the two rings in parallel

$$\mathcal{R} = \frac{\mathcal{R}_1\mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2} = 0.0035 \text{ oersted.}$$

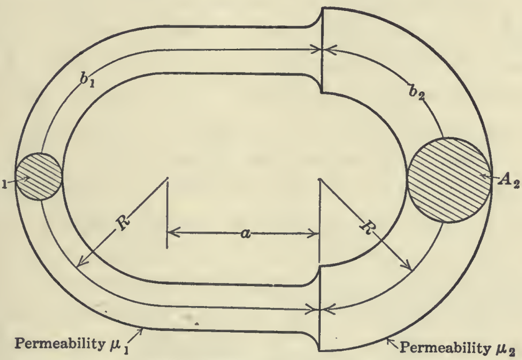

FIG. 60. A magnetic circuit made of different materials and having a non-uniform cross-section.

Prob. 9–6. The soft-steel piece of Fig. 60 has the following dimensions: $r_1 = \frac{5}{16}$ inch, $r_2 = \frac{1}{2}$ inch, $a = \frac{3}{4}$ inch, $R = 1\frac{1}{2}$

inches. The permeability of the part of smaller cross-section is 500 and of the larger cross-section part 1100. What is the total reluctance of this magnetic circuit?

Prob. 10–6. A magnetic circuit is composed of two parts in series. One part is of cast steel with a permeability of 1000 and has the following dimensions: total length, 21 inches; cross-section, 6 square inches. The other part is an air gap $\frac{1}{10}$ inch long and of 8 square inches cross-section. Find the reluctance of (*a*) the steel part of the circuit; (*b*) the air gap; (*c*) the complete circuit.

Prob. 11–6. If the air gap of Prob. 10–6 were put in parallel with the steel circuit, what would be the reluctance of the combination? Assume the permeability to remain constant.

Prob. 12–6. Make a sketch of each of the magnetic circuits in Prob. 10–6 and 11–6, marking the dimensions of parts and showing flux paths in each.

Prob. 13–6. If the flux density in the steel path in Prob. 11–6 is 9000 gausses, how many maxwells are there in the air gap?

63. Variation of Permeability. In solving an electric circuit, at least one made in the ordinary manner of metal wire, it is found that the resistance to be used is strictly constant, as long as the temperature of the circuit remains unchanged. If the temperature varies, we discover that the resistance also varies with it, but at any fixed temperature the resistance is constant. If, however, we attempt to solve an electric circuit including an electric arc, we soon find that Ohm's law in its simple form does not altogether hold. The resistance of the arc itself is not a fixed value but depends upon the amount of current flowing through it.

In the study of magnetic circuits, we are, in a similar way, soon impressed with the fact that the permeability of iron is not a fixed quantity, but depends upon the flux density at which the permeability is measured. This relation between flux density and permeability is not a formal one, but is experimental only, and hence is determined by actual tests on the iron.

In Fig. 62 is shown a set of curves which give the relation

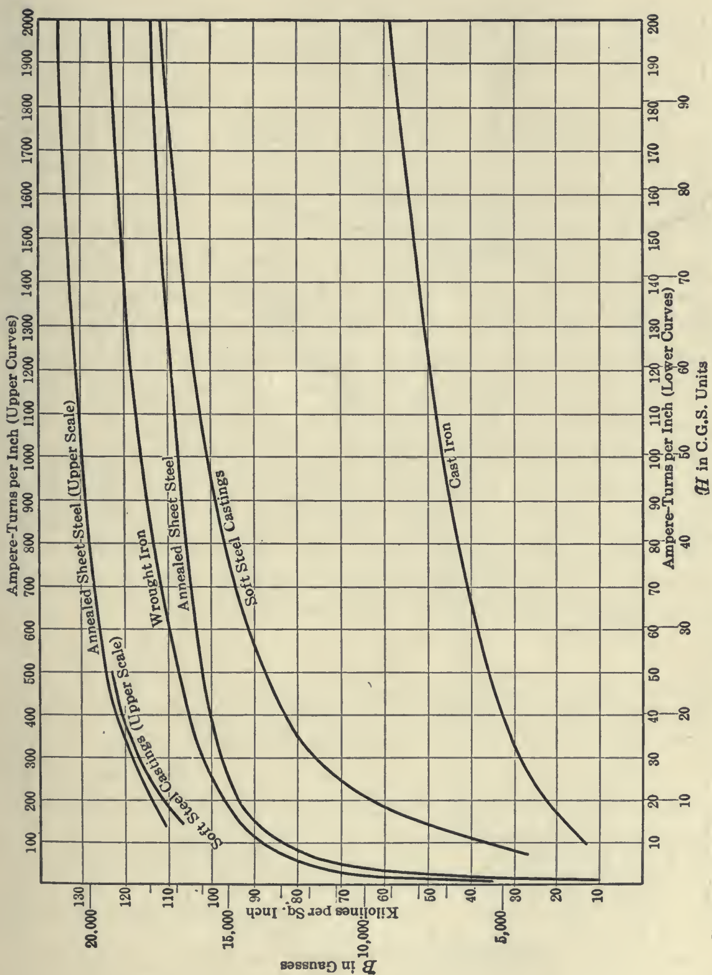

FIG. 61. Curves showing the relation between magnetizing force and flux density for various irons and steels.

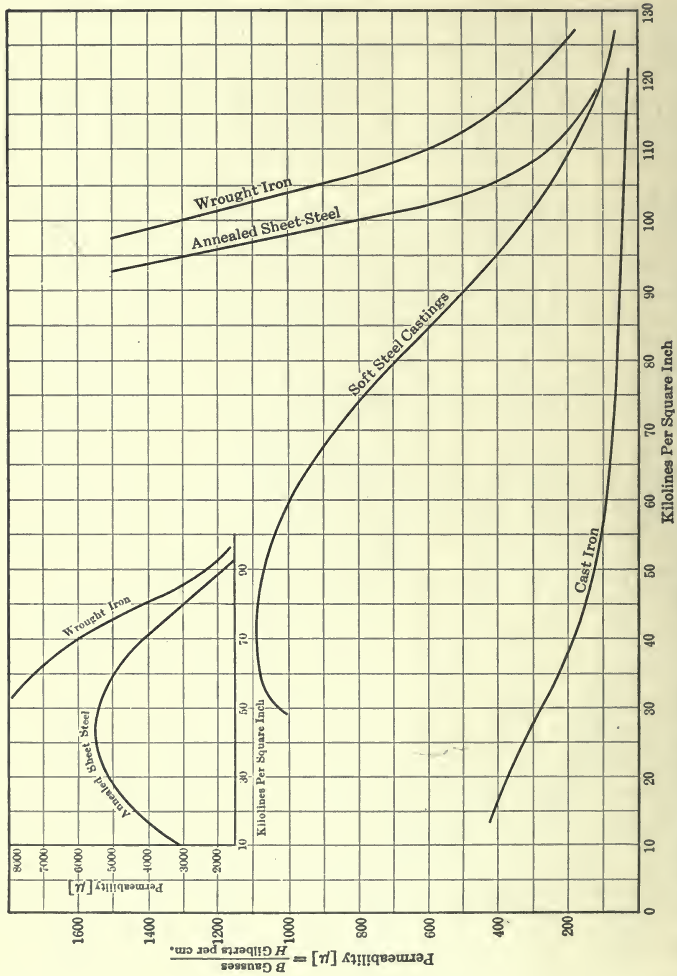

FIG. 62. Curves showing the change in permeability of various irons and steels when the flux density is varied.

between the permeability and the flux density of various materials. It will be noted that at low flux density the permeability of iron is fairly low, that there is a certain flux density at which maximum permeability is reached, and that above this flux density the permeability is decreasing. It is somewhat more convenient for work of computation to plot immediately the flux density against the magnetizing force. Such a curve is called the *B–H* curve of the iron used. In Fig. 61 is shown a set of curves thus plotted for various kinds of iron such as are usually used in magnetic circuits, the curves giving directly the flux density in gausses, when the magnetizing force is known in gilberts per centimeter. Such curve sheets will be found in handbooks, and will also be supplied by steel manufacturers in order to give accurate data concerning the electrical steel which they manufacture. For practical use, such curves are often plotted in other units, for instance, the flux density in lines per square inch against the magnetizing force in ampere-turns per inch. The flux density in lines per square inch is equal to 6.45 times the density in gausses. The gilberts per centimeter magnetizing force is 0.4π times the ampere-turns per centimeter, or $0.4\pi/2.54$ times the ampere-turns per inch.

Prob. 14–6. Find from the curves the permeability of sheet steel when the magnetizing force is 5, 10 and 15 ampere-turns per inch. What is the magnetizing force in gilberts per centimeter in each case?

Prob. 15–6. What magnetizing force is necessary to produce a flux density of 108,000 lines per square inch in soft cast steel? A density of 100,000 lines per square inch? Repeat for 66,000 and 58,000 lines per square inch. What is the flux density in gausses in each case?

64. Ampere-turns to Produce a Given Flux. The fact of the dependence of the permeability of iron upon the flux density often introduces a difficulty into magnetic-circuit calculations. We cannot apply Ohm's law for the

magnetic circuit unless we know the permeability of the material. We cannot tell what the permeability will be unless we know the flux density. One sort of problem is quite simple. For instance, if the total flux is given, we can compute the flux density, look up the corresponding value of permeability for the material used on the curve sheet, and then use Ohm's law for computing the required magnetomotive force and hence the number of turns and the current necessary in a magnetizing coil in order to produce this flux.

A simpler proceeding is to make use of the *B–H* curves and determine directly the ampere-turns per centimeter (*H*) necessary to produce the desired flux density (*B*). By multiplying this value (*H*) by the length of the magnetic circuit, the total number of ampere-turns (*NI*) is obtained.

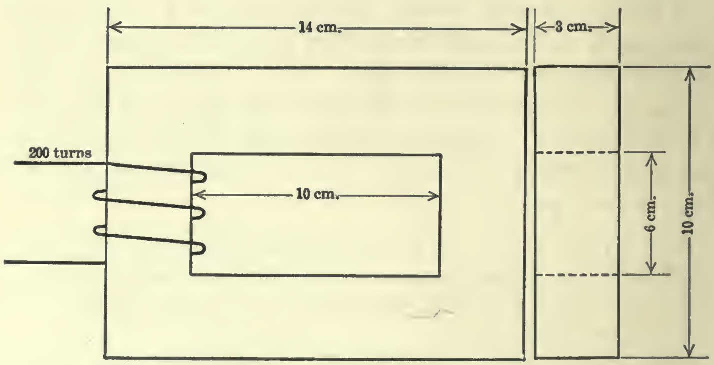

Fig. 63. A current in the coil will set up a magnetic flux in the iron circuit.

Referring to Fig. 63, suppose it is required to find the current necessary in the magnetizing coil of 200 turns in order to produce a total flux in the circuit of 60,000 lines. The core we will assume to be similar to that used in a transformer and built up of steel laminations. The cross-sectional area of the core is six square centimeters, and the

desired flux density is hence 10,000 gausses. Referring to the curve for sheet steel in Fig. 62, we find the corresponding permeability to be 4500. The mean length of the magnetic circuit is approximately 40 centimeters. Therefore the reluctance is

$$\mathcal{R} = \frac{40}{6 \times 4500} = 0.00148 \text{ oersted}$$

and the necessary magnetomotive force is

$$\mathcal{F} = \mathcal{R}\phi = 0.00148 \times 60{,}000 = 88.8 \text{ gilberts.}$$

The current flowing in the magnetizing coil must therefore be

$$I = \frac{\mathcal{F}}{0.4\pi N} = \frac{88.8}{0.4\pi\, 200} = 0.353 \text{ ampere.}$$

The same result may be obtained more directly as follows. From the B–H curve for sheet steel, Fig. 61, we find that for a flux density (B) of 10,000 gausses there are required 4.50 ampere-turns per inch of magnetic circuit. If the length of the path is 40 centimeters, the total number of ampere-turns is

$$NI = \frac{40}{2.54} \times 4.50 = 70.8 \text{ ampere-turns.}$$

The necessary current is then

$$\frac{70.8}{200} = 0.354 \text{ ampere.}$$

Prob. 16–6. Referring to Fig. 63, what current must flow in the coil if the flux is to be 50% greater than the value used in the above example?

Prob. 17–6. A certain magnetic circuit is made up of annealed-steel sheets. The length of the circuit is 2 feet and the cross-section is 5 square inches. How many ampere-turns are necessary in order to set up a flux of 500,000 maxwells in this circuit?

Prob. 18–6. (*a*) If the cross-section of the magnetic path in Prob. 17–6 were twice as great, how many ampere-turns would be needed to set up twice as many maxwells? (*b*) How do you account for your answer, inasmuch as there is twice as much iron being magnetized to the same flux density as in Prob. 17?

Prob. 19–6. The U-shaped magnet of Fig. 64 must have 100,000 maxwells in the air gaps at ends A and B. The core C of sheet steel has a mean length of 4 inches and cross-section

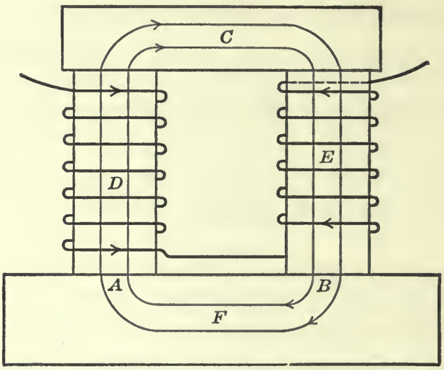

Fig. 64. A U-shaped electromagnet.

of 1 square inch. The pieces D and E are of cast steel, each having a length of 3 inches and a cross-section of $1\frac{1}{4}$ square inches. The length of the air gaps at A and B is 0.04 inch each, and the area of each gap may be taken as $1\frac{1}{2}$ square inches. The length of the magnetic path through the cast-steel piece F is 4 inches and the mean cross-section is $1\frac{3}{4}$ inches. How many turns must be wound on each of the cores D and E if the coils are to carry 0.45 ampere?

65. Flux Produced by a Given Number of Ampere-Turns. The reverse problem to those in the above article is also very simple when the magnetic circuit is uniform, as is the case, for instance, in the transformer core shown in Fig. 63. Suppose, for example, that it is required to find the flux which will be produced in this core when the magnetizing coil carries a current of 1 ampere. With this current

there are 200 ampere-turns in the magnetizing coil, which accordingly has a magnetomotive force of

$$0.4\pi\,200 = 251 \text{ gilberts.}$$

The magnetizing force is hence

$$\frac{251}{40} = 6.27 \text{ gilberts per centimeter.}$$

Referring to Fig. 61, this is found to correspond to a flux density of 13,300 gausses. The total flux produced in the core will thus be 13,300 times 6 or 79,800 maxwells.

When, however, the magnetic circuit is not uniform, it is necessary to resort to a cut-and-try process of solution in order to find the amount of flux which will be produced by a

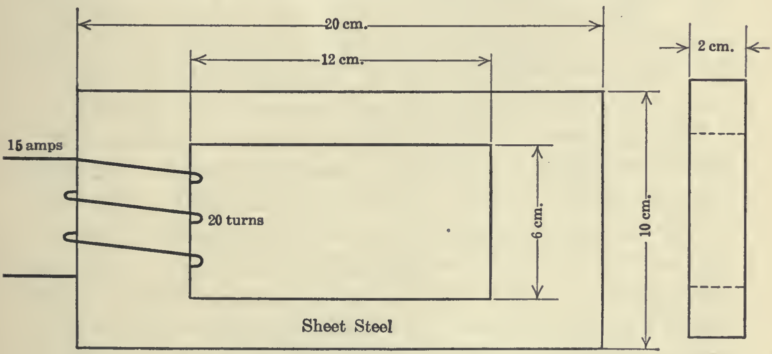

FIG. 65. The magnetic circuit has not a uniform cross-section.

given magnetomotive force. It will be best to illustrate what is meant by a problem. In Fig. 65 is shown again a simple transformer core, but in this case the cross-section of the core is not uniform. It is required to find the flux density produced by a magnetizing coil as shown. The magnetomotive force is evidently

$$0.4\pi\,20 \times 15 = 377 \text{ gilberts.}$$

The magnetic circuit can be divided into two parts, the first

being approximately 30 centimeters long and of 4 square centimeters cross-section, the second 14 centimeters long and of 8 square centimeters cross-section. In estimating these lengths, it is necessary to allow for a certain amount of rounding of flux at the corners of the core. The amount to be allowed in each case depends to a considerable extent upon what experience has shown to be correct for similar problems. A fairly accurate estimate can usually be made by laying out the core to scale and drawing in the probable flux paths.

Since there are two parts to the circuit and we do not know the permeability, it is now necessary to start at the other end and work back. Let us assume as a first guess that there is a total flux of 60,000 lines. Referring to our curve sheet, and multiplying out, we find that this corresponds to a magnetomotive force of 390 gilberts for the first part of the circuit and of 14 gilberts for the second part; that is, a total magnetomotive force of 404 gilberts.

This first guess was evidently too high, for we have 377 gilberts available. It is thus necessary to decrease our estimate of the flux lines. We should naturally make the next guess by decreasing the first guess approximately by a percentage of difference between the computed and available gilberts. In this manner we obtain as a second guess 59,000 lines for the flux. Computing as before we find that this corresponds to 374 gilberts. Since this is a close enough check to our available gilberts for ordinary purposes, we may consider the problem solved and the second estimate for flux as correct.

By using the above method of calculation, and bearing in mind always the analogous solution of an electric circuit, it is possible to solve even rather complicated magnetic circuits.

A type of problem whicn often comes up is such as is illustrated in Fig. 66, where the iron circuit is the core of a three-phase transformer. Suppose it is required to find

the total flux in leg A of this core, due to a known magnetomotive force applied by a coil as shown. It is necessary to use the cut-and-try process which may be outlined as follows. Assume a certain flux density in leg C. By the use of curves, as in the preceding problem, compute the magneto-

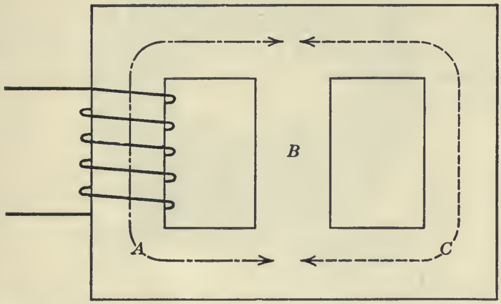

FIG. 66. The magnetic circuit of a three-phase transformer.

motive force necessary to force this flux density through a length of circuit shown by the dotted line in the figure. This same magnetomotive force may then be considered to be applied to core B, for as far as the magnetic circuit is concerned, the legs B and C are parallel paths for the flux, with the same magnetomotive force applied to each. Dividing this magnetomotive force by the length of the core B, we find the magnetizing force on B in gilberts per centimeter. The curves, Fig. 61, will then give us the flux density in B, and multiplying by the cross-sectional area we get the total flux in leg B. Adding the flux in legs B and C we have the flux in A. This gives us the flux density in A, and computing as before, we find the magnetomotive force for the part of the path shown by the dot-dash line. Adding together these two magnetomotive forces, we have the total magnetomotive force. If this corresponds with the applied magnetomotive force, then our estimate of the flux density was correct; otherwise the assumed flux den-

sity should be corrected in proportion to the error found in the computed magnetomotive force.

It will often be found in solving problems of this sort that an allowance may be made for the curvature of the magnetization curves which are being used, and the work thus shortened by rendering successive estimates more accurate. This should be clear from Fig. 67, for if we find that ϕ_C is too great and ϕ_B too small, and we take a mean, *OE*, of the magnetomotive force, the flux will be greater than our expectation by *FG*.

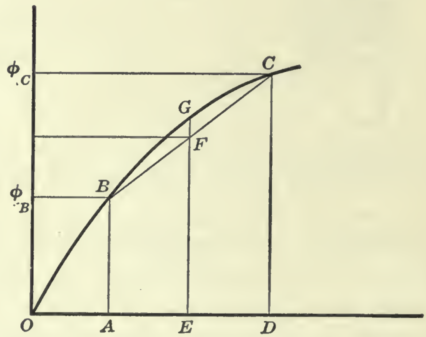

FIG. 67. The magnetization curve of iron.

The work may also be shortened by making the first guess by neglecting the effect of the center leg and considering the simple remaining circuit. The resulting flux density gives a point to start from in the cut-and-try process.

Prob. 20–6. In Fig. 66 the magnetizing coil is of 200 turns and carries a current of 2 amperes. The material of the core is annealed sheet steel. The paths *A* and *C* are each 50 cm. long, and leg *B* is 20 cm. long. The cross-section of the magnetic circuit is uniform throughout, and of 16 sq. cm. area. What will be the flux density in each part of the circuit? (If the computations check the given magnetomotive force within 4%, the results are as accurate as can be expected in practice in view of probable variations in the quality of the steel.)

66. Air Gaps. In the above consideration, the magnetic circuit was entirely in iron. It is very common, as we know, for air gaps to be interposed in an iron magnetic circuit.

FIG. 68. Frame of a two-pole motor showing field coils and poles.

Such, for instance, is the case in the magnetic circuit of the dynamo shown in Fig. 68. The magnetic circuit of this machine is indicated in Fig. 69 by the dotted lines, and

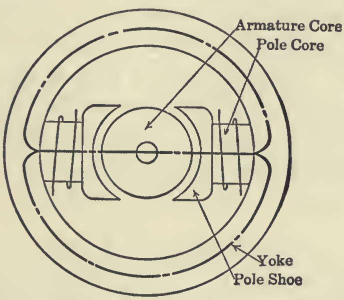

FIG. 69. The magnetic paths of the motor of Fig. 68.

it will be noted that it is made up of the following parts: an armature core, an air gap, a pole shoe, a pole core, two half yokes in parallel, a second pole core, a second pole shoe and a second air gap, all of these parts being in series. To compute the magnetomotive force necessary to pro-

duce a given flux in such a magnetic circuit, we compute the magnetomotive force necessary for each part, and add. In computing the air gap, we have noted above that the permeability is unity. The formula for the magnetomotive force necessary for the air gap is thus simply

$$\mathcal{F} = Bl \text{ gilberts,} \tag{13}$$

and for the ampere-turns for the air gap

$$Ni = \frac{1}{0.4\pi} Bl \text{ ampere-turns,} \tag{14}$$

where B is the flux density in gausses and l is the length in centimeters.

The correct cross-section to use for an air gap in computation work, however, is sometimes a matter of doubt. This is for the reason that the flux in the air gap spreads out. It is exactly as if we were studying an electric current in an electric circuit composed of carbon with a small air gap in the carbon conductor, the whole being immersed in a salt solution. Between the ends of the carbon rod, the current would spread out somewhat. Similarly, in an air gap in an iron magnetic circuit, the flux spreads out as shown in Fig. 70. Where the air gap is short, it is usually sufficiently accurate to increase each dimension used in the computation by the length of the air gap. For Fig. 70, the air gap would hence be considered as of length δ, and cross-sectional area

$$A = (b + \delta)(a + \delta).$$

When an air gap is introduced into a magnetic circuit, there are further effects noticeable by which the magnetic flux immediately makes its presence felt. Pieces of iron near the gap are attracted. In fact, there is a strong attraction between the two parts of a magnetic circuit separated by the gap, the amount of which we will shortly compute. The two ends of the iron part of the magnetic

circuit act like the ends of the familiar permanent magnet, except that they may be made much more powerful. One of these will be a north pole and the other a south pole.

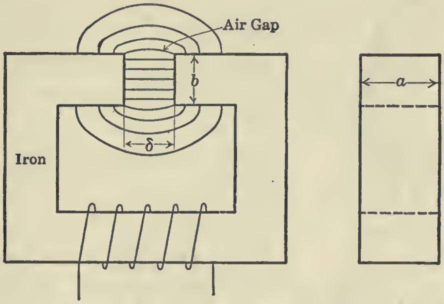

FIG. 70. Note that the flux spreads out in the air gap.

In order to distinguish, it is convenient to draw arrowheads on the flux lines in such a manner that the pole at which the flux lines leave the iron will be a north pole.

It will now be found that there is a **right-hand-screw relation** between the direction of the current in the mag-

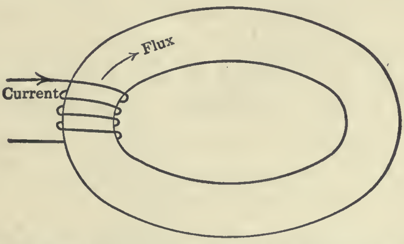

FIG. 71. Showing the relation between the direction of the magnetic flux and the electric current producing it

netizing coil and the direction of the flux produced in a magnetic circuit. This relation is shown in Fig. 71. It may be expressed as follows.

The flux produced by a current in a magnetizing coil will bear the same relation to the current that the motion of a nut (Fig. 72) on a **right-hand screw** bears to the rotation of

the nut. If the nut is turned in the direction of the current in the coil, its progress along the screw will be in the direction of the flux in the core.

It is, of course, possible for a coil carrying current to be threaded by flux in the direction reverse to the above. This may be accomplished if we overcome the magnetomotive force of the coil by a second opposed and more powerful magnetomotive force. When the flux is produced by the magnetomotive force of the coil itself, however, it will have the right-hand-screw relation to the direction of the current as above outlined.

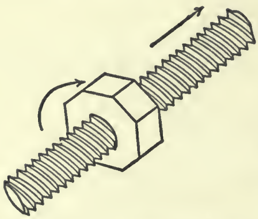

FIG. 72. The right-hand screw.

As has been noted, the permeability of iron is several hundreds to several thousands, while the permeability of air is unity. If the air gap is any large part of a magnetic circuit, the reluctance of the air gap will hence constitute almost the entire reluctance of the entire circuit. Suppose that we have a magnetic circuit of total length ten inches with an air gap one-tenth of an inch long. The length of the air gap is one percent of the length of the magnetic circuit. Yet if the permeability of the iron is a thousand and the cross-section of the magnetic circuit is uniform throughout, the reluctance of the air gap will be ten times that of the remainder of the circuit.

This leads to a convenient simplification in the cut-and-try process for solving a magnetic circuit. When such a circuit contains an air gap of fair size, so that its reluctance is a considerable proportion of the reluctance of the magnetic circuit, a first estimate of the flux density may be found by considering the magnetomotive force to be applied to the air gap alone.

Prob. 21–6. In the magnetic circuit of Fig. 63, an air gap is now introduced in the middle of one of the long legs by cut-

ting a space 1 millimeter wide entirely through with a hacksaw. What current will now be necessary in the magnetizing coil to force a flux of 60,000 lines through the magnetic circuit?

Prob. 22–6. If the gap in Prob. 21–6 were only $\frac{1}{10}$ millimeter wide, what current would be necessary?

Prob. 23–6. If, with this narrow gap of Prob. 22–6, a current of 0.4 ampere flows in the coil, what total flux will be produced?

67. Leakage Flux. In an electric circuit, practically none of the electric current leaks off the electric circuit under ordinary conditions, that is, with a metallic conductor and ordinary insulating materials. The conductivity of the metal may be ten million mhos per inch cube, while the conductivity of the surrounding insulating material is one ten-millionth of a mho per inch cube. These values are so far apart that at ordinary differences of potential and ordinary values of current the amount of leakage is entirely negligible. The current is the same, therefore, in all parts of the circuit, and no conduction takes place outside the circuit except when the potential gradient becomes abnormally high.

In the magnetic circuit an entirely different condition prevails. The permeability of the iron used for the magnetic flux path may be several thousands, but the permeability of the surrounding material will be approximately unity. The leakage paths may be short and of large cross-section, and their reluctance hence fairly small. Therefore in all ordinary magnetic-circuit calculation, it is necessary to take account of the amount of flux which passes in leakage paths. The total flux will not pass completely around the circuit, but some of it will leak out, and hence this loss of flux must be taken into account in computing the flux density of the circuit itself. There is no magnetic insulator. By this we mean simply that there is no material which has a permeability so small that practically no flux can be passed through it. All materials except a few

magnetic materials such as cobalt, nickel and iron, have permeabilities very close to one. Even a vacuum has a permeability differing from that of air by an amount so small that the difference is detected with difficulty.

Magnetic leakage is, of course, particularly high when conditions in the circuit are such as to produce large magnetic differences of potential between portions of the circuit in close proximity. Such large differences of magnetic potential may be produced by introducing an air gap into the circuit, or by an arrangement where the magnetomotive forces of two coils are large and opposed to each other.

In the magnetic circuit of a dynamo, as shown in Fig. 69, the principal reluctance of the magnetic circuit occurs at the air gaps. The pole shoes are hence at a large difference of magnetic potential. This means simply that there is a large magnetomotive force acting between pole shoes, and for this reason there will be a large leakage flux between the pole shoes; that is, considerable flux will pass from shoe to shoe without going through the armature. Only that portion of the flux is useful which passes through the armature.

The leakage flux between two points of a magnetic circuit may be computed by finding the magnetic difference of potential between the two points, and also the reluctance of the leakage path joining them. The magnetic difference of potential between two points of a magnetic circuit may be found in exactly the way that we find the electric difference of potential between two points on an electric circuit. It is equal to the reluctance drop in the circuit joining it, unless there is a magnetomotive force in that part of the circuit. In such a case, the magnetic potential difference is the magnetomotive force acting taken together with the reluctance drop. Kirchhoff's law can be used for the magnetic circuit.

The magnetic potential difference between the points A and B, Fig. 73, can thus be written as

$$\mathcal{F} - \phi \mathcal{R}_{AB}, \tag{16}$$

or

$$0.4\pi NI - \phi R_{AB}, \tag{17}$$

because $0.4\pi NI$ is the magnetomotive force applied to that part of the circuit and $\phi\mathcal{R}_{AB}$ is the reluctance drop of that part of the circuit. The expression $(\mathcal{F} - \phi\mathcal{R}_{AB})$ is thus the rise in potential from B to A. If the points A and B are at the terminals of the coil, the expression $\mathcal{F} - \phi\mathcal{R}_{AB}$ might be called the terminal magnetic potential of the coil. This

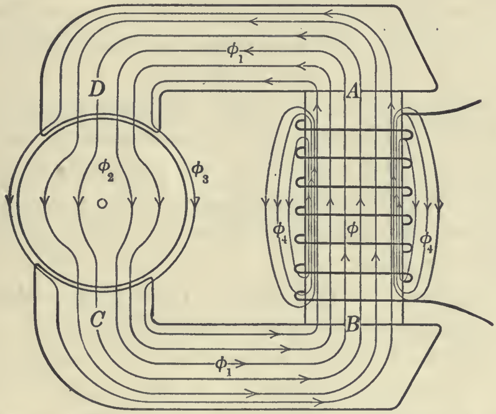

FIG. 73. The flux lines ϕ_3 and ϕ_4 are called leakage lines.

is analogous to the terminal electric potential of a generator, which is equal to the e.m.f. of the generator minus the internal resistance drop of the generator, or $E - IR$.

The magnetic potential difference between the points A and B can also be computed from the fall of magnetic potential from A to B along the path $ADCB$, because the fall in potential from A to B must equal the rise in potential from B to A as computed above.

The potential difference of A and B must thus equal

$$\phi_1\mathcal{R}_{AD} + 2\phi_2\mathcal{R}_{gap} + \phi_2\mathcal{R}_{arm} + \phi_1\mathcal{R}_{CB}, \tag{18}$$

where ϕ_1 = flux through the iron from A to D; $\mathcal{R}_{AD}$ =

reluctance of the iron from A to D; ϕ_2 = flux through the armature; R_{arm} = reluctance of the armature, etc.

In addition to the quantities of flux ϕ, ϕ_1, and ϕ_2, there will, however, be other quantities of flux through the air between certain points on the iron part. For instance, the flux, ϕ_4, leaks from the region of the pole core to the end, B, of the yoke, because there is a difference of potential between these points of $(.4\pi NI - \phi_1\mathcal{R}_{AB})$ gilberts. To determine the amount of this flux, ϕ_4, the magnetic potential difference must be divided by the reluctance of the leakage path from A to B. This reluctance is very difficult to compute.

In a similar manner, some flux, ϕ_3, leaks between the pole tips D and C without going through the armature. The amount of this flux is equal to the magnetic potential difference of the pole tips $(2\phi_2\mathcal{R}_{gap} + \phi_2\mathcal{R}_{arm})$ divided by the reluctance of the leakage path, which again is difficult to determine accurately.

The reluctance of the leakage path in air between two parts of a magnetic circuit is usually difficult to compute. The flux in the air spreads out, and as a result the cross-section of the gap is usually a matter which is somewhat difficult to estimate. In a few cases which are simple in geometric form, the value of the reluctance has been accurately computed. In most cases which arise in practice, the geometry of the circuit is so complicated that an accurate solution is impossible and approximation must be used.

As an example illustrating this proposition, the alternating-current transformer will serve excellently. This is one of the most important pieces of electrical apparatus and the flexibility which it gives to the commercial circuit is the principal reason that a wide use of alternating currents prevails. The transformer consists of two and sometimes more coils wound on a single magnetic circuit. One of these, the primary, supplies a current which magnetizes the core, and since this current is alternating or varying, the flux in

the core will vary with it. This varying flux produces a voltage in the secondary coil. By varying the number of turns on the secondary, we can vary the voltage produced, and so obtain any desired ratio of voltages in the unit. When a current is drawn from the secondary, more current will flow in the primary. As a result, the two coils may each be carrying a heavy current at the same time. Their magnetomotive forces will be opposed but not quite equal. The difference between them is the magnetomotive force which forces the flux about the circuit. Such an arrangement is shown in Fig. 74. In a transformer the leakage flux is of great importance, since it determines

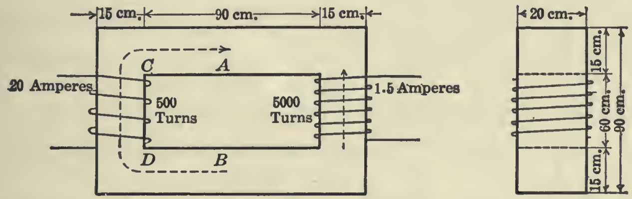

FIG. 74. The diagrammatic representation of a transformer. The currents in the two coils tend to set up opposing fluxes in the core.

almost entirely the regulation of the transformer. In practice, the coils are split up, and part of the primary and also part of the secondary put on each leg in order to keep the leakage flux small. It will be more convenient, however, for us to examine a more simple arrangement in which the primary is wound on one leg and the secondary on the opposite leg. The arrangement shown in the figure will hence have a much larger leakage flux than that which would be allowable in practice, but is simpler to compute than the actual arrangement.

We have chosen an instant when the primary coil of 500 turns is carrying a current of 20 amperes and the secondary coil of 5000 turns is carrying 1.5 amperes. The magnetomotive forces will be in the directions shown by the dotted

arrows, and are therefore opposed. The net ampere-turns useful in forcing the flux about the circuit will be

$$(20 \times 500) - (1.5 \times 5000) = 2500 \text{ ampere-turns.}$$

The magnetic circuit, allowing for curvature at the corners, is about 350 centimeters long. The magnetizing force is therefore

$$0.4\pi \frac{2500}{350} = 8.98 \text{ gilberts per centimeter.}$$

Neglect for a moment the variation in density of the flux at different parts of the core due to leakage and refer to the curves for sheet steel, and it is found that this magnetizing force corresponds to a flux density of 14,500 gausses, which is about the maximum usually used in transformer practice. The cross-section of the iron is 300 square centimeters. Thus there are 4,350,000 maxwells total flux.

Let us now examine the magnetomotive force acting on the air path from point A to point B on the core. The left-hand coil supplies 10,000 ampere-turns. About 1250 of these are used up in forcing the flux through the reluctance of half of the electric circuit as shown by the dotted lines; that is, the reluctance drop around this half of the circuit is 1250 ampere-turns or 1570 gilberts. The net ampere-turns back from A to B is hence

$$10{,}000 - 1250 = 8750 \text{ ampere-turns.} \qquad (33)$$

This value might also have been found by taking the magnetomotive force of the secondary, 7500 ampere-turns, and adding to it the reluctance drop of 1250 ampere-turns in the right-hand half of the core. The reluctance drop in this case is added to the magnetomotive force of the coil since the flux is being forced through the coil against its magnetomotive force. This is entirely analogous to the potential drop caused in a battery in an electric circuit in which the current is being forced through the battery in a

direction opposite to its generated potential. The magnetomotive force between points C and D will similarly be the magnetomotive force in the primary minus the reluctance drop in the left-hand leg. The magnetomotive force across the air path is thus seen to vary linearly along the core, and since the flux produced is directly proportional to the magnetomotive force, it will be accurate for us to use the mean value, that is, the 8750 ampere-turns acting at the center, as shown in Fig. 75.

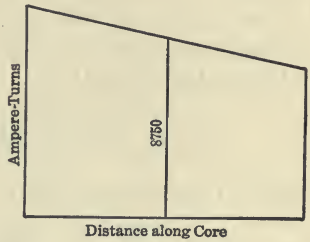

FIG. 75. Showing the values of the magnetomotive forces between different points on the core in Fig. 74.

We now need to estimate the reluctance of the leakage path. In Fig. 76 is shown a cross-section of the transformer with the flux lines drawn in their approximate

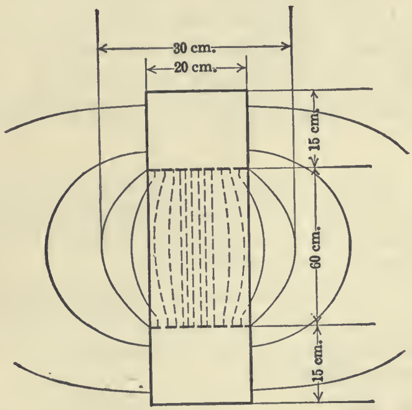

FIG. 76. Showing the flux distribution in the leakage paths of the transformer of Fig. 75.

positions for the leakage flux. The value to be chosen for the cross-section, that is, the amount to be allowed for

fringing, is largely a matter of experience, and we will consider it more fully below. At present we will take the value of 30 centimeters as a reasonable width, and 90 centimeters as a reasonable length of the cross-section of the leakage path. As the average length of the leakage path we might take 65 centimeters. Accordingly, the reluctance of this leakage path is

$$\mathcal{R} = \frac{1}{\mu A} = \frac{65}{1 \times 30 \times 90} = 0.024 \text{ oersted},$$

and the total leakage flux is

$$\phi = \frac{0.4\pi NI}{\mathcal{R}} = \frac{11{,}000}{0.024} = 458{,}000 \text{ maxwells}.$$

The flux in the core was estimated at 4,350,000 maxwells. The leakage flux is thus

$$\frac{458{,}000}{4{,}350{,}000} \text{ or about 10 percent of the main flux.}$$

In an accurate computation it would now be necessary to recalculate the main flux, for since the flux is not exactly uniform throughout the core, the assumptions we made in our first calculation were not entirely accurate. The correction will be small, however, and we will not estimate it at this time.

Prob. 24–6. In the transformer shown in Fig. 74, the secondary current is increased from 1.5 to 1.75 amperes, the primary current remaining as before. What will be the new values for the main and leakage fluxes?

Prob. 25–6. Fig. 77 represents a magnetic path in a 4-pole generator. On each pole are wound 770 turns carrying 5 amperes. The reluctance drops for various parts of the magnetic circuit are as follows:

Each air gap,..............2582 ampere-turns;
Armature,................ 298 ampere-turns;
Each pole,................ 775 ampere-turns.

(*a*) What is the reluctance drop for the yoke? (*b*) What is the magnetic potential between tips of adjacent poles?

Prob. 26–6. The average distance, l, between nearest edges of adjacent poles of the generator in Fig. 77 is 3.28 inches.

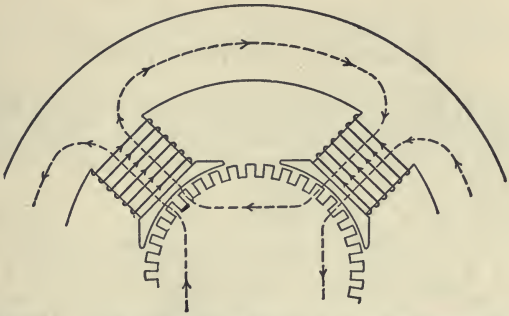

Fig. 77. The magnetic paths in a four-pole generator.

The average cross-section of the path of the leakage flux from tip to tip is 7×0.5 inches. How great is the leakage flux from tip to tip?

SUMMARY OF CHAPTER VI

AN ELECTRIC CIRCUIT ALWAYS LINKS ONE OR MORE MAGNETIC CIRCUITS and sets up magnetic fluxes.

NO ENERGY IS CONSUMED by a magnetic circuit in which the flux is not changing.

A CHANGE IN THE FLUX in a magnetic circuit induces an electromotive force in any interlinking circuit.

WHEN A CHANGE IN THE FLUX INDUCES ONE AB-VOLT in an interlinking electric circuit of one turn, the rate of change is said to be ONE MAXWELL PER SECOND.

THE MAXWELL, sometimes called a LINE, is the unit of magnetic flux. The letter ϕ is used to denote magnetic flux.

THE GAUSS is the unit of flux density and equals one maxwell per square centimeter. The letter B is used to denote flux density.

MAGNETIC FLUX IS SET UP BY MAGNETOMOTIVE FORCE much as an electric current is set up by an electromotive force.

OHM'S LAW FOR THE MAGNETIC CIRCUIT:

$$\text{Magnetic flux} = \frac{\text{Magnetomotive force}}{\text{Reluctance}},$$

or

$$\phi = \frac{\mathcal{F}}{\mathcal{R}},$$

where ϕ = the magnetic flux in MAXWELLS,
$\mathcal{F}$ = the magnetomotive force in GILBERTS,
$\mathcal{R}$ = the reluctance in OERSTEDS.

The quantity $\phi\mathcal{R}$ is called the RELUCTANCE DROP and is analogous to IR, the resistance drop.

THE MAGNETOMOTIVE FORCE can be found from the following equation:

$$\mathcal{F} = \frac{4\pi NI}{10},$$

where $\mathcal{F}$ = the magnetomotive force in GILBERTS,
N = the number of turns around the magnetic circuit,
I = the electric current flowing in the turns in AMPERES.

THE RECIPROCAL OF THE RELUCTANCE OF A UNIT CUBE OF A MATERIAL IS CALLED ITS PERMEABILITY. THE RELUCTANCE CAN BE FOUND from the equation

$$\mathcal{R} = \frac{l}{\mu A},$$

where $\mathcal{R}$ = the reluctance in OERSTEDS,
l = the length of the magnetic path in CENTIMETERS,
μ = the permeability of the substance in GAUSSES DIVIDED BY GILBERTS PER CENTIMETER,
A = the cross-section area in SQUARE CENTIMETERS.

SERIES AND PARALLEL ARRANGEMENTS OF RELUCTANCES are treated in a manner similar to that used for series and parallel arrangements of resistances.

THE MAGNETOMOTIVE FORCE PER CENTIMETER of the magnetic circuit is called THE MAGNETIZING FORCE and is represented by the letter H.

THE PERMEABILITY may be thought of as B/H. For all non-magnetic substances (including a vacuum) it has substantially the value of unity. For magnetic substances it varies from unity to several thousand depending upon the nature of the substance and the degree of magnetization.

MAGNETIZATION CURVES plotted between simultaneous values of B and H are of great value in determining the ampere-turns necessary to produce a given flux in a given magnetic circuit.

IN MAGNETIC CIRCUITS CONTAINING AIR GAPS, the greater part of the magnetomotive force is usually consumed by the gap.

THERE IS NO MAGNETIC INSULATION. Thus there is always a certain amount of FLUX LEAKAGE from the magnetic path in which it is desired to confine the flux.

LEAKAGE FLUX can be computed by dividing the magnetomotive force across the leakage path by the reluctance of the leakage path. This reluctance is usually difficult to determine and is generally estimated.

PROBLEMS ON CHAPTER VI

Prob. 27–6. A D'Arsonval galvanometer, such as is used in the laboratory, has been calibrated and it is found that one centimeter deflection corresponds to 2.1×10^3 abvolt-seconds. A measuring coil of 25 turns is placed on an iron core and the ends of the coil are connected to the ballistic galvanometer. The core is now suddenly magnetized, and a deflection of 20.2 centimeters is obtained. The core area is 0.8 square inch. What is the flux density produced in the core?

Prob. 28–6. A telephone transformer is in common use with the following combinations of turns:

Primary Taps		Secondary Taps	
No.	Turns	No.	Turns
1–2	150	5–6	1200
2–3	300	6–7	2400
3–4	600	7–8	4800

If the flux in the core were changing at a uniform rate, with a voltage of 1.2 appearing on taps 1 and 2, what would be the voltage measured on the various pairs of secondary taps?

Prob. 29–6. Determine the ampere-turns necessary to produce a flux density of 70 kilolines to the square inch in the core A of the cast-steel chuck of Fig. 148. (Mean lengths and cross-sections of flux paths must be estimated.)

Prob. 30–6. Find the number of ampere-turns necessary to produce a flux density of 65 kilolines in the central core of the annular chuck of Fig. 149 if a disk of machined steel one inch thick and of the same diameter as the chuck is laid on the face of the chuck and in intimate contact with it.

Prob. 31–6. Work Prob. 20–6 using 3000 ampere-turns. Use the cut-and-try method.

Prob. 32–6. Cut a $\frac{1}{8}$ inch gap in leg C of Fig. 66 and recalculate Prob. 31–6.

Prob. 33–6. A transformer has a continuous winding of 700 turns. At every one hundred turns there is taken out a tap or lead. Numbering these leads 1, 2, 3, 4, 5, 6, 7, 8, if the flux is changing at such a rate that a voltage of 110 is read between leads 1 and 7, what voltages will be read at the same instant between leads 1 and 2, 6 and 7, 2 and 5?

Prob. 34–6. What is the magnetic potential difference between the centers of the pole cores in Prob. 25–6?

Prob. 35–6 Fig. 78 and 79 represents a simple 2-pole magnetic chuck, frame and cover. The pole pieces are of cast steel, the "work" is of wrought iron. The length of each core is $2\frac{1}{4}$ inches, the cross-section of each core is $2 \times 4\frac{1}{2}$ inches. The cover and frame are $\frac{3}{4}$ inch thick. The space between poles is 1 inch wide. The "work" is $4\frac{1}{2} \times 5 \times 1\frac{1}{2}$ inches. How many ampere-turns must be wound on each core in order to set up a flux of 540,000 maxwells through a cross-section at the center of the "work"? Assume no leakage and no air gaps between "work" and cover or between cover and cores.

Prob. 36–6. From the data in Prob. 35–6, what magnetic difference of potential exists between (*a*) points *b* and *c* of the chuck in Fig. 78? (*b*) Points *a* and *d*? (*c*) Points *g* and *h*? (*d*) Points *k* and *l*?

Prob. 37–6. Compute the leakage between core and shell in Fig. 78, disregarding that which leaks between "work" and shell.

Prob. 38–6. Compute the leakage between cores in Fig. 78.

Prob. 39–6. Leakage flux leaves and enters magnetic material nearly at right angles to the surface. This causes the leakage lines of flux between the "work" and cover to have the shapes shown in Fig. 78. The lines may be assumed straight in the space between *fa* and *ed*, and arcs of concentric circles with *a* as a center between *fa* and *am*. Compute the reluctance of the path thus formed between the $4\frac{1}{2} \times 1\frac{1}{2}$ sides of the "work" and the cover of the chuck. (Note: integrate between limits O and arc *mf*.)

Prob. 40–6. Compute the leakage from the $4\frac{1}{2} \times 1\frac{1}{2}$-inch sides of the "work" to the cover of the chuck in Fig. 78.

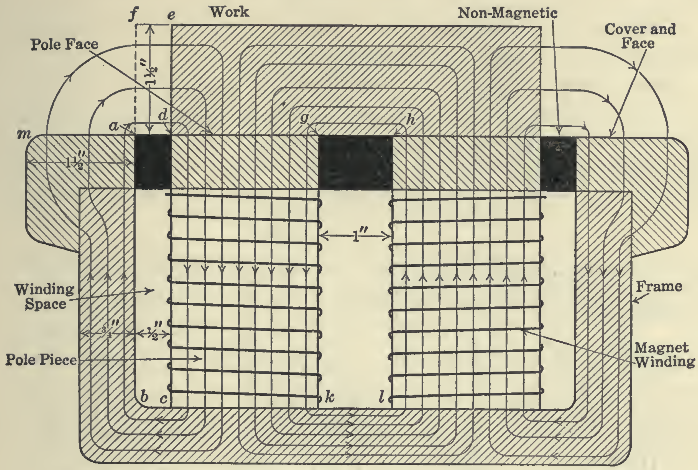

Fig. 78. Vertical section of a magnetic chuck and "work."

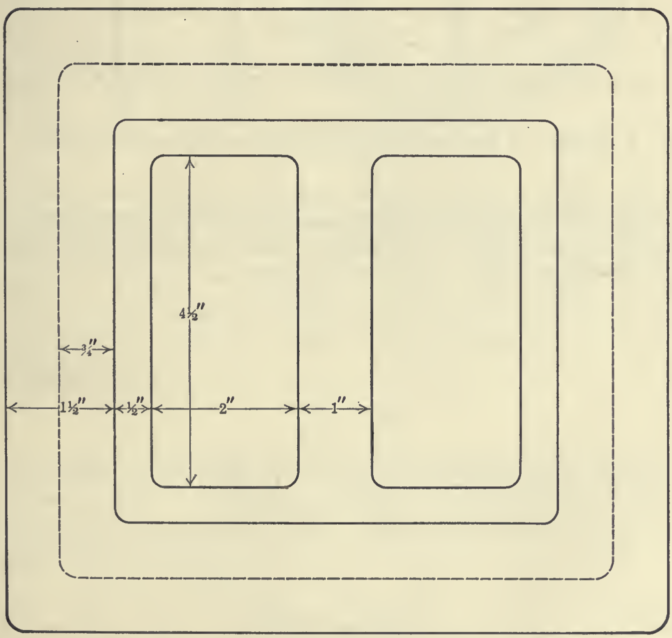

Fig. 79. Top view of magnetic chuck of Fig. 78.

Prob. 41–6. Compute the leakage from the $5 \times 1\frac{1}{2}$-inch sides of the "work" to the cover in Fig. 78.

Prob. 42–6. Taking into account the leakage flux as found in Probs. 37–6, 38–6, 40–6 and 41–6, make a closer approximation of the ampere-turns necessary to set up 540,000 lines through a cross-section at the center of the "work" than was possible in Prob. 35–6, where the leakage flux was neglected.

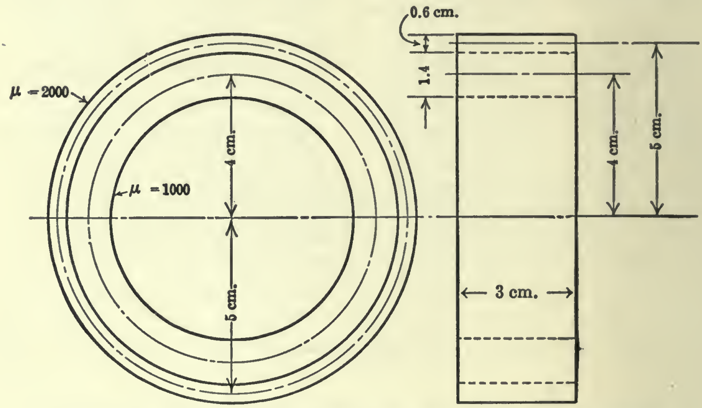

FIG. 79a. A magnetic circuit made up of two parallel paths.

Prob. 43–6. By what percentage will the reluctance of the ring of Fig. 79a be changed if the material of higher permeability is placed on the inside? The volume of each material and the overall dimensions of the ring remain the same.

CHAPTER VII

THE MAGNETIC FIELD

In this chapter will be discussed methods for determining the flux density at various points in magnetic fields produced by electric currents.

68. The Line Integral Law. Kirchhoff's law applied to magnetic circuits states that the sum of the reluctance drops around a closed magnetic circuit is equal to the sum of the magnetomotive forces acting on the circuit. If the magnetic circuit is uniform, this is simply

$$\mathcal{F} = \mathcal{R}\phi, \tag{1}$$

which is another way of writing the familiar expression

$$\phi = \frac{0.4\pi NI}{\mathcal{R}}. \tag{2}$$

This expression, we have seen, is true whether the magnetic circuit is uniform or not. In the case of a non-uniform magnetic circuit, we take the several parts which are in series and multiply the flux by the reluctance of each and add, in order to obtain the total magnetomotive force accurately; that is,

$$\mathcal{F} = \Sigma\mathcal{R}\phi. \tag{3}$$

In the case of a magnetic circuit varying uniformly in cross-section, such as a circuit made of a conical piece of iron, where the reluctance varies continuously from point to point, the law becomes

$$\mathcal{F} = \int \phi d\mathcal{R}. \tag{4}$$

Inserting for ϕ its value

$$\phi = BA,$$

and for $\mathcal{R}$

$$\mathcal{R} = \frac{l}{\mu A}$$

and canceling, we obtain

$$\mathcal{F} = \int \frac{B}{\mu} dl. \tag{5}$$

Then since

$$B = \mu H$$

this may be written

$$\mathcal{F} = \int H dl. = 0.4\pi NI. \tag{6}$$

This expression may be stated in words: the **line integral** of H about a closed circuit is equal to $\mathcal{F}$; or, the line integral of the magnetizing force about a closed magnetic circuit is equal to 0.4π times the number of ampere-turns linking the circuit.

In the above, we have assumed that H is always in the direction of the length of the magnetic circuit. The law above, however, has been found to be true in a more general way than this. In fact, if we take into account the direction of H, that is, consider the above integral to be a line integral, the general proposition follows that the line integral of H about any closed curve is equal to 0.4π times the number of ampere-turns linking the curve. It will now be necessary to explain carefully and in detail just what is meant by this expression, for it is important.

First, consider what is meant by a line integral. If a point, as it moves along a path, is acted upon by a variable force, the line integral of this force along the path is equal to the length of the path times the average value of the component of the force in the direction of the path. In other words, if at every point of the path we take the force multiplied by the cosine of the angle between the force and the direction of the path, and integrate this product over

the entire distance, then the result is the line integral of the force along the path. This can be made clear by an example.

Suppose that we attach a rubber band to a fixed circular post as shown in Fig. 80, and fasten the other end to a pencil point. The rubber band may be wrapped around

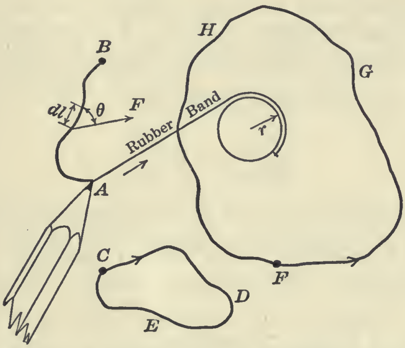

FIG. 80. The pencil is attached by a stretched rubber band to the post. The line integral of the force exerted by the rubber band as the pencil moves from B to A is the work done as the pencil moves from B to A.

the post and fastened at some convenient point. The pull of the rubber band exerts a force on the pencil in the direction shown by the arrow. The line integral of this force along the path AB is the work done on the pencil point as it moves from B to A. If we represent the force by F and the work by W, this becomes

$$W = \int_A^B \bar{F} dl, \tag{7}$$

which is to be understood as a line integral equation. The line over the F indicates that F is a vector and is to be considered as such; that is, it has direction as well as magnitude. This line integral may also be written

$$W = \int_A^B F \cos\theta \, dl. \tag{8}$$

This means simply that the path is divided into elementary lengths, dl, the magnitude of the force at each one of these points is multiplied by the cosine of the angle which it makes with the direction of the path, and the limit of the sum of all these products is taken. The line integral above from A to B is evidently the work done, that is, the energy given up by the rubber band as the pencil point passes from A to B.

Suppose now that the pencil point is moved around a closed path such as CDE. The line integral around this closed path must be zero; that is,

$$\int_{CDEC} \overline{F} dl = 0. \tag{9}$$

We know that this must be true, for the work done by the rubber band is evidently zero, since it returns to exactly its original condition. To state the matter in another way, if the above integral were not zero, that is, if there were a total amount of energy produced by the rubber band during the operation, then it would be possible to construct a perpetual motion machine by arranging the point to continually trace this closed circuit, and such a machine we know to be impossible. We thus know that the line integral of a force such as the above around such a closed path is zero.

Consider now the integral around the closed path FGH, which links the post to which the rubber band is attached. This integral is not zero, for when the point has returned to point F, the rubber band has been stretched by an additional amount, since it has been wound one more turn around the post. The amount that it has been wound up, that is, $2\pi r$ times the force necessary to stretch it a unit length, is the total amount of work done. The line integral is equal to this work, and if we represent by K the force necessary to stretch the rubber band one unit of length, we have the expression

$$\int_{FGHF} \overline{F} dl = 2\pi r K. \tag{10}$$

The law for the magnetic field is very similar to this. H is a force, called the magnetizing force. The line integral of this force about a closed path which links no coils carrying current is zero; that is,

$$\int_l H dl = O. \tag{11}$$

The line integral of H about a closed path linking a coil carrying current is equal to the magnetomotive force in the coil, that is, $4\pi NI$. This is expressed by

$$\int_l H dl = 4\pi NI. \tag{12}$$

The magnetomotive force F must therefore be *work* rather than *force*. Similarly, we have found that the electromotive *force* E is really *work* and not a force in the usual meaning of the term.

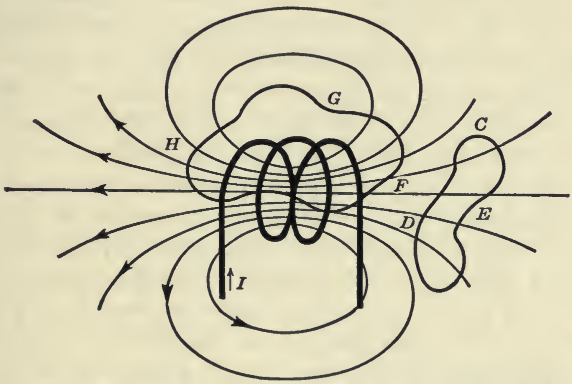

FIG. 81. The line integral along any path in the magnetic field is the work done by the magnetizing force, H, acting upon a particle moved along that path.

This law will enable us to examine the flux distribution in a magnetic circuit of very irregular shape, in particular for a magnetic circuit in air, where the lines spread out. When a coil of wire carrying current is simply suspended in

the air, with no iron present, the field spreads out and extends a considerable distance from the coil, taking somewhat the shape shown in Fig. 81. We may compare this with the electric circuit in the following manner. Suppose that an ordinary flashlight battery of cylindrical form is dropped into a bucket of salt water. The battery will be short-circuited by the salt solution, and lines of current flow will extend from one end of the battery to the other, spreading out through the solution much in the same way as above. It is difficult, and in fact almost impossible, to figure the reluctance of the path for a solenoid such as is shown in Fig. 81. We know, however, that the line integral of H along such a path as DCE is zero. We know also that the line integral of H along such a path as FGK which links the coil is equal to 0.4π times the product of the current in amperes and the number of turns in the coil. By utilizing this law, we are enabled to find the flux density in the vicinity of wires and coils carrying current when there is no iron present.

69. Field About a Long Wire. We will first examine the flux density in the vicinity of a long straight wire carrying current. This problem is important in the examination of transmission lines, telephone lines and so on. We have all heard the sound in a telephone receiver caused by the variation in the current taken by the motors of a trolley car. This sound is produced by the fact that the flux surrounding the trolley wire induces a voltage in nearby telephone lines and causes a hum. The regulation of a transmission line, that is, the drop in voltage from one end to the other, on alternating currents, is largely dependent on the magnetic field surrounding the wire. For these and similar reasons it is important that we have an accurate formula for this problem.

When a wire carrying current is situated at a distance from magnetic materials and from other wires, the magnetic flux about the wire is arranged in circles concentric with the wire. This must be true from symmetry, but we can check

it with the arrangement shown in Fig. 82. A long wire is passed through a hole in a card. The latter is sprinkled with iron filings, and the wire caused to carry a current. When the card is tapped, it is found that the filings arrange themselves in circles as shown, proving the field to be circular. In order for a current to flow, there must, of course, be a return wire somewhere to complete the circuit, but we will consider this return wire to be so far distant that its magnetic effect may be neglected. The whole electric circuit constitutes one turn. An application to this turn of the right-hand rule of the previous chapter shows that the flux lines must have the direction shown by the arrows. This rule may now be stated another way. The flux about a long wire has a direction the same as the rotation of the head of a right-handed screw when turning in such a direction as to progress in the direction of the current along the wire.

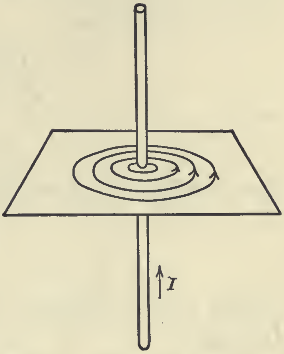

Fig. 82. Showing the relation between the direction of the current in a wire and that of the magnetic field set up by the current.

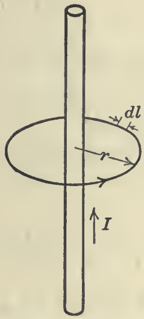

Fig. 83. The magnetizing force at a distance r from a wire carrying a current I.

We will now examine the strength of this circular field. In Fig. 83, we draw a circle of radius r about the wire. The magnetizing force H will be the same at each point on this circle, since the flux lies in concentric circles about the wire, and it will be everywhere tangent to this circle. Hence the line integral for H about this circle is simply equal to H times the circumference of the circle. This line integral

must be equal to $0.4\pi NI$, and since the entire electric circuit makes only one turn, N may be taken as unity. We have therefore

$$\int_l Hdl = 0.4\pi NI, \tag{13}$$

or

$$2\pi r H = 0.4\pi I, \tag{14}$$

from which it follows that

$$H = \frac{0.2I}{r}, \tag{15}$$

where I is in amperes. If I is in abamperes, the formula becomes

$$H = \frac{2I}{r}. \tag{16}$$

If the wire is in air, where the permeability is everywhere unity, B would always be equal numerically to H, as given by

$$B = \frac{0.2I}{r}; \tag{17}$$

that is, the flux density at any point near a long wire in air carrying current is equal to 0.2 times the current in amperes, divided by the distance from the wire in centimeters.

If the wire is surrounded by a material of a permeability other than unity, we have

$$B = \mu\frac{0.2I}{r}. \tag{18}$$

Such a case arises when a conductor is at the center of a cable with an iron sheath as shown in cross-section in Fig. 84. If the thickness of the sheath is a, the total flux per centimeter of length in the sheath will be

$$\phi = \mu a\frac{0.2I}{r}. \tag{19}$$

It will be instructive to derive the formula, $H = \dfrac{2I}{r}$,

in a slightly different way. Referring to Fig. 85, the magnetizing force at the point P at the distance r from a long

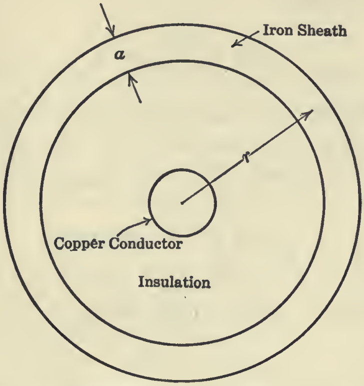

Fig. 84. A copper conductor within an iron sheath.

wire carrying current I may be found by integrating the effect of small elements of the wire of length dx, provided

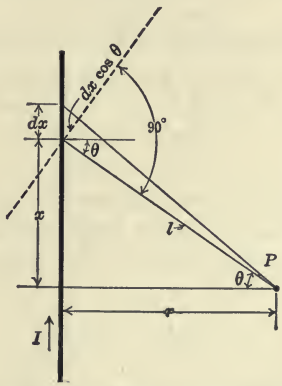

Fig. 85. To find the magnetizing force at P produced by current I in conductor X.

we make an assumption as to the effect of each of these elements. Such an assumption is that the effect of an element of length dx carrying a current I at a point distant l

is equal to the product Idx divided by l^2 and multiplied by the cosine of the angle θ which a perpendicular to the wire makes with the length l

$$dH = \frac{Idx}{l^2}\cos\theta.* \tag{20}$$

Under this assumption, the effect of the total wire will be found by adding the effects of all the elements, that is, integrating for x along the total length of the wire. This gives

$$H = \int_{-\infty}^{+\infty}\frac{I\cos\theta}{l^2}dx. \tag{21}$$

From the geometry of the figure we have

$$x = r\tan\theta, \tag{22}$$

and differentiating

$$dx = r\sec^2\theta\, d\theta. \tag{23}$$

Also

$$l = \frac{r}{\cos\theta}\cdot \tag{24}$$

Inserting these values in the above integral, we have

$$H = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{I\cos\theta\, r\sec^2\theta}{\frac{r^2}{\cos^2\theta}}d\theta, \tag{25}$$

where, since the variable has been changed to angle θ, the limits of the integral are the limits of this angle, or from minus to plus a right angle. This integral simplified becomes

$$H = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{I\cos\theta}{r}d\theta, \tag{26}$$

which, integrating, gives

$$H = \frac{I}{r}\Big[\sin\theta\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cdot \tag{27}$$

* Note from Fig. 85 that the quantity $dx\cos\theta$ is the projection of dx upon a line perpendicular to l.

Inserting the values of the limits, we have

$$H = \frac{2\,I}{r} \tag{28}$$

which is the same result as obtained previously by a different method. Our expression for the effect of an element of circuit is therefore justified, namely, that the magnetizing force varies inversely as the square of the distance from the element; that it is proportional to the cosine of the angle of projection of the element and proportional to the length of the element and to the current carried. We may therefore use the expression with assurance in subsequent calculations.

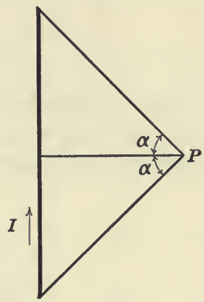

FIG. 86. The conductor subtends an angle of 2α at P.

This expression for the magnetizing force near a long wire carrying current holds strictly only for a wire of infinite length. For ordinary accuracy it holds for a wire which is fairly long compared with the distance from the wire considered. For example, if we are examining the field strength ten centimeters from a wire which is ten meters long, the formula will hold to much greater than engineering accuracy. We may, however, obtain the effect of a short wire. This may be done by simply changing the limits on the integral above. Thus, referring to Fig. 86, if the wire subtends at the point P a total angle of 2α, we put the limits of integration from $-\alpha$ to $+\alpha$, and we have

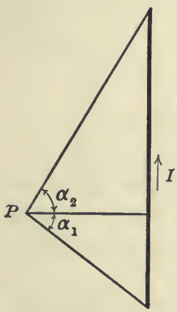

FIG. 87. The conductor subtends an angle at P of $\alpha_1 + \alpha_2$.

$$H = \int_{-\alpha}^{+\alpha} \frac{I}{r} \cos\theta \, d\theta, \tag{29}$$

which integrated gives

$$H = \frac{2\,I}{r}\sin\alpha. \tag{30}$$

This formula may be used for the magnetic effect of a straight wire of finite length. If the point P is not situated opposite the midpoint of the wire, as for instance in Fig. 87, the formula evidently becomes

$$H = \frac{I}{r}(\sin\alpha_1 + \sin\alpha_2). \tag{31}$$

The objection may properly be raised that it is not correct to speak of the magnetic effect of a short straight wire alone since electric current must always flow in a closed

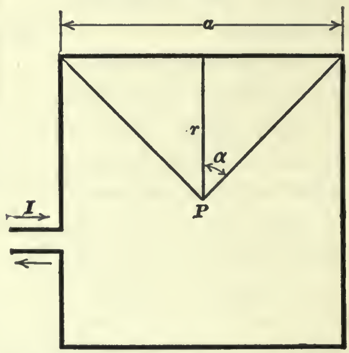

Fig. 88. To find flux density at P, the center of a square of wire carrying a current.

circuit. A short straight wire may, however, become a part of a closed magnetic circuit, and the total effect of the whole circuit is found by the sum of the effects of its parts. Suppose, for example, that we wish to find the flux density at the center of a closed square of wire carrying a current I, as shown in Fig. 88. This will evidently be four times the effect of one side, or

$$B = H = \frac{8\,I}{r}\sin\alpha.$$

If the length of the side of the square is a, we shall have

$$r = \frac{a}{2},$$

or since α is 45° when point P is at the middle of the square, the resulting expression for the flux density at the middle point of such a square will be

$$B = \frac{16\,I}{a\sqrt{2}}$$

when the square of wire is not in the vicinity of iron. In this problem the effect of the leads by which the current enters the square cancel, since they are close together and the current flowing in them is in opposite directions.

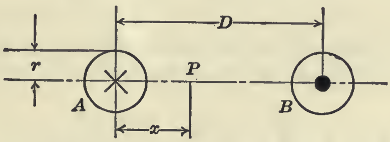

FIG. 89. A and B represent a pair of parallel conductors carrying current. The current goes out on A and returns on B.

It is now easy to compute the total effect due to a pair of wires such as are shown in Fig. 89, where the current goes out along one and returns by the other. This is a common arrangement in the transmission of power, and the consideration of the field is therefore of much importance. We will assume that the wires are separated by a distance D, their radius being r. The flux density at any point on the line joining the wires at a distance x from one of the wires may immediately be found. The magnetizing effect of the left-hand wire we have just found to be equal to

$$H_A = \frac{2\,I}{x}. \tag{32}$$

Since the wire B is carrying equal current in the opposite direction, its magnetizing force at the point P adds directly

to that of A, since both magnetizing forces are perpendicular to the line joining the centers of the two wires. The magnetizing force of B is

$$H_B = \frac{2\,I}{D - x} \tag{33}$$

since the current in the two wires is I. The total magnetizing force is hence

$$H = \frac{2\,I}{x} + \frac{2\,I}{D - x}. \tag{34}$$

Since the wires are in air, the same expression holds for the flux density; that is,

$$B = 2\,I\left(\frac{1}{x} + \frac{1}{D - x}\right). \tag{35}$$

The total flux passing between the two wires may be found by integrating this expression, thus

$$\phi = 2\,I\int_r^{D-r}\left(\frac{1}{x} + \frac{1}{D - x}\right)dx \tag{36}$$

$$= 2\,I\Big[\log_e x - \log_e(D - x)\Big]_r^{D-r} \tag{37}$$

$$= 4\,I \log_e \frac{D - r}{r} \text{ maxwells per centimeter of line,}$$

$$= 2\,I \log_e \frac{D - r}{r} \text{ maxwells per centimeter length of wire.} \tag{38}$$

This expression will be of use to us later.

The field at some point not on the line joining the two wires may be found by combining the values of H at the point in accordance with the laws of vectors as shown in Fig. 90. Since the two forces now are not in the same direction, they must be added vectorially instead of algebraically.

By finding the value of H and then the value of B, at every point P in the vicinity of the two wires, we may map out the entire field. This distribution of the flux lines about two parallel wires carrying current in opposite directions

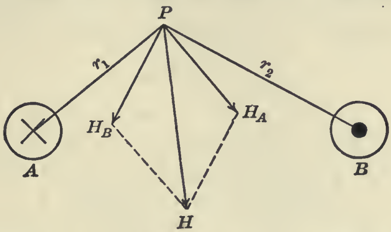

FIG. 90. The field at a point P not on a line joining A and B is the resultant of H_B and H_A.

may thus be found to be as in Fig. 91; that is, a series of circles with their centers all on the line joining the centers of the wires but not concentric with the wires. In similar

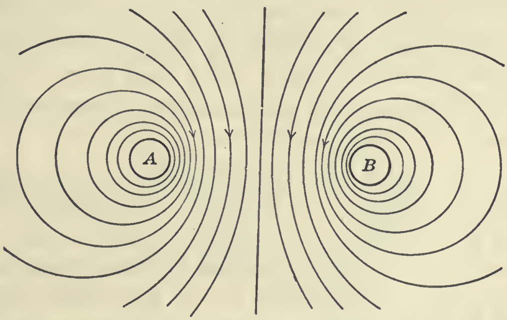

FIG. 91. The shape of the magnetic field about two parallel wires carrying current in opposite directions.

manner we may find the distribution about a pair of wires carrying current in the same direction, and it will be found to be much as is shown in Fig. 92. We know that two parallel wires carrying current in opposite directions repel each other, and that two parallel wires carrying current in the same direction tend to be drawn together. By referring

to the figures showing the distribution of flux about such wires, it is seen that this action may be interpreted by stating that flux lines tend to crowd apart when running parallel to each other, and on the other hand tend to shorten in length. The first effect tends to push the coils of wires apart when they carry current in opposite directions, and the second

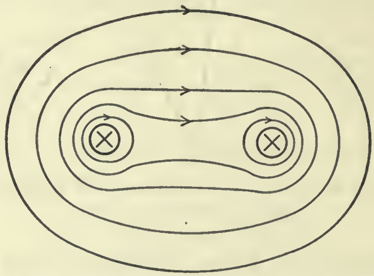

FIG. 92. The shape of the magnetic field about two parallel conductors carrying current in the same direction.

effect tends to draw wires together when they carry current in the same direction, much as if they were surrounded by a series of rubber bands. We shall later compute the amount of this force and show its great importance in electrical work.

Prob. 1–7. What is the line integral of H at a radius of 2.5 inches about a straight wire carrying 25 abamperes?

Prob. 2–7. In order to produce a flux density $B = 6$ gausses at a point 3 inches from the center of a straight wire in air in a plane perpendicular to the wire, what must be the current in amperes?

Prob. 3–7. Plot the flux density for points on a line perpendicular to the axis of a wire which is carrying a current of 1 abampere.

Prob. 4–7. A rectangle 3 by 5 inches carries a current of 3 amperes. What is the flux density at the intersection of the diagonals if the effect of the leads may be neglected?

Prob. 5–7. What flux is enclosed by a pair of No. 000 copper wires spaced 6 feet between centers and of one mile length each when $I = 30$ amperes? Neglect the end effects.

Prob. 6–7. In Fig. 93, A and B represent the "line and return" of a single-phase power line carrying a maximum of 48 amperes. x and y are the "line and return" of a telephone line running parallel to the power line. How much flux links the telephone line per mile when the maximum current is flowing in the power line?

FIG. 93. A and B represent the line and return of a power line, x and y a pair of telephone wires.

Prob. 7–7. The current in the power line of Prob. 6–7 reverses 120 times per second from a maximum current of 48 amperes in one direction to the same value in the opposite direction. What average voltage is induced in the telephone circuit if it is 20 miles in length?

Prob. 8–7. If the telephone wires of Prob. 6–7 were placed in a vertical plane 40 feet from B, with x at a distance of 9 inches above the level of the power line and y at a distance of 9 inches below it, what maximum flux per mile would link them?

70. Field Inside a Conductor. When a wire carries a current, there is a magnetic field produced not only external to the wire but also in the interior of the wire. We will now examine the strength of a field on the inside of a cylindrical wire carrying a current.

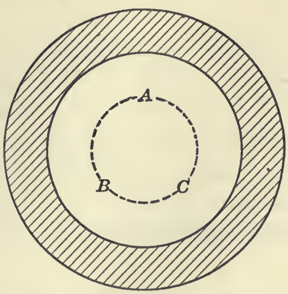

FIG. 94. A tubular conductor.

First, let us consider a tubular conductor such as is shown in Fig. 94. We will assume that the current density is uniform across the cross-section of the conductor. For any circle inside of the tube, such as ABC, the line integral of H will be zero, for this circle links no conductor carrying current, and we found that the line integral

of H about a closed path is always equal to zero when the path links no current. There is no current linkage in this case, for all the current-carrying material is external to the path which we have chosen. Since the line integral of H is zero about this circle, it follows from symmetry that H itself must be zero along the circle. This holds for any circle which is entirely inside of a tube. We may therefore conclude that the magnetic field inside of a tubular conductor carrying uniform current is everywhere zero.

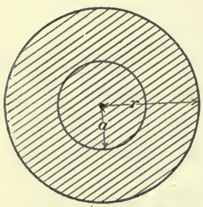

FIG. 95. A solid conductor of radius r.

Consider now a solid cylindrical conductor carrying a current uniformly distributed across its cross-section. Such a conductor of radius r is shown in Fig. 95. Let us examine the field strength at a distance a from the center.

Draw a circle with radius a. By symmetry the field strength is the same at every point on this circle and is everywhere tangent to the circle. The line integral of H_a around this circle is accordingly equal to simply H_a times the circumference of the circle; that is,

$$\int H dl = 2\pi a H_a. \tag{39}$$

We know, however, that this line integral is equal to $0.4\pi NI$, where N is the number of turns, (in this case one), and I is the current inside of the path of integration. The current to be taken in this case is hence the fraction of the total current which lies within the radius a. If the total current in the wire is I abamperes, the amount within the circle of radius a will be

$$\frac{a^2}{r^2} I \text{ abamperes.} \tag{40}$$

Therefore we have

$$2\pi a H_a = 4\pi \frac{a^2}{r^2} I \tag{41}$$

or

$$H_a = \frac{2\,I}{r}\,\frac{a}{r}. \tag{42}$$

If I is in amperes and the material of the conductor is copper of permeability unity, we have

$$B = \frac{0.2\,I}{r}\,\frac{a}{r}\text{ gausses.} \tag{43}$$

On the other hand, if the wire is, for instance, of iron, of permeability other than unity, we must take the permeability into account and write

$$B = \frac{0.2\mu I}{r}\,\frac{a}{r}. \tag{44}$$

We are now in a position to give the field strength at any point inside or outside of a long wire carrying current. This may be shown in a diagram, as in Fig. 96, where the flux

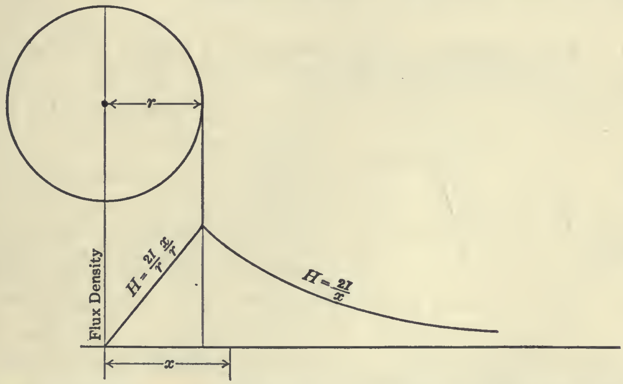

FIG. 96. The flux density is greatest at the surface of the wire as shown by the curve.

densities are plotted as ordinates and the distances from the center of the wire as abscissas. It will be noted that the flux density follows one law in the material of the wire and a second law outside of a wire, so that there is a

break in the curve. Note also that when $x = r$, that is, at the surface of the wire, the results obtained from these two formulas coincide for a wire of unity permeability.

We shall see later that the distribution of flux inside a large wire is an important factor in alternating-current work. The variation of this flux as the current changes gives rise to what is known as skin-effect. By this effect, the current is crowded toward the surface of the wire and there appears a much larger resistance to high-frequency alternating currents than there is to direct currents.

Prob. 9–7. In the wire of Prob. 3–7, plot the flux inside the wire with the center as an origin, magnifying the wire so as to make the plot reasonable in size.

Prob. 10–7. Derive the formula for the total flux inside a solid round wire carrying current.

Prob. 11–7. Derive the formula for the total flux linkages inside a solid round wire carrying current.

71. Flux Density at the Center of a Single Turn of Circular Form Carrying Current. It will now be necessary for us to compute the flux density existing at the center of a single turn of wire of circular shape carrying a current I, as shown in Fig. 97. The other view of this coil is shown in Fig. 98, with the direction of the flux lines. The flux at P at the center of the turn is perpendicular to the plane of the turn.

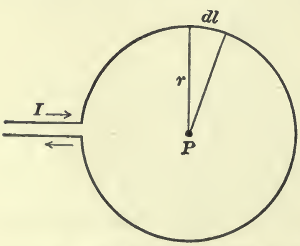

Fig. 97. A single circular turn of wire carrying a current I.

The element of length, dl, in Fig. 97, exerts an amount of magnetizing force at P which by equation (20) is

$$dH = \frac{Idl}{r^2}\sin\theta.$$

Since this element is perpendicular to the radius r, θ is a right angle and the sine of θ is unity, this becomes

$$dH = \frac{I}{r^2}\,dl. \tag{45}$$

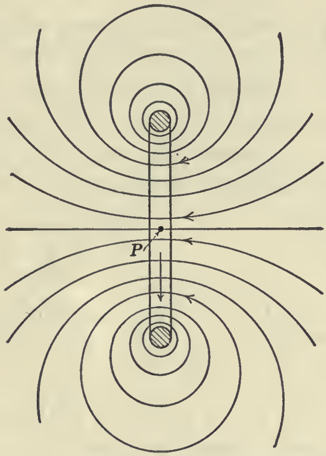

FIG. 98. A side view of the wire in Fig. 97.

The total effect of the current is the integral of dH; that is,

$$H = \int dH = \int_0^{2\pi r} \frac{I}{r^2}\,dl, \tag{46}$$

where the integration is to be carried once around the circle. This integrated gives

$$H = \frac{2\pi I}{r}. \tag{47}$$

If the single turn is in air, the flux density is also given by the expression

$$B = \frac{2\pi I}{r}, \tag{48}$$

where, of course, I is in abamperes. If I is in amperes, this becomes

$$B = \frac{0.2\pi I}{r}. \tag{49}$$

Let us now examine the magnetizing force produced at the center of an arc of one centimeter radius and one centimeter length, as shown in Fig. 99, when a current of one abampere flows. Since the leads are carried off on radii they will exert no magnetizing force at P. The magnetizing effect of the arc is found by an integral similar to that used for the effect of a single turn, except that the limits of integration will now be simply zero to one; that is,

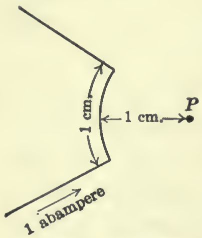

FIG. 99. Field at P is produced by 1 cm. arc of 1 cm. radius with P as center.

$$H = \int_o^1 \frac{Idl}{r^2}, \tag{50}$$

which gives for $I = 1$, $r = 1$, the simple expression

$$H = 1.$$

The effect of this arc of a circuit is therefore to produce a unit magnetizing force at the center. We are thus led to a new statement, which if we wish may be used as a definition of unit current:

One abampere is that current which flowing in a unit length of an electric circuit bent into an arc of unit radius will produce unit magnetizing force at the center of the arc.

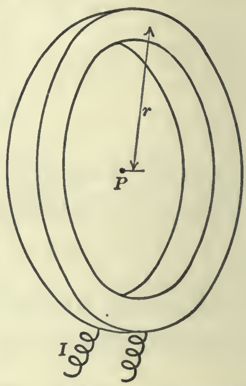

FIG. 100. A circular concentrated coil of many turns.

The flux density produced in air by a concentrated coil of wire as shown in Fig. 100 of N turns and carrying a current of I amperes, is evidently

$$B = \frac{0.2\pi NI}{r}. \tag{51}$$

Prob. 12–7. (*a*) A concentrated coil of wire having 36 turns and carrying a current of 0.3 ampere produces what field intensity at the center of the coil? (*b*) What flux density? (*c*) What effect would a different permeability of the medium surrounding the coil have on flux density? Radius is r cm.

72. Flux Density at a Point on the Axis of a Circular Coil. We have found the flux density at the center of a circular coil. We are also interested in the flux density at some other point on the axis of the coil. Such a coil, of a single turn, is shown in Fig. 101 and 102, and we will proceed to compute the flux density at point P at a distance of a centimeters from the center of the coil.

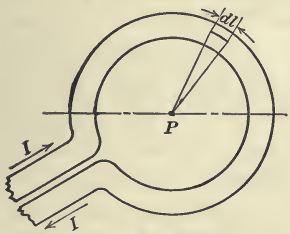

FIG. 101. A single circular turn of wire carrying current.

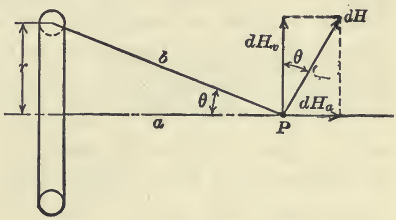

FIG. 102. Side view of wire in Fig. 101.

Consider an element of the coil dl and call b its distance from P. Call θ the angle between a and b. By the formula which we have previously used for the magnetic effect of an element of a circuit carrying current, we may write the value of the magnetizing force dH due to the element of the circuit dl. This will be

$$dH = \frac{Idl}{b^2}; \tag{52}$$

that is, it is proportional to the current and the length of the segment, and inversely proportional to the square of the distance from the point to the element. Since b is perpendicular to dl, the cosine of θ, which previously appeared

in the formula for the effect of an element, is in this case unity.

This magnetizing force dH will be perpendicular to b. It may be resolved into two components, dH_v perpendicular to a and dH_a parallel to a. From the geometry of the figure, the vertical component will be

$$dH_v = dH \cos \theta, \tag{53}$$

and the horizontal component

$$dH_a = dH \sin \theta. \tag{54}$$

The total magnetizing force at P will be the vector sum of all of the components of magnetizing force due to the element dl taken completely around the circle. We may note that to every element dl on the top of the circle, having a vertical component of magnetizing force dH_v directed upward, there will be an opposite element on the other side of the circle which will have an equal vertical component of magnetizing force directed downward, and these two components will cancel. Hence the vertical components of magnetizing force for the whole circle cancel out completely. On the other hand, since the horizontal components are all in the direction of the axis and all directed toward the right, they will add algebraically.

In order to find the total magnetizing force at P, we must therefore add the effect of the horizontal components due to the element dl taken completely around the circle; that is,

$$H = \int_o^{2\pi r} dH \sin \theta = \int_o^{2\pi r} \frac{I \sin \theta}{b^2} dl. \tag{55}$$

Integrating this expression and inserting the limits, we have

$$H = 2\,\pi r \frac{I \sin \theta}{b^2}. \tag{56}$$

Since

$$\sin \theta = \frac{r}{b}, \tag{57}$$

we have

$$H = \frac{2\pi I}{r}\sin^3\theta; \tag{58}$$

or since

$$b = \sqrt{a^2 + r^2}, \tag{59}$$

this equation becomes, upon substitution,

$$H = \frac{2\pi r^2 I}{(a^2 + r^2)^{3/2}}, \tag{60}$$

which is our final formula for the magnetizing force at the point P. If the coil is surrounded by air, the flux density will also be given by

$$B = \frac{2\pi r^2 I}{(a^2 + r^2)^{3/2}}. \tag{61}$$

If for the single turn we substitute a concentrated coil of N turns, the total flux density at a point on the axis will be given by

$$B = \frac{2\pi r^2 NI}{(a^2 + r^2)^{3/2}}. \tag{62}$$

Prob. 13–7. In Fig. 101, if the coil has 5 concentrated turns carrying a current of 4 amperes, what is the flux density at a point P where $a = 6$ inches and $r = 6$ inches? $\mu = 1$.

73. The Air-Core Solenoid. A solenoid is a coil of wire usually wound with circular cross-section and of a length great in comparison with its diameter. Solenoids are used on relays, circuit-breakers and similar apparatus, for operating the mechanism when a certain amount of current flows through the coil. They are also used in measuring instruments. At the present time, we will confine ourselves to a consideration of a solenoid with an air core, or with a core made of non-magnetic material. Such a solenoid is shown in Fig. 103.

It may have one layer or several layers. For simplicity we will consider at the present time a single-layer solenoid. If the conducting wire were rectangular and covered with

very thin insulation so that a cross-section had the appearance of Fig. 104, the current around the coil would closely

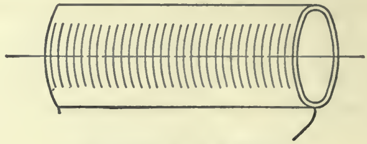

FIG. 103. A solenoid.

approximate a "current sheet." The following treatment assumes such a current sheet to flow, although most coils are

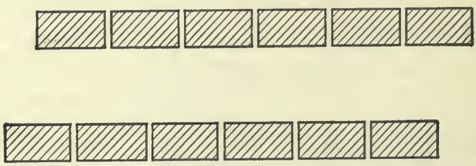

FIG. 104. A solenoid may be considered to be made up of square wires having a very narrow space between them. This would mean that practically a sheet of current flows around the coil.

constructed of round wires and thus depart from these conditions. For this reason the following treatment is an approximation.

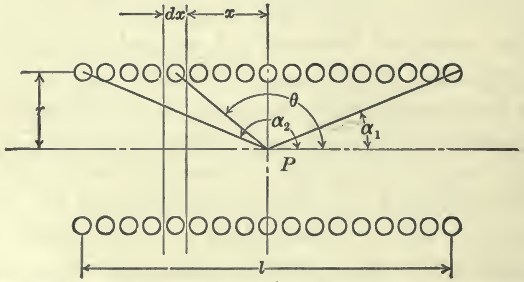

FIG. 105. Find the magnetizing force at point P, the center of the solenoid.

We will first investigate the field strength at the center point of the solenoid. Point P, Fig. 105, is the central point of the solenoid.

Consider that the solenoid is l centimeters long and has N turns, which carry a current of I abamperes. Consider a belt of conductors the plane of which is at a distance x from point P, and of width dx. The total number of ampere-turns comprised in the belt of conductors included in the length dx will then be

$$NI\frac{dx}{l}.$$

The effect of this belt upon the point P is the effect of a circular coil upon a point in the axis. Call this effect dH to indicate the portion or the magnetizing force at P which is due to the belt of conductors within the region dx. Then from the formula which we have derived for such a circular coil, we shall have

$$dH = \frac{2\pi NI}{rl}\sin^3(\pi - \theta)dx. \tag{63}$$

We may reduce this to an expression entirely in terms of θ and the dimensions of the coil, as follows. From the triangle in Fig. 105,

$$x = r\cot(\pi - \theta) = -r\cot\theta, \tag{64}$$

and differentiating, we have

$$dx = r\csc^2\theta\, d\theta. \tag{65}$$

Substituting this in the equation above, it becomes

$$dH = \frac{2\pi NI}{l}\sin\theta\, d\theta. \tag{66}$$

To find the total value of H, we integrate this expression to cover the entire coil. If θ increases from the angle α_1 to angle α_2, the entire coil will be included. This leads to the following integration

$$H = \frac{2\pi NI}{l}\int_{\alpha_1}^{\alpha_2}\sin\theta\, d\theta$$

$$= \frac{2\pi NI}{l}\Big[(-\cos\theta)\Big]_{\alpha_1}^{\alpha_2}$$

$$= \frac{2\pi NI}{l}(\cos\alpha_1 - \cos\alpha_2). \tag{67}$$

This resulting expression is the formula for the magnetizing force, H, at a point on the axis of a long solenoid. If the entire surroundings are non-magnetic, that is, if there is no iron present, the formula gives also the flux density at any point on the axis of a solenoid in terms of the angles α_1 and α_2, which are subtended by the ends of the coil.

In the case where P is at the midpoint of the coil,

$$\alpha_2 = \pi - \alpha_1. \tag{68}$$

From this

$$\cos\alpha_2 = -\cos\alpha_1 \tag{69}$$

and

$$(\cos\alpha_1 - \cos\alpha_2) = +2\cos\alpha_1. \tag{70}$$

The magnetizing force at the center point of a solenoid hence becomes

$$H = +\frac{4\pi NI}{l}\cos\alpha_1, \tag{71}$$

or in terms of the radius and length of the coil

$$H = \frac{4\pi NI}{\sqrt{l^2 + 4r^2}}. \tag{72}$$

If the solenoid is very long in proportion to its diameter, say twenty times its diameter in length, the angle α is very small and approximately

$$\cos\alpha_1 = 1, \tag{73}$$

which gives

$$H = \frac{4\pi NI}{l}. \tag{74}$$

For an air-core solenoid in which no magnetic material is

present, the flux density will be equal to the magnetizing force and we may write

$$B = \frac{4\pi NI}{l}. \tag{75}$$

This formula gives the flux density at the center point of a long solenoid on the assumption that the current flows in a sheet around the coil.

We will now examine the flux density at some other point not on the axis but in the cross-section of the solenoid at the center. In Fig. 106 and 107, P is such a point. At P pass two planes each parallel to the axis of the solenoid and making an angle of $d\phi$ with each other, as shown in Fig. 107. These will cut out on the surface of the solenoid a

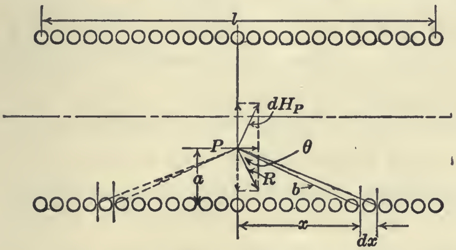

Fig. 106. Find the flux density at P, a point on the center plane but not on the long axis of the coil.

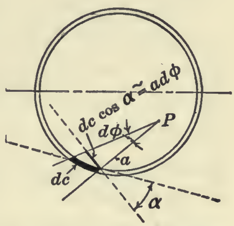

Fig. 107. End view of coil in Fig. 106.

certain strip of the conductor material parallel to the axis of the coil and of width dc.

We will now examine the portion of the magnetizing force at P produced by the current in an element of the winding of length dc and width dx, as shown in Fig. 107.

The magnetizing force due to an element we have previously seen is proportional to the current in the element and to the length of the projection of the element perpendicular to a line joining it with the point in question, and inversely proportional to the square of its distance from the point. In the c.g.s. system, the proportionality factor is one.

In the width of winding dx there will be $\frac{NI}{l}\,dx$ amperes. This is the current in the element. The projection of dc on a line perpendicular to a line joining it with P is

$$dc \cos \alpha,$$

which is equal to $a d\phi$ since $d\phi$ is a differential angle. The distance from the element to P is equal to b.

Accordingly for the portion of the magnetizing force due to this element we have

$$dH_P = \frac{a d\phi}{b^2} \times \frac{NI}{l} dx \tag{76}$$

where $b^2 = (a^2 + x^2)$.

This magnetizing force will not be parallel to the axis of the solenoid, but will have a component which is parallel and one which is perpendicular to the axis. The perpendicular component need not be considered because it will cancel out when the entire length of the strip on the surface of the solenoid is taken into account; for to each element dx at a distance x, there corresponds an element on the opposite side of the center and at an equal distance, in which we have the magnetizing force directed as indicated by the dash-line R and hence with an equal and opposite component perpendicular to the axis. We need therefore consider only the horizontal component. To obtain the component parallel to the axis, we have only to multiply the value of the force $d\,H_P$ (Equation 76) by the cosine of the angle θ, Fig. 106. Call this component $dH_{a\phi}$. Thus the magnetizing force parallel to the axis of the coil at any point P within the coil and due to current in an element of the winding is

$$dH_{a\phi} = \frac{a d\phi}{b^2} \times \frac{NI}{l} \cos \theta \; dx. \tag{77}$$

The effect of a strip of dc width and running the entire length

of the coil can be found by integrating this expression between the limits $x = -l/2$ and $x = +l/2$.

Thus

$$H_{d\phi} = \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{NI}{l} \times \frac{ad\phi}{b^2} \cos\theta\, dx. \tag{78}$$

This integral may be reduced to be all in terms of θ by making substitution from

$$x = a \tan\theta, \tag{79}$$

$$dx = a \sec^2\theta\, d\theta,$$

$$b = a \sec\theta. \tag{80}$$

Since the length of the solenoid is great compared with the diameter, the limit of the angle θ is $\pi/2$. The expression then becomes

$$H_{d\phi} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{NI}{l} d\phi \cos\theta\, d\theta, \tag{81}$$

which, integrated in exactly the same manner as above, gives

$$dH = \frac{2NI}{l} d\phi. \tag{82}$$

This is the magnetizing force at P produced by that portion of the conductor which lies within the angle $d\phi$. In order to obtain the total effect of the solenoid, we need now simply to integrate ϕ from 0 to 2π; that is,

$$H = \int_0^{2\pi} \frac{2NI}{l} d\phi = \frac{4\pi NI}{l}. \tag{83}$$

We thus find that the magnetizing force, and also (in air) the flux density, at the point P, are just the same as at a point in the center of the solenoid. It follows that the flux density across the mid-section of a long solenoid is

uniform, the lines are parallel to the axis of the solenoid at this point, and the total flux is

$$\phi = BA = \frac{4\pi NI}{\frac{l}{A}} \cdot \tag{84}$$

We are led to the remarkable conclusion that a long solenoid acts as if it had a reluctance $\frac{l}{A}\cdot$ The flux produced at the center of a long solenoid is exactly the same as would be produced if all of the lines passed completely through the length of the solenoid, thus producing uniform flux density, and as if there were no reluctance whatever in the path outside the solenoid itself.

We know, of course, that the path outside the solenoid has a certain small reluctance. It is true also that the lines do

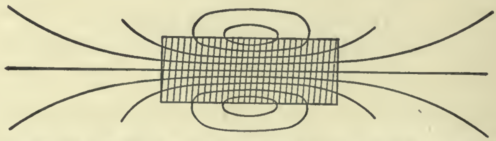

Fig. 108. Half of the flux which passes through the center of the coil leaves the coil before reaching the ends of the coil.

not all completely thread a solenoid, but half of them leave through the sides of the solenoid as shown in Fig. 108. It happens, however, that these two effects exactly offset each other. As far as the flux at the mid-section of the solenoid is concerned, we may therefore treat the solenoid as if we needed to consider only the reluctance of the air path inside of the winding.

In the above we have assumed that the wires were small in diameter and closely spaced, so that the effect was to produce practically a uniform current density along the surface of the solenoid. Our formula will be found to hold, therefore, for only reasonable distances from the sides of the solenoid. For a point close to the wires, the effect of

the size of the wires will enter, and corrections will be necessary to the formula as above derived. These corrections are too involved to be introduced here. A careful treatment of this matter will be found in the Bulletin of the U. S. Bureau of Standards No. 1, vol. 3, page 3.

Prob. 14–7. Prove the statement made above that one-half the magnetic-flux lines leave a long solenoid, air-core type, through the sides of the coil.

Prob. 15–7. A solenoid has a length of 11 inches and a radius of 0.5 inch. The number of turns to the inch is 23. A current of 2.6 amperes flows through the coil. What is the field intensity at the center of the solenoid? See Fig. 106.

Prob. 16–7. What is the total flux expressed in maxwells at the center section of the solenoid of Prob. 15–7?

Prob. 17–7. A long solenoid is desired with a flux at the center of 1000 lines. The diameter must be 1.7 inches and the number of turns to the inch 25. What current must the wire carry?

74. Calibration of a Ballistic Galvanometer. The above formula for the flux density at the center of a long solenoid is convenient in calibrating a ballistic galvanometer for use in measuring flux. It was noted in Art. 57 that the de-

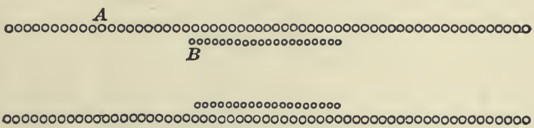

FIG. 109. Coil B is a measuring coil.

flection of such a galvanometer is proportional to the total change of flux linkages in a measuring coil connected with it. In order to be able to find the total flux by this method, however, it is necessary to calibrate the galvanometer, that is, find its deflection for some known change of linkages.

For this purpose we may use an arrangement such as is shown in Fig. 109. A is a long solenoid with $\frac{NI}{l}$ ampere-turns per centimeter of length when connected with the

battery H as shown in Fig. 110. B is a measuring coil of a number of turns N' and cross-sectional area S, placed inside and close to the center of A.

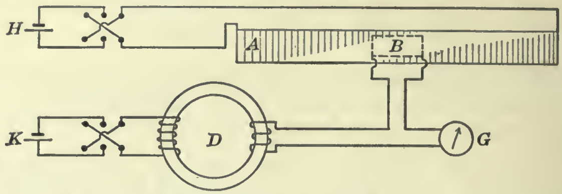

FIG. 110. Showing how coils A and B are used to measure flux in ring D.

We know that when a current of I abamperes is flowing in A, the flux density produced at the center will be

$$B = 4\pi \frac{NI}{l}. \tag{85}$$

The total flux passing through B is accordingly

$$\phi = 4\pi \frac{NI}{l} S, \tag{86}$$

and the flux linkages thereby produced

$$N'\phi = 4\pi \frac{NN'I}{l} S. \tag{87}$$

In practice, the coil B is connected in series with the galvanometer and also in series with the coil D enclosing the flux which it is desired to measure.

When the switch is closed, forcing a current of I abamperes through A, there will be a deflection of G. Call this deflection d. By a similar battery arrangement, the flux may now be caused to vary through coil D, which we will assume of N'' turns. The galvanometer will again deflect and we will call this second deflection d'. Then if ϕ' is the flux in the specimen under test, we shall have

$$\frac{d}{d'} = \frac{4\pi NN'IS}{\phi' N'' l}, \tag{88}$$

from which

$$\phi' = \frac{d'4\pi N N' I S}{d N'' l}. \tag{89}$$

All of the quantities on the right-hand side of this equation can be counted or measured by convenient means. By the ratio of two deflections, we thus find the total flux in the specimen, here shown as a ring, which links the measuring coil *D*.

It will be noted that reversing switches are used. In practice, we will throw each of these switches in turn first in one direction and then in the other, so that the galvanometer deflection will be twice what it would be for a single change of flux. The reason for this is to avoid any effect of retentivity in the sample, an effect which will be considered in the next chapter.

It will be noted also that *B*, *D* and *G* are connected permanently in series. This is in order to keep the constants of the circuit of the ballistic galvanometer unchanged while it is subjected to the two processes successively. The deflection of the galvanometer will depend upon the resistance of the circuit to which it is connected. For this reason it is necessary to keep the entire galvanometer circuit unchanged while making the calibration and the actual measurements in order that the deflection of the galvanometer may be proportional simply to the two values of flux under consideration, one of which we can compute and the other of which we wish to measure.

Prob. 18–7. If in the center of the solenoid of Prob. 17–7 there is placed a coil *B*, as in Fig. 109, having a cross-section of 1.4 sq. inches and 200 turns of wire, what will be the flux linkages in coil *B*?

Prob. 19–7. If a specimen is under test and the galvanometer shows a deflection of 12.3 centimeters for the test specimen and a deflection of 4.2 centimeters for the test solenoid of Prob. 18–7, what is the flux in the specimen?

Prob. 20–7. What is the constant of the galvanometer in Prob. 19–7 as a flux meter?

75. The Toroid. A toroid is a solid in the shape of a doughnut. For reasons explained later, this is the shape of the core of the repeating coils used to connect telephone circuits with one another. If a core of this form is uniformly wound with a magnetizing winding, as shown in Fig. 111, the resulting field may be computed as follows.

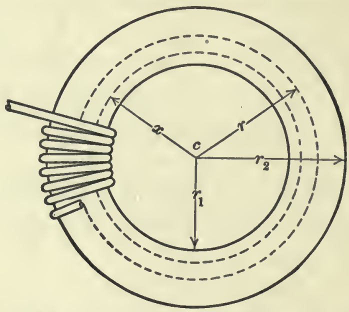

FIG. 111. A toroid, consisting of an iron ring. It is magnetized by the coil.

By symmetry, the lines of force must all be circles about the axis through the center C. Moreover, the magnetizing force, provided the toroid is uniformly wound, must be uniform along any one of these circles.

Consider such a circle of radius x, where x lies between the inside and outside radii r_1 and r_2. The line integral of the magnetizing force, H_x, around this circle is equal to 4π times the number of abampere-turns linked. If there are N turns on the toroid carrying I abamperes, this gives the equation

$$2\pi x H_x = 4\pi NI; \qquad (90)$$

that is,

$$H_x = \frac{2NI}{x}. \qquad (91)$$

Since the magnetizing force around a circuit is in this case uniform, we may use a core of permeability other than unity

without distorting the field. If the toroid is made of iron of permeability μ, the flux density at a radius x will thus be given by

$$B_x = \mu \frac{2NI}{x}. \tag{92}$$

The total flux, considering a toroid of cross-section A, will then be obtained by integrating this flux density over the cross-sectional area of the core A. If the thickness of the core is small compared with its diameter, we shall not produce great error if we write the total flux as equal to the flux density at a mean radius multiplied by the total cross-sectional area. Thus

$$\phi = \mu \frac{2NIA}{r}. \tag{93}$$

However, this equation is true only in the case where μ is constant throughout the iron and for the values used for I. If we wish, we may write it in the form

$$\phi = \frac{4\pi NI}{\frac{l}{\mu A}}. \tag{94}$$

Note that this last formula is the usual Ohm's law for the magnetic circuit. If, as in the case of a toroid, we know accurately the path of the flux and the cross-section of the path at every point, it is possible for us to write this equation immediately. The reluctance of the path is, of course, $l/\mu A$, where l is the mean flux path, that is, approximately the length of the circumference at the mean radius r; or

$$l = 2\pi r.$$

The total flux would then, as we should expect, be the total magnetomotive force, $4\pi NI$, divided by this reluctance.

It is important to realize, however, that this is an approximation, for the lengths of the paths of the various flux lines differ. We have seen above, moreover, that the flux den-

sity varies inversely as the distance from the axis of the toroid. It is therefore not correct for us to use an average value of flux density and multiply by the cross-section. It is permissible to do this in problems where an exact solution cannot be made or where we know that an approximate solution is correct to sufficient engineering accuracy. We must be careful how averages are used, however, and in each case be sure either that an average will give an accurate result or that the errors thereby introduced are permissible. It will accordingly be instructive for us to compute the exact amount of flux passing through a uniformly wound toroid under the above conditions and compare it with the approximate formula obtained by using a mean length of path as above.

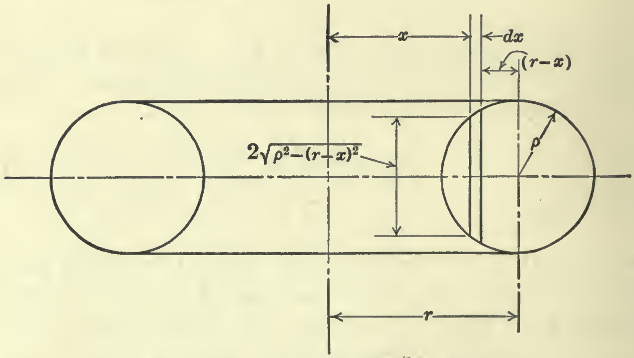

FIG. 112. A cross-section of a toroid.

In Fig. 112 is shown a cross-sectional view of the toroid. Assume a distance x, and an element of this cross-section cut out by the width dx. The area of this element is evidently

$$2\sqrt{\rho^2 - (r - x)^2}\,dx. \tag{95}$$

Since we have found that the flux density at this distance from the center line x is

$$B_x = \mu\frac{2NI}{x}, \tag{96}$$

the total flux passing through this element of the cross-section is

$$d\phi = 4\mu \frac{NI}{x} \sqrt{\rho^2 - (r - x)^2}\, dx. \tag{97}$$

We may thus obtain the total flux by integrating this expression from $x = r_1$ to $x = r_2$; that is,

$$\phi = 4NI \int_{r_1}^{r_2} \sqrt{\frac{\rho^2 - (r - x)^2}{x^2}}\, \mu dx. \tag{98}$$

The evaluation of this integral is somewhat involved, but it may be worked out when μ is a constant by means of a table of integrals.* Evaluating, inserting the limits and making a few simple reductions, it becomes

$$\mu\pi\,(r - \sqrt{r^2 - \rho^2}). \tag{99}$$

Inserting this value in our expression for the total flux, we obtain

$$\phi = 4\mu NI\pi\,(r - \sqrt{r^2 - \rho^2}). \tag{100}$$

This is the exact expression for the total flux in a toroid, but here again μ must be a constant.

When ρ is small compared with r, that is, when the toroid is of large diameter and correspondingly small cross-section, we may obtain an approximate formula by expanding the radical by the binomial theorem, thus:

$$(r^2 - \rho^2)^{1/2} = r - 1/2\frac{\rho^2}{r} + \cdots \tag{101}$$

If we neglect terms beyond the second in this, our formula becomes

$$\phi = 4\mu NI\pi\,(1/2\frac{\rho^2}{r}); \tag{102}$$

which, using the abbreviations

$$l = 2\pi r$$

* See Pierce, Table of Integrals, 187, 183, 160.

and
$$A = \pi\rho^2,$$
becomes
$$\phi = \frac{4\pi NI}{\dfrac{l}{\mu A}}, \tag{103}$$

the same formula which we derived for the toroid from the simple assumption that the reluctance could be computed from the length of path at mean radius.

In cases where ρ is not negligible in size compared with r, we must use the exact formula

$$\phi = 4\pi NI\mu\,(r - \sqrt{r^2 - \rho^2}). \tag{104}$$

A good idea may be formed of the approximation involved in using the approximate formula by working out an example.

Assume a toroid of mean radius 4 centimeters, the radius of the cross-section being 2 centimeters. We shall then have

$$\begin{aligned} r &= 4, \\ \rho &= 2, \\ l &= 8\pi, \\ A &= 4\pi. \end{aligned} \tag{105}$$

Using the approximate formula for the flux in this case, it becomes

$$\phi = 2\pi NI\mu. \tag{106}$$

The exact formula, on the other hand, gives

$$\phi = 4\pi NI\mu\,(4 - \sqrt{4^2 - 2^2}). \tag{107}$$

This becomes, upon reduction

$$\phi = 2\pi NI\mu\,(1.07). \tag{108}$$

We thus see that an error of about 7% would be introduced by assuming that the approximate formula was nearly enough correct. This approximate formula is based upon an average. Thus averages can be used only where linear relations hold, that is, where averages will yield exact

values; or, on the other hand, where the amount of error introduced by including an average can be estimated and tolerated.

In this latter case, if the core of the toroid were of iron, say of permeability 1000, an error of even 6% in the formula would be tolerable, for the reason that the value of μ would in many cases be in doubt to this extent and the formula holds only when μ is constant. If, however, such a toroid were being used for finding an exact value for the permeability of a sample of iron, this 6% error might be very serious. It is thus evident that the question of whether or not an error can be tolerated depends entirely upon the conditions under which the computation is being made.

Referring back to Fig. 111, suppose we assume the value of x less than r_1. The line integral of H around the circumference of this radius must then be zero, for such a circumference does not link any circuit carrying current. By symmetry, therefore, H all the way along the circumference must be zero. This means practically that outside of a uniformly wound toroid there is no field whatever. In other words, such an arrangement has no leakage flux. For this reason and others, cores of this sort are used in the repeater coils of telephony, where a leakage flux would cause cross-talk, which is naturally to be avoided where-ever possible. In a practically wound toroid, there will, however, always be a certain minute amount of leakage flux, since it is impossible to wind a coil uniformly. The wires must have insulation, which means that as we pass around the circumference of the core, we encounter first a space carrying current and then a space of insulation. In order to have an absolutely uniformly wound coil, the insulation would have to be entirely eliminated and the wires made rectangular in form and closely spaced, as for instance is shown in the cross-sectional view of Fig. 113. Such an arrangement is known as a " current sheet." In a practical arrangement with round wires, as shown in Fig. 114, there

will always be a certain amount of leakage flux around each individual wire.

Prob. 21–7. A cast-iron toroid with the dimensions $r = 3$ inches, $\rho = 0.25$ inch, has 2000 turns of wire wound uniformly about the core. If the current is 0.1 ampere, what will be the flux in the core? See Fig. 111.

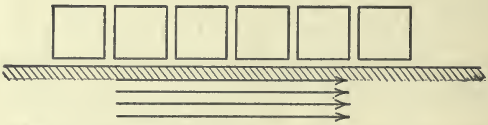

FIG. 113. Closely wound coils are considered to be constructed of rectangular wire with thin insulation between turns.

Prob. 22–7. How many turns would be required to produce a flux density of 65,000 lines to the square inch in the cast-iron ring of Prob. 21–7?

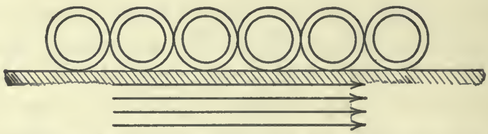

FIG. 114. Coils are usually constructed of round wire and some leakage takes place between turns.

Prob. 23–7. (*a*) What is the magnetomotive force acting upon each line of force in the ring of Prob. 22–7?

(*b*) What is the magnetizing force in ampere-turns per inch acting upon the lines of force at the outer edge?

(*c*) At the inner edge?

(*d*) At the center?

(*e*) Compute the average magnetizing force and compare it with the magnetizing force at the center.

76. Flux About a Conductor in a Slot. We have now examined magnetic circuits in two forms; first, those in which the magnetic circuit was entirely in iron; second, those in which the magnetic circuit was entirely in air or other non-magnetic materials. We have even examined a circuit in which a short air gap was introduced into a

circuit otherwise of iron. In all of these cases it has been possible to arrive at accurate solutions.

There is another class of problems, however, which in general are more difficult and require approximations for their solutions. These involve cases where the magnetic circuit is partly in iron but to a considerable extent in air.

FIG. 114a. Section of a motor frame showing method of installing coils in slots. *The General Electric Co.*

In such cases the reluctance of the magnetic circuit is usually almost entirely in the air path. This is because of the fact that there is such a great difference between the permeability of air and the permeability of iron. If the length of the air path is a considerable percentage of the length of the total path of the flux, the reluctance of the path of iron may usually be entirely neglected. In cases of this sort, the difficulty of computation arises by reason of the fact that it is usually difficult to map out the path of the flux lines and accordingly to compute the reluctance of the air path.

One important problem of this sort is that of a conductor carrying current and imbedded in a slot in iron. Such an arrangement is shown in Fig. 114a, each slot of the armature core containing two conductors. Fig. 115 is a drawing of one conductor in a slot, the cross-hatched area being the conductor of rectangular form, which we will assume carrying a current of I abampercs. The outlined area is of iron. This figure is a cross-section, the conductor lying in a trough in the iron. Such an arrangement as this is found in almost all electrical machinery. The amount of flux produced by the conductor is of importance in considering the regulation of alternators, in determining the proper commutation of direct-current machines and so on. We will hence examine the methods of computing the amount of flux produced by such a conductor.

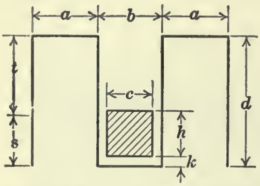

FIG. 115. A conductor lying at the bottom of a slot in iron.

The flux will surround the conductor somewhat as shown in Fig. 116. This flux may be divided into two parts, a part A which passes directly across from tooth to tooth, and a part B which passes around between the ends of the teeth. We will first examine the part A.

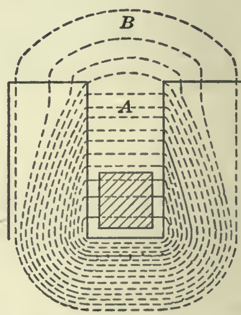

FIG. 116. The flux distribution about a conductor in a slot in iron.

For a flux line which crosses the slot above the conductor, as shown in Fig. 116, the computation is simple. Since the magnetizing force is almost entirely used up in the reluctance of the air path, we may write the line integral of H around this path as simply the product of H times the width of the gap b. This line integral is equal to the magnetomotive force of

the conductor, which will be $4\pi\ I$, since there is only one turn involved. We have, therefore,

$$Hb = 4\pi\ I \text{ gilberts.} \tag{109}$$

In the air, of permeability 1, the flux density will be equal to H, and hence we have for the flux density in the portion of the space above the top of the conductor and below the top of the slot a flux density

$$B = \frac{4\pi\ I}{b} \text{ gausses.} \tag{110}$$

Consider now a flux path such as 2 in Fig. 117. This crosses the conductor at a distance x above the bottom of the conductor. The line integral of this path is the same as

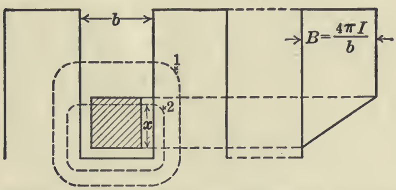

Fig. 117. Distribution of flux density about a conductor lying in a slot. Tooth-tip flux is not included in this figure.

before, namely Hb. This path, however, does not link the entire current, but only that part of the current in the conductor lying below the distance x, that is, the fraction x/h of the total current. We have, therefore, for this path the formula

$$Hb = 4\pi \frac{x}{h} I \text{ gilberts,} \tag{111}$$

which gives immediately

$$B = \frac{4\pi}{b}\frac{x}{h} I \text{ gausses.} \tag{112}$$

The flux which passes across the actual conductor thus has a density which is proportional to the distance x, that is, to

the distance above the bottom of the conductor. If we plot the flux density against the distance above the bottom of the slot, we shall accordingly obtain a figure such as is shown in the right-hand half of Fig. 117.

The area between this curve and the vertical axis is evidently the total amount of flux per centimeter length of slot passing directly across from tooth to tooth. We therefore obtain for the portion A of the total flux

$$\phi A = \frac{4\pi I}{b} t + \frac{4\pi I}{b}\frac{h}{2}$$

$$= \frac{4\pi I}{b}\left(t + \frac{h}{2}\right) \text{ lines per cm. length of slot.}$$

$$= \frac{4\pi I}{b}\left(t + \frac{h}{3}\right) \text{ linkages per cm. length of slot.} \quad (113)$$

In order to compute the amount of flux, E, passing between the ends of the teeth, it will be necessary to assume some shape for the path of these lines which seems reasonable and which admits of computation. A path such as shown in Fig. 118 is a reasonable assumption. The flux E at a distance from the edge of the tooth y is assumed to pass in a path made up of two arcs of circles and a straight line. The length of such a path is

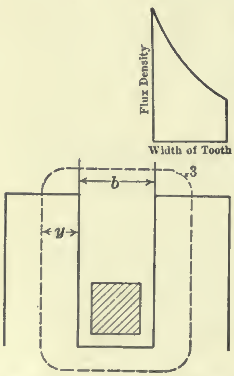

FIG. 118. The distribution of the tooth-tip flux density set up by a conductor in a slot.

$$b + \pi y$$

and since this path links the entire flux, we may write

$$H(b + \pi y) = 4\pi I, \quad (114)$$

which gives

$$B = \frac{4\pi I}{b + \pi y} \text{ gausses.} \quad (115)$$

This distribution of flux density is shown by the diagram

of Fig. 118. The total flux passing between the teeth will be the area under this curve, or

$$\phi_E = \int_0^a \frac{4\pi I}{b + \pi y} dy \text{ maxwells,} \tag{116}$$

which integrated gives

$$\phi_E = 4I \log (b + \pi y) \Big]_o^a. \tag{117}$$

This, when the limits have been substituted, becomes

$$\phi_E = 4I \log \frac{b + \pi a}{b} \text{ maxwells.} \tag{118}$$

The total flux produced by the conductor A will then be the sum of ϕ_A and ϕ_E, or

$$\begin{aligned} \phi &= \phi_A + \phi_E \\ &= \frac{4\pi I}{b}\left(t + \frac{h}{2}\right) + 4I \log \frac{b + \pi a}{b} \text{ lines per centi-} \\ &\qquad \text{meter length of slot.} \end{aligned} \tag{119}$$

We have thus derived a formula for most of the flux produced by a conductor in a slot. The flux thus computed is confined to the air and to the iron of the core and the adjacent teeth. Additional flux may be set up in the core and in teeth somewhat removed from the slot considered. Such problems are of common occurrence in electrical design and calculation. It will generally be found that they can be solved in the manner of the above problem to obtain at least an approximate result, by assuming a reasonable path for the flux as a basis for the calculation.

Prob. 24–7. Taking the following dimensions as applying to Fig. 115:

a = 0.75 inch
b = 0.80 "
c = 0.75 "
d = 2.50 "
h = 1.15 "
k = 0.05 "
s = 1.20 "
t = 1.30 " ,

plot the flux density curve and determine the total flux (excluding flux passing between the tips of the teeth) for a current in the conductor of 500 amperes.

Prob. 25–7. With the data of Prob. 24–7, plot the flux passing between the ends of the teeth and find the total value.

Prob. 26–7. Compute the total value of the flux set up by a current of 500 amperes in a conductor in the top of a slot. Slot and conductor have the dimensions of Fig. 115, *k* being the distance which the top of the conductor is below the ends of the teeth, and the values of *s* and *t* being interchanged.

77. Introduction of Iron into a Uniform Field. Engineers occasionally have to deal with a problem in which a piece of iron is introduced into a magnetic field initially approximately uniform. Such a problem is presented, for example, by an iron-core solenoid as shown in Fig. 119. This is a cross-sectional diagram, and shows simply a coil of wire into the center of which we have projected an iron plunger. In such sectional drawings it is usual to represent the cross-sectional area of the winding by a double cross-hatch. When current is applied to this winding, the iron plunger will be drawn toward the center of the coil. Solenoids are often used in electrical appliances in the place of electromagnets in which the core is stationary and an armature moves, because of the longer distance of movement which can be obtained with the solenoid arrangement. Relays such as are used in a power station for opening the main oil switches, in case of short circuit, are usually solenoids. The over-load release of a direct-current starting box is another example.

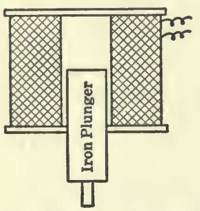

Fig. 119. Plunger type of solenoid.

These appliances present problems which are usually difficult of solution on account of the doubt existing as to the path of the magnetic flux. We have seen that an air-core solenoid may be fairly accurately computed, on the

simple assumption that all of the reluctance of the path of the flux lies within the winding and that the reluctance of the return path may be neglected. When, however, iron of high permeability fills part of this internal path, the outside reluctance can no longer be neglected in comparison with the reluctance inside of the coil. In order to solve such an arrangement, it is therefore necessary to map out the actual flux paths.

In engineering calculations it is usually sufficiently accurate to map out these paths in the same manner as was done in the last section in the case of a conductor in a slot. The accurate solution of a problem such as an iron-core solenoid involves difficult mathematics and is seldom undertaken by engineers. In fact, the more accurate solution must always be based upon some assumption in regard to the permeability of the material used. For this reason, the mathematically accurate solution, based as it is upon an incomplete premise, is likely to be nearly as much in error as a solution practically based upon reasonable assumption in regard to simple paths for the flux lines.

It will not be worth while for us to analyze such problems completely, for to treat such cases with any great degree of thoroughness would require a complete book.* There are, however, several simple propositions in regard to the paths taken by flux lines which if kept in mind will aid the engineer in making sufficiently accurate estimates. Such simple propositions taken together with engineering experience will usually enable him to solve his problems with sufficient engineering accuracy. The usual practical method of designing a solenoid, in fact, is to build several of approximately the size and shape desired, measure their characteristics and make a final design based upon this experience. Such a proceeding is likely to be quicker and less expensive than an attempted complete mathematical analysis. An accurate knowledge of some of the simple rules in regard

* For example, "Solenoids," by Underhill.

to the flux paths is, however, of great service in analyzing and comparing the results of such cut-and-try tests.

In the first place, it must be realized that we deal always with magnetic circuits, just as we deal always with electric circuits. We write

$$B = \mu H \text{ gausses}$$

as a general law, and we have found that in the neighborhood of a long conductor carrying current we may also write

$$H = \frac{2I}{r} \text{ gilberts per centimeter,} \tag{120}$$

which tells us that at a distance of 1 centimeter from such a center of a long wire carrying a current of 1 abampere, the magnetizing force will be 2 gilberts per centimeter. This does not mean, however, that in the case of a wire situated in an iron trough of semicircular cross-section, as shown in Fig. 120, we can find the flux density at a point in the iron trough by multiplying this value of H by the permeability of iron.

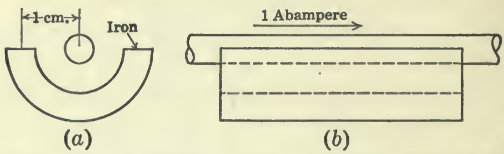

FIG. 120. A copper conductor in an iron trough.

The permeability of iron may be 1000, but the flux density in the iron will not by any means be 2 times 1000. Such a fallacy is one often committed by engineers who lack experience. It is exactly the same thing as if we dealt with an electric circuit consisting of a battery giving an electromotive force of 1 volt, with an internal resistance of 0.1 ohm, applied to an external resistance of 10 ohms. The electromotive force in the battery itself is 1 volt, and its internal resistance 0.1 ohm. We might say that the conductivity of the battery is 10 mhos. It would be foolish,

however, to assume that the current in the battery is 10 times 1 ampere. We know that we must treat the entire electric circuit. In the same way, we must treat an entire magnetic circuit.

If the trough in Fig. 120 is removed, we know that the flux density at a distance of 1 centimeter from the center of the wire will be 2 gausses. The total magnetizing force is 4π gilberts, and the length of the magnetic circuit 2π centimeters. This magnetizing force is uniformly applied along the magnetic circuit, and hence we have everywhere the magnetizing force of 2 gilberts per centimeter. Since the permeability of air is unity, the resulting flux density will then be 2 gausses. Moreover, if we have a complete cylindrical sheath about the wire, composed of iron, as indicated in Fig. 121, the permeability of iron being taken as 1000, we may say immediately that the flux density in this sheath will be 2000 gausses. We deal again with a complete magnetic circuit, the magnetizing force is uniformly distributed along this circuit, and therefore the flux density can be found by multiplying this uniform magnetizing force by the permeability.

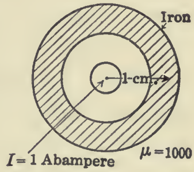

FIG. 121. A copper conductor in an iron sheath.

In the arrangement shown in Fig. 120, however, a complete magnetic circuit is made up partly of iron and partly of air. The iron part is of low reluctance and the air part of high reluctance. The total magnetomotive force will hence be used up almost entirely in the air part of the circuit. The magnetomotive force acting across the iron will be very small. The magnetizing force in the iron, that is, H, will thus also be small. If we can find this small magnetizing force, and multiply it by the permeability, we shall have the resulting flux density in the iron. The computation of the actual value of H is, however, very difficult.

Another illustration of this principle is of importance.

When there is no iron about, we have a very uniform field, due to the earth. The intensity of this field is about 0.6 gauss. We have everywhere acting a magnetizing force of 0.6 gilbert per centimeter. Suppose we place a cannon ball of permeability 3000 in such a field. If to find the flux density we simply multiplied μ by H, we should obtain 1800 gausses for the flux density in the cannon ball. With such a flux density, the ball would be very highly magnetized indeed. In fact, if two of them were placed side by side on the floor, each thus magnetized, they would roll around violently. We know, of course, that such a thing does not occur. The resulting magnetization in the iron ball can with difficulty be detected. The explanation is simple. The iron ball is introduced into a uniform field, and since its reluctance is very low, there is very little reluctance drop from one end of it to the other.

FIG. 122. A uniform parallel field.

The resulting H in its interior is accordingly very small. The field before the introduction of the ball is shown in Fig. 122. After the introduction of the ball, it is much as shown in Fig. 123. A few of the flux lines are diverted and pass through the ball. The flux density at its interior will be about 0.4 gauss. This flux density is obtained by multiplying μ by the resulting magnetizing force, and not by the original force.

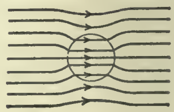

FIG. 123. The field of Fig. 122 after an iron ball has been introduced.

An analogy will show the reason why the flux density in the cannon ball merely doubles. Suppose that we take a square yard of thin rubber sheet, attach sticks to the two opposite sides and stretch it. We have a uniform strain in the rubber which can be represented, if we wish, by a series of parallel strain lines. Now let us cut a circular hole in the rubber. In this hole the resistance of the air to stress

is zero, and yet it is not deformed very much. The hole becomes elongated so that its length is about twice its width. It stretches, in fact, until there is no stress left in its interior. The resistance of the sheet to strain is analogous to the reciprocal of the permeability. The hole corresponds to a cannon ball of infinite permeability. It will be seen from this illustration why, if the permeability of the ball is many times that of air, almost the same effect will be produced as if the permeability were actually infinite.

During the recent war, one method that was tried for detecting the presence of enemy submarine boats was to observe by various means the distortion in the earth's magnetic field produced by the shells of the submarine. The permeability of the material of the submarine boats' plates was unknown, and also their thickness. Practically the same effect on the earth's field was obtained, however, as if the submarine had been solid and of infinite permeability. The effect was small, however, and could be detected at distances of a few hundred yards only with extreme difficulty.

It will be noted in Fig. 123 that the flux lines where they enter and leave the ball are drawn perpendicular to the surface. It is a general proposition that flux lines entering or leaving iron of high permeability will pass in or out almost exactly perpendicular to the surface. This fact is of great assistance in mapping out magnetic fields where iron is involved.*

The reason that this must be true can readily be made clear. In Fig. 124, suppose that the area above the dividing line is air, and that below the line is iron of high permeability. H inside the iron is very small. It is equal to B/μ, and since μ is very large, the resulting H can practically be

* When the flux density in the iron is very high, and in the air very small, this rule has exceptions; for example, on armature teeth under certain conditions. Such cases are, however, extreme. See Rogofsky, Archiv. Für. Elek., Band 9, 1920.

neglected in comparison with the value of H_a, for the air. Consider a closed path drawn in the magnetic field as shown by the dotted line. The line integral of H around this path will evidently be zero. The line integral across *ab* is zero, since the path here is perpendicular to the flux. The part of

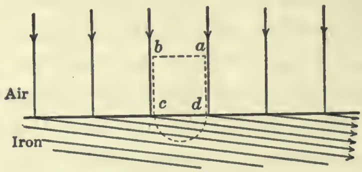

Fig. 124. The correct method of representing the angle at which magnetic lines enter iron from air.

the line integral along *cd* is practically zero, for this path is entirely in iron. H in the iron, we have seen, is so small it can be neglected without serious error. The integral along the paths *bc* and *da* will cancel, for these paths are

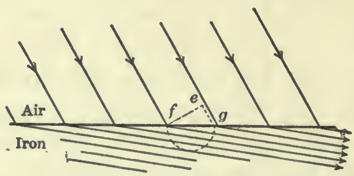

Fig. 125. The lines are incorrectly shown entering the iron at less than right angles to the surface.

equal in length and opposite in direction. The line integral around this closed path of H is accordingly zero; and this is as it should be, for the path links no conductors carrying current.

Suppose, however, that we consider Fig. 125, where the flux lines are incorrectly drawn entering the surface at an angle much less than 90°. It will always be possible to draw a path such as *efg*, around which the line integral of H is not zero. The integral along *ef* and *fg* is zero as before. That along the path *ge*, however, is not zero. The total

line integral is therefore not zero. We know this to be incorrect, for the line integral around such a closed path must be zero if it links no conductors carrying current. This arrangement, therefore, must be incorrect, and it is not generally possible to have flux lines entering a material of high permeability from a material of low permeability at an acute angle with the surface. In drawing flux diagrams in which air and iron are both involved, we should be careful to draw the flux lines always perpendicular where they enter the iron surface from the air, and where they leave the iron for the air.

There is one further principle which will assist in the proper mapping of flux fields and in making approximate estimates of the reluctance of air gaps, etc.

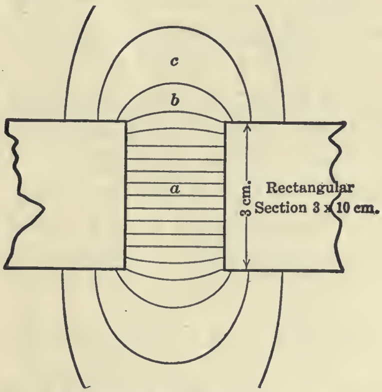

FIG. 126. The lines leave and enter the iron perpendicularly to the surface.

Fig. 126 shows a pair of iron pieces which compose a part of a magnetic circuit, with an air gap between them. The field in this air gap is drawn, fringing widely at the edge of the gap. It is, of course, necessary to draw only a few lines. The flux density in the air gap may amount to thousands of lines per square centimeter, but only a few of these need be drawn to show the direction of the field. If these are drawn at regular intervals, they divide the gap between the iron pieces

into a number of unit volumes. Since these filaments are all in parallel, there must be the same reluctance drop across all of them. Neglecting the small reluctance drop in the iron pieces themselves, the reluctances of these air filaments must all be equal. They should therefore be drawn in such a manner that the quotient of the length divided by the cross-sectional area of each filament is the same. If we are considering fringing in one direction only, that is, if in Fig. 126 the iron pieces extend a long distance in the direction perpendicular to the paper, this means that the width of each space between lines would on the average be proportional to the length of the lines. If filament b is twice as long as a, its width would thus be twice as great as the width of a. Filament c, which is about three times as long as a, would also be about three times as wide.

After we have mapped out the field in this manner, the total reluctance of the air gap may be found by computing the reluctance of one filament, and dividing by the number of filaments. Thus in Fig. 126, suppose filament a is 2 centimeters long and 0.3 centimeter wide. Suppose the iron pieces are 10 centimeters broad in the direction perpendicular to the paper. The reluctance of filament a will

$$R_a = \frac{2}{0.3 \times 10} = 0.667 \text{ oersted.}$$

There are 17 such filaments in the figure, if we do not consider the fringing around the ends of the pieces. The total reluctance of the air gap is accordingly

$$R = \frac{0.667}{17} = 0.039 \text{ oersted.}$$

Compare this with the reluctance of the air gap on the assumption that the flux goes straight across without any fringing.

$$R' = \frac{2}{3 \times 10} = 0.067 \text{ oersted.}$$

The actual reluctance thus has the ratio to this uncorrected reluctance of

$$\frac{R}{R'} = \frac{0.039}{0.067} = 59\%.$$

The fringing in this case decreases the reluctance of the air gap to a little over half of what it would be if there were no fringing.

As a rough rule to cover this case, we have previously proposed to add to the width of the flux path an amount equal to its length. Applying this rule to our present gap, we should have a length of 2 centimeters and a width of $3 + 2$ or 5 centimeters. The reluctance, disregarding the end fringing, would then be

$$R'' = \frac{2}{5 \times 10} = 0.040,$$

which is quite close to the value which we obtained by mapping the field. This rule is usually sufficiently accurate

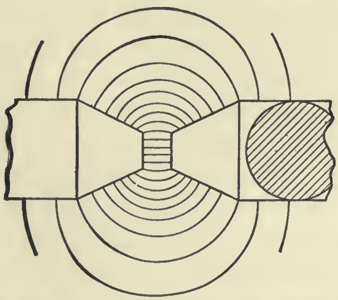

FIG. 127. The lines enter and leave the poles at right-angles to the surface.

for short air gaps. In cases where the air gap is not simple, or where it is long, the above process of mapping the field will be found to be of service.

Sometimes we must consider the fringing in two directions. Consider the field shown in Fig. 127, where the pole

pieces are circular in cross-section. In this case we must remember that the filaments of air bounded by the flux lines we draw have two dimensions, and the second dimension varies for different filaments. The lines we have drawn show a cross-section of the field. We may, if we like, consider the air gap to be divided into surfaces of revolution such as would be obtained by revolving the figure shown about its axis. The total cross-sectional area of a filament thus generated would be proportional to the length of its boundary. The resulting figure will, of course, not show a proper cross-sectional view of the distribution of flux lines, but it may be more convenient as a means of computation.

78. Magnetic Poles and Pole Strength. The subjects of magnetic poles and magnetic pole strength have been purposely left out of the above discussion. In many texts it will be found that some of the above matters are computed in a different manner, by finding the strength of the poles and assuming that they contribute to a resulting magnetizing force. This method has long been in use by physicists. It is historically much older than the method that is here presented. However, engineers now depend almost entirely upon the laws and methods of computation which have been used in this chapter. Magnetic circuits are treated by them in exactly the same way that electric circuits are treated. Ohm's law and Kirchhoff's laws can be applied to the magnetic circuit as well as to the electric circuit. In cases where the flux paths are in doubt because of the introduction of large air gaps into circuits otherwise of iron, approximate methods are used which depend upon estimations of the paths and the consequent reluctances. In view of the fact that such cases cannot be solved accurately by any known means, this method is much preferred to other and more complicated methods of analysis.

Prob. 27–7. Map the field as in Fig. 126, about two iron bars in the same plane one inch in width, one-quarter inch in

thickness and ten inches long, whose ends are separated by one inch. Calculate the reluctance of a filament, also the total reluctance.

Prob. 28–7. What would be the approximate reluctance of an air gap of $\frac{1}{8}$ inch between the two bars of Prob. 27–7?

SUMMARY OF CHAPTER VII

THE MAGNETOMOTIVE FORCE, $\mathcal{F}$, is the LINE INTEGRAL of the magnetizing force, H, about a closed circuit and equals 0.4π times the number of ampere-turns linking the circuit. As an equation, this may be expressed

$$\mathcal{F} = \int_l \text{Hdl} = 0.4\pi\text{NI}.$$

TO DEFINE A LINE INTEGRAL, consider a point acted upon by a variable force and moving along a path, and integrate over the entire length of the path the products obtained by multiplying the force at every point in the path by the cosine of the angle between the direction of force and the path. The result is really the energy expended on the point by the force in traveling the length of the path.

THE MAGNETIZING FORCE at a point outside a long wire carrying a current is expressed by the equation

$$\text{H} = \frac{0.2\text{I}}{\text{r}},$$

where

H = the magnetizing force in GILBERTS PER CENTIMETER,
I = the current in AMPERES,
r = the distance of the point from the center of the wire in CENTIMETERS.

THE FLUX DENSITY at a point in a material of a permeability μ completely surrounding a long wire which is carrying a current may be expressed by

$$\text{B} = \frac{0.2\mu\text{I}}{\text{r}}.$$

IN THE VICINITY OF A SHORT WIRE carrying a current the magnetizing force is

$$\text{H} = \frac{\text{I}}{\text{r}}(\sin\alpha_1 \pm \sin\alpha_2),$$

where

H = the magnetizing force in GILBERTS PER CENTIMETER,
I = the current in ABAMPERES,
r = the distance of the point from the wire in CENTIMETERS,
α_1 and α_2 are angles between the perpendicular from the point to the wire and lines drawn from the point to the ends of the wire.

THE TOTAL FLUX BETWEEN THE TWO PARALLEL WIRES (line and return) of a transmission line can be found from the formula

$$\phi = 4\, I \log_e \frac{D - r}{r} \text{ maxwells per centimeter of line,}$$

where

I = the current in ABAMPERES,
D = the distance between the centers of the wires in CENTIMETERS,
r = the radius of the wires in CENTIMETERS.

THE FIELD AT A POINT WITHIN A CIRCULAR CONDUCTOR equals

$$B = \frac{2\mu I}{r}\,\frac{a}{r},$$

where

I = the current in ABAMPERES,
μ = the permeability of the material of the conductor,
r = the radius of the conductor in CENTIMETERS,
a = the distance of the point from the center of the conductor in CENTIMETERS.

AT THE CENTER OF A CIRCULAR COIL OF A SINGLE TURN the magnetizing force equals

$$H = \frac{2\pi I}{r}.$$

AT THE CENTER OF AN ARC OF ONE UNIT IN LENGTH and with a radius of one unit

$$H = 1.$$

UNIT CURRENT can thus be defined as that current which flowing in a unit length of an electric circuit bent into an arc of unit radius will produce unit magnetizing force at the center of the arc.

ON THE AXIS OF A CIRCULAR COIL OF ONE TURN the magnetizing force equals

$$H = \frac{2\pi I}{r}\sin^3\theta,$$

where

θ = the angle between the axis and a straight line joining the point and the wire.

AT ANY POINT ON THE AXIS OF A SOLENOID

$$H = \frac{2\pi NI}{l}(\cos\alpha_1 - \cos\alpha_2),$$

where

α_1 and α_2 are the angles between the axis and lines joining the point to each end turn.

AT THE CENTER OF A SOLENOID

$$H = \frac{4\pi NI}{\sqrt{l^2 + 4r^2}}.$$

AT ANY POINT ON THE MID-SECTION OF A LONG SOLENOID

$$H = \frac{4\pi NI}{l}.$$

THE TOTAL FLUX THROUGH THE MID-SECTION OF A LONG SOLENOID with an air core equals

$$\phi = BA = \frac{4\pi NI}{\frac{l}{A}}.$$

A long air-core solenoid acts as if practically all the reluctance were within the coil and equal to $\frac{l}{A}$. The fact that the flux density at the center of a long coil is uniformly distributed over the cross-section is used in calibrating a ballistic galvanometer to be used to measure magnetic flux.

THE EXACT EQUATION FOR THE FLUX IN A TOROID of constant permeability equals

$$\phi = 4\mu NI\pi \left(r - \sqrt{r^2 - \rho^2}\right),$$

where

r = the radius of the toroid in CENTIMETERS,
ρ = the radius of the stock of the toroid in CENTIMETERS.

THE FLUX SET UP BY CONDUCTORS IMBEDDED IN IRON can be found approximately by plotting the approximate path in the air, computing the reluctance of this path and applying the line-integral law for the magnetic circuit. **IN USING THE LINE-INTEGRAL METHOD** for circuits of non-uniform permeability, great care must be exercised always to include the magnetic whole in the calculations. The value of H computed for a point in the air will not hold for iron placed at that point of the circuit.

PROBLEMS ON CHAPTER VII

Prob. 29–7. A solenoid for use in connection with flux measurements has a primary (air-core) coil 7 centimeters in diameter and 75 centimeters in length wound with 525 turns of cotton-covered copper wire, which can safely carry 3 amperes. What will be the magnetizing force in gilberts and what flux density will be produced at a point at the center of the axis of the coil if this current is flowing?

Prob. 30–7. A Leeds and Northrup wall-type galvanometer (R_g = 117 ohms) was found by experiment to have the following calibration as a flux meter:

$$\phi N = d(4.625R + 868.75),$$

where

ϕN = flux linkages in the galvanometer circuit,
d = galvanometer deflection,
R = total resistance in the galvanometer circuit.

A search coil of 100 turns of No. 36 wire (mean length of turn 1.1 inches) was placed so that it linked all the flux lines of a standard bipolar telephone-receiver magnet. The coil was suddenly moved to the point where it had cut all the flux and the deflection was 30.35 centimeters. What was the flux in maxwells in the receiver magnet?

Prob. 31–7. A test solenoid used in connection with a wall galvanometer and made by Robert W. Paul of London has a length of 20 inches. This coil is so wound that the magnetomotive force in gilberts per centimeter is four times the current carried by the solenoid winding. The secondary coil is wound in concentrated form around the solenoid and at the midpoint of the length. What must be the number of turns on the solenoid and what will be the flux linkages of the secondary coil neglecting leakage?

Prob. 32–7. From an analysis of a telephone receiver, (Research Bulletin II, Mass. Inst. Tech.), the equivalent magnetomotive force of the permanent magnet was found to be 181 gilberts. The average useful polar flux density

produced by this magnetizing force is 1369 gausses. Find the equivalent reluctance of the circuit. On the spool of the receiver magnets are wound approximately 1300 turns of wire. What increase in flux would a current of 2.56 milliamperes produce assuming the permeability to remain constant through the small change? The dimensions of the polar surfaces of this bipolar receiver are each 1.14×0.199 centimeters.

Prob. 33–7. Find the flux density at the polar surfaces of the receiver of Prob. 32–7 when the additional magnetomotive force due to the current in Prob. 32–7 acts. (Assume constant permeability.)

Prob. 34–7. A cast-iron toroid of circular section, (see Fig. 111), $r = 6$ inches, $r_1 - r_2 = 1.5$ inches, has cut in it a radial slot by means of a hacksaw. The width of the slot is 0.12 inch. If there are 410 ampere-turns on the core, what will be the total flux in the core? Plot the flux density in the core in a circumferential direction from r_1 to r_2, neglecting fringing. Plot the flux with the same ampere-turns but without the air gap. What will be the difference in determining the total flux by the exact and by the approximate formulas?

Prob. 35–7. In the equation for the flux density at any point in the toroid

$$B_x = \mu \frac{2NI}{x},$$

(see page 219) why is it not accurate to find an average flux density by finding the flux at $x = r_1$ and $x = r_2$ and taking the mean?

Prob. 36–7. The telephone line of Fig. 93 is to be strung with the same spacing between x and y on the same poles as the power line. Compute the maximum flux linking per mile of the telephone line if strung as follows:

(*a*) In a vertical plane passing through the center of a line joining A and B, and perpendicular to it, with the top wire 10 feet below the line joining A and B;

(*b*) In a horizontal plane, with the center of a line joining x and y directly beneath the center of the line joining A and B, and 10 feet distant.

Prob. 37–7.

(*a*) In a solenoid 100 centimeters long and having an average diameter of 20 centimeters, what percent error is made in

computing the flux density at the center by the use of the formula

$$B = \frac{4\pi NI}{l}?$$

(*b*) If the solenoid were 40 centimeters in diameter, what percentage error would have been made?

Prob. 38–7. Derive the equation for the magnetizing force at any point on the axis of a solenoid but outside of the solenoid.

Prob. 39–7. An air-core solenoid 100 centimeters long and of 15 centimeters average diameter is wound with 1200 turns, carrying 4 amperes. Compute

(*a*) The field intensity at the center;
(*b*) The flux density at the ends;
(*c*) The internal reluctance;
(*d*) The external reluctance.

Prob. 40–7. From the data in Prob. 39-7, compute

(*a*) The internal reluctance drop;
(*b*) The magnetic difference of potential between the ends;
(*c*) The external reluctance drop.

What is the flux density at a point outside the coil on the axis of the solenoid in Prob. 39–7, 20 centimeters from the nearer end?

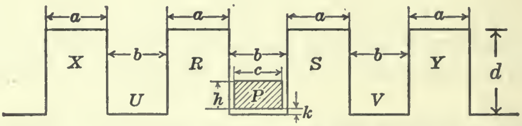

FIG. 128. A conductor *P* placed at the bottom of a slot.

Prob. 41–7. Derive a formula for computing the flux set up by a current of *I* abamperes in the conductor *P* in Fig. 128;

(*a*) Leaving the bottom of the slots *U* and *V*;
(*b*) Leaving the ends of the teeth *X* and *Y*.

Prob. 42–7. In Fig. 128,

a = 0.82 inches,
b = 0.90 " ,
c = 0.82 " ,
d = 2.80 " ,
h = 1.25 " ,
k = 0.075 " .

Find the total flux set up

(*a*) In the slot in which the conductor lies;
(*b*) In the air at the ends of the teeth R and S;
(*c*) In the slots u and v;
(*d*) In the air at the ends of the teeth x and y.

CHAPTER VIII

INDUCED VOLTAGES

We have seen in previous chapters that when flux is varying through a coil, there is a voltage produced in the coil. In this chapter we will consider some of the immediate laws governing voltages and currents induced in this way.

79. Change of Linkages. Lenz's Law. The general law may be stated as follows. Whenever the flux linkages with a coil are changing, there is a voltage induced in the coil.

We have learned that this induced voltage is proportional to the rate of change of the flux. In fact, we defined the maxwell as that amount of flux which would produce one abvolt-second impulse. If the flux through a coil changes by one maxwell, the time for the change occupying one second, there will be produced in the coil one abvolt.

The voltage produced in a coil is thus equal to the rate of change of flux linkages through it; that is,

$$E = N\frac{d\phi}{dt}\text{ abvolts,} \tag{1}$$

where

E = the voltage produced in abvolts,
N = the number of turns,
ϕ = the flux in maxwells.

If E is to be expressed in volts, we must introduce the factor 10^{-8} to convert from the c.g.s. units to the practical system, thus:

$$E = N\frac{d\phi}{dt}\ 10^{-8}\text{ volts.} \tag{2}$$

The direction of the induced voltage may be determined by the following law: an induced voltage is always in such

a direction that it tends to produce a current which will oppose the change of flux.

This was stated by Lenz as follows and is called **Lenz's Law.**

Whenever there is a change in the amount of magnetic flux linking an electric circuit, voltage is set up tending to produce a current in such a direction as to oppose the change in flux.

The voltage thus set up is directly proportional to the rate of change of flux linkages.

In Fig. 129, which shows a transformer, suppose that the primary current is in the direction shown by the arrow, thus producing a flux in the core in the direction shown by the dotted arrow, and suppose that this flux is decreasing. There will be produced in the secondary coil a voltage in the direction of the arrow shown on the secondary winding, that is, in such a direction that a current flowing in this direction would increase the flux, or in other words, in a direction such as to tend to prevent any decrease of flux. Conversely, if the primary current is increasing, the secondary voltage will be in the reverse direction, and hence tend to oppose an increase of flux.

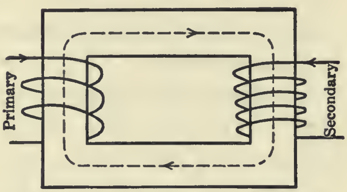

FIG. 129. Diagram of a transformer.

Since the induced voltage is proportional to the rate of change of flux, a very high voltage may be obtained by causing the flux to change rapidly. This principle is made use of in an induction coil such as is shown in Fig. 130. An iron core is wound with a heavy winding capable of carrying a large current and thus magnetizing it strongly. This winding is called the primary. The core is usually made open, rather than a closed ring, for reasons which will appear more fully in the next chapter. The main reason

may be briefly stated to be in order that the flux may collapse rapidly. It is also made of fine iron wire to reduce eddy currents. Over this primary winding is wound the secondary of a large number of turns. The primary circuit flows through a vibrator, which operates in the same manner as the clapper of an electric bell. When the current is interrupted by this vibrator, the flux decreases rapidly from a maximum value to nearly zero. The rapid change of flux linking the secondary produces a high voltage in the secondary.

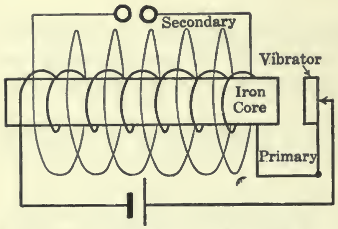

FIG. 130. Diagram of an induction coil.

As an example, suppose that the core is 4 square centimeters in cross-section and is magnetized to a maximum of 6000 gausses, thus producing a total flux of 24,000 maxwells. Assume that the vibrator reduces this flux to zero in one-one-thousandth of a second. Let there be 20,000 secondary turns. The voltage produced in the secondary will then be

$$E = 20{,}000 \times \frac{24{,}000}{0.001} \times 10^{-8} = 4800 \text{ volts.} \tag{3}$$

Induction coils can be produced in this manner which will induce voltages of several hundred thousand volts. The principal commercial use of the induction coil at present is in the ignition system of automobiles. The vibrator is here replaced by an interrupter which is usually mechanically driven by the engine. The adjustment of this interrupter contact determines the period in the cycle at which the circuit is opened and thus the point at which the spark takes place. The primary current is obtained from the storage battery of the car.

A condenser is placed across the interrupter to reduce

sparking at the contacts. The size of this condenser is chosen to give maximum rate of decrease of flux.

Much more important from a commercial standpoint is the transformer, which operates on the same principle. A primary winding of heavy wire, carrying a large current at low voltage, produces a large flux in a core, which in turn produces a high voltage in a secondary winding. This is the step-up transformer, which transforms from a low voltage to a higher voltage. Conversely, we may have a primary of a large number of turns carrying a small current which produces a flux, giving a heavy current at low voltage in a secondary winding of few turns of heavy wire. This is the step-down transformer. The voltage ratio of the transformer is approximately the same as the ratio of the number of turns on the primary and secondary.

In the transformer the flux does not vary abruptly, as it does in the induction coil, but changes smoothly and periodically. In fact, alternating currents are usually very nearly sinusoidal and the flux in the core of a transformer is also nearly a sinusoid. This means that it varies harmonically with the time. In other words, if we plot the flux density in the core against the time, we shall obtain a sine curve, for instance Fig. 131, the equation of which is

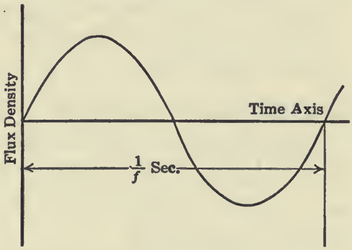

FIG. 131. The flux density has a sine wave form.

$$B = B_{\max} \sin 2\pi ft. \tag{4}$$

The symbol f is the frequency, which in American practice is usually 60 cycles per second. The voltage produced in the secondary will be

$$E = N\, 10^{-8} \frac{d}{dt}(AB_{\max} \sin 2\pi ft), \tag{5}$$

where

N = the number of secondary turns,
A = the cross-sectional area of the core.

This becomes upon differentiating

$$E = 2\pi \; NA \; B_{max} f \cos 2\pi ft \; ; \tag{6}$$

which it will be noted contains a cosine instead of a sine. This equation is plotted in Fig. 132. Note that the voltage is at its maximum when the flux is zero, that is, when the flux is changing at its maximum rate. Note also that when the flux is at maximum it is changing at zero rate, and the secondary voltage is accordingly at that time zero. We say that the voltage in such a case is a quarter of a period out of phase with the flux, that is, in quadrature with the flux.

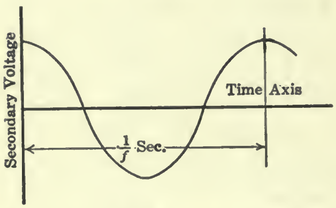

FIG. 132. When flux varies according to the sine curve of Fig. 131, the induced voltage varies according to the cosine curve shown here.

The complete study of the action of a transformer is an interesting problem, but it must be postponed until the laws of the electric and magnetic circuits are more thoroughly understood.

Prob. 1–8. The strength of the earth's magnetic field at a given point is 0.6 gauss and the angle of dip is 70° with the horizontal. If a coil of 900 turns is held vertically with its plane perpendicular to a north-and-south line and then rotated about its vertical axis through 90° in 0.02 second, what average voltage will be produced in the coil by this movement? What would be the average voltage if the coil were moved through only 30° at the same speed? Through 50°? Area = A sq. cm.

Prob. 2–8. If the primary coil of Fig. 130 is wound with 200 turns of wire to produce a total flux of 24,000 maxwells, what average voltage will appear at the points of the vibrator when the circuit is opened if the flux dies down to zero in 0.001 second?

Prob. 3–8. If in a transformer core $B_m = 70{,}000$, plot the flux density with time for one cycle where $f = 60$, assuming a sinusoidal variation.

Prob. 4–8. If the cross-section of the core of the transformer of Prob. 3–8 is 1.5 square inches, plot E (the secondary voltage) against time for one cycle, where $N = 2500$ (secondary turns). Use the same scale for time as in Prob. 3–8 and superimpose this plot on the plot of Prob. 3–8.

80. Self-Induction. Coefficient. A voltage is induced in a coil whenever the number of flux linkages is changing. This is true whether there is a current in the coil or not. If we have only a single coil, which is carrying a current and is thus producing a flux, it will have a voltage produced in it whenever this flux (which it is itself producing) is changing in amount. Such a voltage we call a voltage of **self-induction.**

We have seen that an induced voltage is always in such a direction as to tend to oppose the cause that produces it. A voltage of self-induction is therefore always in such a direction as to oppose a change in current. If the current is building up in a coil, that is, increasing, the self-induced voltage will be opposite to the current in direction. If the current in a coil is decreasing, the self-induced voltage will be in the same direction as the current.

The number of flux linkages which are set up by a coil with its own turns, when the coil is carrying unit current, is called the **coefficient of self-induction** of the coil. The c.g.s. unit of the coefficient of self-induction is therefore the number of linkages per abampere in the coil, and is called the abhenry. The practical unit is 10^{-9} times as large, and is called the **henry.** A coil accordingly has a coefficient of self-induction, or more briefly, simply an inductance, of one henry when a current of one ampere flowing in the coil will produce 10^8 linkages. That is,

$$L = \frac{N\phi}{I} 10^{-8} \text{ henries,} \tag{7}$$

where

L = the inductance in the coil in henries,
N = the number of turns,
ϕ = the total flux produced by a current of I amperes flowing in the coil.

This assumes that all of the flux links every turn. If the flux does not link all of the turns, we should, of course, write the actual number of linkages produced instead of the product $N\phi$.

The inductance is a constant which depends only upon the dimensions of a coil. No iron is used in the construction of the coil. The flux is then proportional to the current and L will have the same value no matter what number of amperes is used in finding

$$\frac{N\phi}{I} 10^{-8}.$$

The same is not true when an iron core is used. Then the flux is not strictly proportional to the current, although for small magnetizing forces it may be nearly enough proportional so that the formulas may still be used with engineering accuracy.

In speaking of the inductance of a coil or a circuit we mean at present a coil or a circuit in which no iron is present or in which the flux density in the iron used is small and hence the flux is proportional to the current. In the ordinary coil with an iron core at high flux densities, the flux is not at all proportional to the current, and the inductance of the coil is not a constant, but depends upon the value of current used in measuring it. In such a case the term "coefficient of self-induction" ceases to have a definite meaning. We will deal with such cases in the next chapter.

Suppose that we have a coil not containing iron and having an inductance of one henry. Assume that this coil is carrying a current which is increasing uniformly at

the rate of one ampere per second. Then, since in such a coil one ampere produces 10^8 linkages, the number of linkages will be increasing at the rate of 10^8 per second. We have seen, however, that when the linkages in a coil are changing at the rate of 10^8 per second, there is an induced voltage in the coil of one volt. We are thus led to another definition of the henry.

A coil has an inductance of **one henry** if there is induced in it one volt where the current is changing at the rate of one ampere per second.

This definition may be extended and expressed as follows. The voltage induced in a coil by self-induction is equal to the inductance times the rate of change of current. In practical units, the voltage induced in a coil by self-induction is equal to the inductance in henries times the rate of change of current in amperes per second; that is,

$$E = -L\frac{dI}{dt}. \tag{8}$$

The minus sign is introduced to indicate that the voltage is in a direction opposite to the current when the current is increasing. This we have seen to be the case, since the voltage is always in such a direction as to oppose the current change.

This very simple formula, "**The induced voltage is equal to the inductance times the rate of change of current,**" will be found of great importance in all electrical theory.

Prob. 5–8. What is the inductance in the primary coil in Prob. 2–8 if the current in the primary is 1.7 amperes and the flux has a value of 24,000 maxwells?

Prob. 6–8. What voltage is induced in a coil having 0.002 henry inductance when the current is changing at the rate of 5×10^4 amperes per second?

Prob. 7–8. A total voltage of 3000 is induced at a certain instant in a coil of 2500 turns of wire. At what rate is the current changing if $L = 0.32$ henry?

Prob. 8–8. A coil of 350 concentrated turns of wire was tested for flux with a ballistic galvanometer. It was found that the coil was linked by a flux of 110,000 maxwells when 3 amperes flowed in the winding. Find L.

81. Transients in Inductive Circuits. We have seen that when a current is varying in a coil, the change of linkages produced causes a voltage to be set up in the coil of value

$$-L\frac{di}{dt}$$

and this voltage of self-induction is opposed to the change of current.

When a coil is switched on a circuit and the current starts to build up through the coil, this voltage of self-induction opposes this increase of current, and therefore causes it to build up slowly. We can readily analyse the way in which this takes place.

Let us consider the simple circuit of Fig. 133. A coil of L henries inductance is connected in series with a resistance of R ohms and a battery giving a constant voltage of E volts. The circuit is closed by a switch. We will assume that the resistance R includes all the resistance of the circuit, that is, the value taken for R will not only be the external resistance but also the value of the resistance of the coil itself and of the battery.

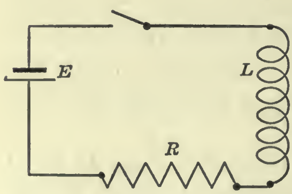

FIG. 133. A circuit containing an inductance L, a resistance R and an e.m.f. E.

We know by Kirchhoff's laws that the sum of the voltages around this circuit is zero. It is, in fact, equal to zero at every instant, provided all of the voltages are taken into account. When we are considering a current which is changing in value, there is, in addition to the battery voltage and resistance drop, a voltage of self-inductance which must be included if our equation is to give a correct value.

Accordingly when the current is building up in a coil we have three voltages to consider, — first, the voltage of the battery, second, the resistance drop, and third, the voltage of self-inductance. The two latter are opposed to the voltage of the battery. Expressing these facts mathematically gives us the differential equation

$$E - Ri - L\frac{di}{dt} = 0. \tag{9}$$

This equation expresses the conditions for any instant in the circuit.

If we solve this differential equation, we shall obtain an expression for the current in terms of the time, that is, an equation which will show the manner in which the current will vary after closing the switch.

Transposing and simplifying, we have

$$(E - Ri)dt = L\,di, \tag{10}$$

which reduces to

$$\frac{di}{i - \frac{E}{R}} = -\frac{R}{L}dt. \tag{11}$$

The variables are now separated and we may integrate both sides of the equation, giving

$$\log_\epsilon\left(i - \frac{E}{R}\right) = -\frac{Rt}{L} + C, \tag{12}$$

where C is the constant of integration. This equation may be put into the form

$$i - \frac{E}{R} = \epsilon^{-\frac{Rt}{L} + C}. \tag{13}$$

The logarithm above is, of course, to the base ϵ.

To determine the constant of integration, we utilize the known fact that at the instant of closing the switch the

current in the circuit is equal to zero. Inserting into our equation the pair of values

$$\begin{aligned} t &= 0 \\ i &= 0, \end{aligned} \tag{14}$$

we thus obtain

$$-\frac{E}{R} = \epsilon^{C}. \tag{15}$$

Inserting this value as the constant of integration in the equation and transposing, we have as a final result

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right). \tag{16}$$

This equation gives the manner in which the current varies after closing the switch. It is plotted in Fig. 134. The curve shows that the current increases at first very rapidly, then more slowly and finally approaches the value $\frac{E}{R}$. In the first instant after the switch is closed, the voltage of self-inductance prevents the current from coming immediately to its final value. As time goes on, however, the current increases more slowly, until it is practically steady and the effect of the inductance disappears. The current then has its steady-state value as given by Ohm's law. We now see why it has been stated several times that Ohm's law in its simple form must be applied only when matters in the electric current have reached their steady condition.

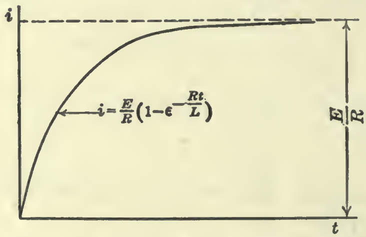

FIG. 134. When the switch in the circuit of Fig. 133 is closed, the current rises according to this curve.

Note carefully that the current follows the curve of Fig. 134, only when E, R and L are all constant. When the circuit links a magnetic circuit in which iron is present, L is not a constant and the growth of the current takes the form

shown in Fig. 134a. Since equation 16 applies only to a curve of the form shown in Fig. 134, it cannot be used in connection with a circuit linked by iron.

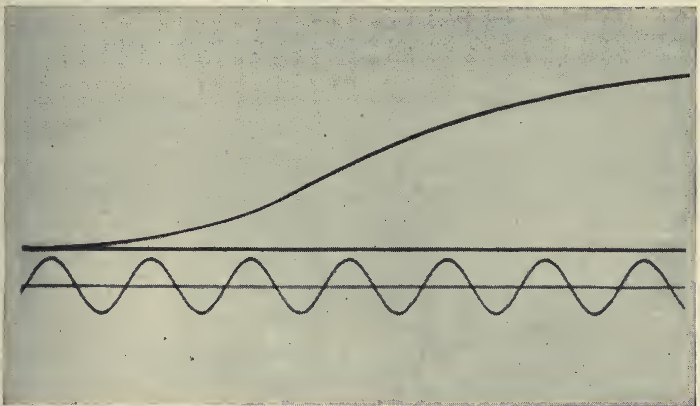

Fig. 134a. An oscillogram of the growth of the current in a circuit linking an iron circuit. The alternating wave at the bottom is a 60-cycle wave for determining the time. *Taken at M.I.T. by Prof. F. S. Dellenbaugh of the Research Division of the Electrical Engineering Department.*

From the above results, we see that it takes time to set up a current through an inductive circuit. The effect is very similar to the corresponding one in mechanics. It takes time to set a heavy car in motion. We say that the car has inertia, and that a force must be applied for a certain period in order to overcome the inertia of the car. In the same way an electromotive force must be applied for a certain period in order to overcome the inductance of the circuit. The **inductance** of a circuit is hence **electrical inertia.**

If after a car has once been set going we remove its driving force, it will coast for a considerable time before stopping. Similarly, if a current is set up in an inductive circuit and the electromotive force of the circuit then removed, the current will persist for a period before coming to rest at

zero. This effect we can analyse in a manner similar to that already used.

Suppose in Fig. 135 that the battery E is furnishing a steady current I to the inductance L and the resistance R. Let us now close a switch which short circuits the battery and cuts it out of circuit, the battery, of course, being protected against short circuit by additional resistance. The current through the inductance will not instantly stop but will die out gradually. There is no e. m. f. in the right-hand loop of this circuit after the switch is closed except the voltage of self inductance. Applying Kirchhoff's law to the instantaneous condition in the circuit, we obtain

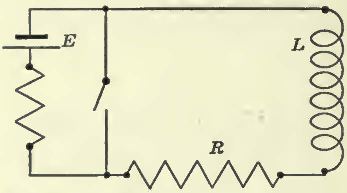

FIG. 135. The switch cuts out the battery from the circuit of L and R and short circuits them.

$$Ri + L\frac{di}{dt} = 0, \tag{17}$$

which can be reduced to

$$\frac{di}{i} = -\frac{R}{L}dt, \tag{18}$$

where the variables are now separated. Integrating, we obtain

$$\log i = -\frac{Rt}{L} + C_2, \tag{19}$$

where C_2 is a constant of integration. This equation may be put into the form

$$i = \epsilon^{-\frac{Rt}{L}}\ \epsilon^{C_2}. \tag{20}$$

We know that at the instant of closing the switch, the current has the value I. Accordingly from the relations

$$\begin{cases} t = 0 \\ i = I \end{cases} \tag{21}$$

inserted into the current equation, we obtain

$$I = \epsilon^{C_2}, \tag{22}$$

and inserting this value for the constant of integration, we obtain

$$i = I\epsilon^{-\frac{Rt}{L}} \tag{23}$$

for the relation between the current and the time, after the switch is closed.

This equation is plotted in Fig. 136. The current dies out exponentially and approaches zero as time goes on. The manner in which the current dies out in a circuit is consequently the exact reverse of that in which the current builds up when the voltage is applied.

It will be noted that mathematically the current never equals zero. No matter what value of t we use, the equation will give a value of the current, which may be, indeed, exceedingly small, but which still exists. The current therefore approaches zero but never reaches it. Practically, however, the current will within a short interval of time become so small that it may be entirely neglected. There are always voltages of small magnitude present in a circuit and hence a small current flowing in one direction or another. These small voltages may be due to thermal or chemical electromotive forces, or some similar cause. A short interval of time after closing the switch above, the current becomes so small that it becomes merged with these minute currents. To all intents and purposes, it is then zero. It will be noted, however, that the time that it takes for the current to reduce to zero cannot be determined experimentally. Neither can we state the exact length of time that it will take for the current to reach its final value when the

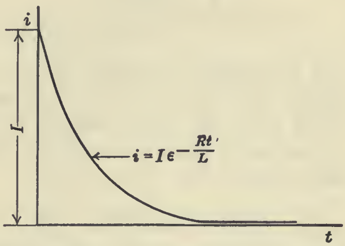

FIG. 136. The current in the circuit of Fig. 135 decays on short circuit according to this curve.

voltage is first applied. The time taken for it to reach any given fraction of its final value is, however, a perfectly definite matter.

Prob. 9–8. In Fig. 133, let $R = 10$ ohms, $L = 0.0001$ henry and $E = 15$ volts. How long after the switch is closed will the current rise to one-half and three-fourths respectively of its final value? Plot the growth of the current by finding five points.

Prob. 10–8. If, in Prob. 9–8, L is made 1.0 henry, find the time as before.

Prob. 11–8. If the circuit of Fig. 135 has the following constants:

$$\begin{aligned} R &= 50, \\ L &= 0.5, \\ E &= 100, \\ r &= 12 \text{ (internal resistance of battery),} \end{aligned}$$

and the switch is closed when the current has reached its final value, at what rate will the current be decreasing when $t = 0.008$ second? What will be the time necessary for the current to reduce to half its original value? What is the initial rate of decrease?

82. Time Constant. The greater the inductance of the circuit, that is, the more inertia it has, the longer it will take for a given electromotive force to set up a given amount of current through it. With a fixed value of inductance, moreover, the greater the value of resistance associated with it, the less will be the final value of current obtained, and accordingly the more quickly will a given electromotive force reach any given fraction of this final value. The addition of a resistance thus masks the effect of the inductance. The less inductance a circuit has, the more quickly a current can be built up through it. The more resistance it has, on the other hand, the less current can be passed through anyway; and so the less will the effect of the inductance be felt.

We can express these facts exactly by saying that the cir-

cuit has a certain **time constant.** This time constant is called T, and has the value

$$T = \frac{L}{R}. \tag{24}$$

The time constant (in seconds) of a circuit is therefore its inductance in henries divided by its resistance in ohms. If a circuit has an inductance of 2 henries and a resistance of 1000 ohms, its time constant will be 0.002 second.

The meaning of the time constant may be made clear by Fig. 137, which shows the curve for the decrease of current through an inductive circuit after the voltage is removed. At the end of the time, T, after removing the electromotive force, the current will have decreased to a certain fractional part of its original value. The amount of the current at this time may be found by inserting

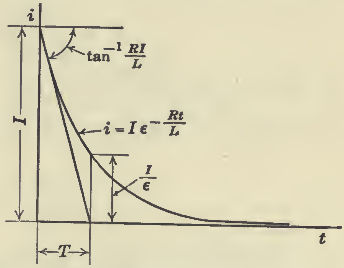

Fig. 137. If the current continued to decrease at the rate $\frac{RI}{L}$, it would become zero in $\frac{L}{R}$ seconds. $\frac{L}{R}$ is the time constant of the circuit.

$$t = T \tag{25}$$

or

$$t = \frac{L}{R} \tag{26}$$

in the equation

$$i = I\epsilon^{-\frac{Rt}{L}} \tag{27}$$

for the current. This gives evidently

$$i = I\epsilon^{-1} \tag{28}$$

or

$$i = \frac{I}{\epsilon}, \tag{29}$$

and since

$$\epsilon = 2.718,$$

it may be written if we wish (using 2.718 as a sufficiently precise value of ϵ)

$$i = \frac{1}{2.718} I = 0.368\, I. \tag{30}$$

The time constant of an inductive circuit is hence the time in which the current will decrease to

$$\frac{1}{\epsilon}$$

of its original value after the electromotive force of the circuit is removed. The curve for the increase of the current, Fig. 138, is exactly the same as the curve of Fig. 137, except that it is turned upside down. We may accordingly state the time constant of a circuit as follows. The time constant of a circuit is the time after the electromotive force of the circuit is changed for the current in the circuit to make approximately sixty-three percent of its complete change.

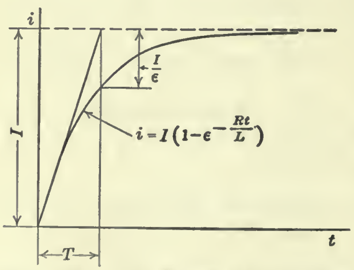

FIG. 138. The current would reach the value $\frac{E}{R}$ in $\frac{L}{R}$ or T seconds if it continued to grow at the same rate at which it started.

For a current starting at zero, it is the time after the electromotive force is applied for the current in the circuit to reach approximately sixty-three percent of its final value. The exact percentages may be worked out to as great an accuracy as desired from the known value of ϵ.

The time constant, however, has a certain further significance which should be understood.

In Fig. 137, let us find the tangent to the curve when t is zero. The slope of this tangent is ·

$$\frac{di}{dt}.$$

But

$$i = I\epsilon^{-\frac{Rt}{L}}$$

because this is the equation for the current when the impressed voltage is removed.

Therefore, differentiating,

$$\frac{di}{dt} = -\frac{RI}{L}\epsilon^{-\frac{Rt}{L}}. \tag{31}$$

When t is zero, this equation for the slope reduces to

$$\frac{di}{dt} = -\frac{RI}{L}. \tag{32}$$

Extend the tangent at the zero point until it cuts the time axis, that is, the axis of abscissas. From the geometry of the triangle of the figure, it will cut it at a point distant from the origin an amount

$$t = \frac{I}{\frac{RI}{L}},$$

that is,

$$t = \frac{L}{R}. \tag{33}$$

We thus see that the tangent to the curve at the I-axis cuts the horizontal axis at a distance from the origin equal to the time constant.

The tangent, however, gives the rate at which the current starts to decrease. This leads us to the following statement.

The time constant of an inductive circuit is the time in

which the current would decrease to zero after the electromotive force was removed, provided it continued to decrease at its initial rate. Similar treatment applies to Fig. 138. The time constant may also be stated thus: the time constant of an inductive circuit is the time in which the current would reach its final value in the circuit after the application of an electromotive force to the circuit, provided that it continued to increase at the rate at which it initially started to increase.

When the switch is closed in an inductive circuit, the current changes first rapidly and then more gradually. If it did not taper out, but continued to change at the initial rate, it would complete the entire change in a time equal to the time constant of the circuit.

As an illustration of these principles, suppose that the field circuit of a dynamo consists of a large coil of wire having a resistance of ten ohms and an inductance of two henries. The time constant of this circuit will be

$$T = \frac{L}{R} = \frac{2}{10} = 0.2 \text{ second.} \tag{34}$$

Suppose that we close the switch which supplies an electromotive force of 100 volts to this circuit. The final current will be $\frac{100}{10}$ or 10 amperes. At the end of two-tenths of a second after closing the switch, the current will have reached 63 percent of this final value, or 6.3 amperes. Upon first closing the switch, the current will increase at the rate of 50 amperes per second, that is, at such a rate that if it continued at this rate it would reach its final value of 10 amperes at the end of 0.2 second. Assume that we wish to know the current in this circuit after a lapse of one second. The value will be

$$\begin{aligned} i &= 10\left(1 - \epsilon^{-\frac{10\times 1}{2}}\right) = 10\left(1 - \epsilon^{-5}\right) \\ &= 10\,(1 - 0.007) \qquad (35) \\ &= 9.93 \text{ amperes;} \end{aligned}$$

that is, 99.3 percent of its final value. At the end of one second, then, the current has, as nearly as we could easily measure, completely reached the value it will maintain continuously.

Prob. 12–8. In Prob. 9–8, what is the value of the current when $t = \frac{L}{R}$?

Prob. 13–8. If a constant voltage is suddenly applied to a circuit containing resistance and inductance, what is the initial rate of increase of the current?

Prob. 14–8. If the rate of growth of current, at the first instant the switch is closed in a circuit containing inductance, resistance and a battery is 20 amperes per second, and the final value of current is 2 amperes, what is the time constant?

Prob. 15–8. In Fig. 133, assuming the resistance of the battery to be negligible and taking the following constants:

$$R = 20,$$
$$L = 0.9,$$

plot the growth of the current after the switch is closed. Also draw the tangent to this curve at $t = 0$. Show on the plot where the tangent intersects the straight line $I = E/R$. Compute the time constant and show its value on the diagram. Do the same for the decay of current after closing the short-circuiting switch in Fig. 135, assuming the same constants.

83. Inertia of an Electric Circuit. When an electric current passes through an electric circuit, there is a movement of electrons through the metal. It takes energy to set the electrons in motion, that is, they have inertia. We have seen that we call the measure of this **inertia** reaction the **inductance** of the circuit.

Similarly the electrons when moving set up a magnetic field which represents kinetic energy. That is, there is energy stored in a circuit due to its inductance. In order to set up a current in a circuit, it is accordingly necessary to apply a certain amount of energy to be stored in the magnetic field, in addition to any energy losses in the resistance. We will

compute the amount of this energy storage and see how it is returned when the current again decreases to zero.

When the switch is closed in the circuit in Fig. 133, the current rises in accordance with the curve of Fig. 134, that is, in accordance with the equation

$$i = \frac{E}{R}(1 - \epsilon^{-\frac{Rt}{L}}). \tag{36}$$

At any instant the power input of the circuit is equal to Ei. Of this power, a part, Ri^2, is being used up in heating the conductor. The remainder, or

$$Ei - Ri^2,$$

is being stored up in the magnetic field. The power going into the magnetic field is therefore at any instant

$$\begin{aligned} P &= Ei - Ri^2 \\ &= \frac{E^2}{R}(1 - \epsilon^{-\frac{Rt}{L}}) - \frac{E^2}{R}(1 - 2\epsilon^{-\frac{Rt}{L}} + \epsilon^{-\frac{2Rt}{L}}) \\ &= \frac{E^2}{R}(\epsilon^{-\frac{Rt}{L}} - \epsilon^{-\frac{2Rt}{L}}). \end{aligned} \tag{37}$$

The total energy put into the magnetic field is the total integral of this power, beginning at the time when the switch is closed; that is,

$$\begin{aligned} W &= \int_0^\infty P\,dt \\ &= \frac{E^2}{R}\int_0^\infty (\epsilon^{-\frac{Rt}{L}} - \epsilon^{-\frac{2Rt}{L}})dt, \end{aligned} \tag{38}$$

which integrated gives

$$W = \frac{E^2}{R}\left(-\frac{L}{R}\epsilon^{-\frac{Rt}{L}} + \frac{L}{2R}\epsilon^{-\frac{2Rt}{L}}\right)\Big]_0^\infty. \tag{39}$$

Inserting the limits of integration, we have

$$W = \frac{E^2L}{2R^2}, \tag{40}$$

or, since in the steady state

$$E = RI, \tag{41}$$

this may be written

$$W = \frac{LI^2}{2} \text{ joules.} \tag{42}$$

Thus the energy stored in an inductance of L henries carrying a current of I amperes is $\frac{LI^2}{2}$ joules.

When the electromotive force is removed from this circuit, the current persists for a certain length of time as shown in Fig. 136. The energy stored in the magnetic field is disappearing, as the current is decreasing to zero. The voltage which forces the current through the resistance during this decrease of current is simply the voltage of self-inductance. We can compute the amount of energy returned to the circuit during the period in which the current is decreasing to zero. The energy returned to the circuit as the current dies to zero exactly equals the energy stored up in the magnetic field as the current builds up from zero. The equation of the dying current is

$$i = I\epsilon^{-\frac{Rt}{L}}. \tag{43}$$

The rate at which energy is being dissipated in heat is

$$Ri^2 = RI^2\epsilon^{-\frac{2Rt}{L}}, \tag{44}$$

and therefore the total amount of energy dissipated in heat as the current dies out is

$$\begin{aligned} W &= \int_0^\infty Ri^2 dt = \int_0^\infty RI^2 \epsilon^{-\frac{2Rt}{L}} dt \qquad (45) \\ &= -RI^2 \frac{L}{2R} \epsilon^{-\frac{2Rt}{L}} \Big]_0^\infty \\ &= \frac{LI^2}{2}. \end{aligned}$$

We thus see that the total amount of energy that is stored in the magnetic field during the time that the current is being built up is returned to the circuit and appears in the form of heat during the time that the current is again decreasing to zero.

We know that in mechanics, wherever there is a mass M moving at a velocity V, there is represented a kinetic energy $MV^2/2$. Entirely analogous to this, wherever in an electric circuit there is an inductance L carrying a current I, there is represented a stored energy $LI^2/2$.

We have seen that the magnetic field is really a strained condition in the ether. This strained condition represents stored energy. The amount of this stored energy we have now learned to compute.

It thus takes energy to set up a magnetic field. On the other hand it does not take any expenditure of energy to maintain a magnetic field. When a steady current is flowing through a coil of wire, the energy input is all used in the ohmic loss of the coil and appears as heat. No additional energy is necessary to maintain the field. The effect of the inductance, so long as the current is steady, is zero.

Confusion sometimes arises by reason of the fact that there is always energy input through a coil even when the current is steady. This energy, however, is used up in losses, and is not used to maintain the field. An analogy will make this matter clear. A ten-pound weight placed on a table represents potential energy. If the table is three feet from the floor, the weight can do thirty foot-pounds of work in falling to the floor. It thus represents a stored energy of amount thirty foot-pounds. If it is resting quietly on the table, there is no expenditure of energy to maintain this potential energy at its fixed value. The ten-pound weight held in the hand at an equal distance above the floor represents also thirty foot-pounds of stored potential energy. No expenditure of energy is necessary to maintain this, as we have seen. Yet if one holds the weight out at arm's

length three feet above the floor for a considerable interval, his arm becomes decidedly tired, and he is conscious of expending a considerable amount of muscular energy. This energy, however, is going into muscular losses in his body, and as we have seen, is not at all expended on maintaining the potential energy in the weight.

Similarly, a magnetic field always represents stored energy. If this field is due to a permanent magnet, it is perfectly evident that no energy input is required to maintain the field. If the field is due to an electromagnet, however, we always have an energy input necessary to force the current through the resistance of the coil. This energy input is entirely taken up in ohmic losses in the coil and no part of it is used in maintaining the stored energy in the field.

If one raises his arm holding the ten-pound weight, and lifts it to six feet above the floor, he puts in an additional thirty foot-pounds of energy, making the stored energy in all sixty foot-pounds. In order to do this he must exert a certain amount of force, which means an expenditure of muscular energy. Besides the losses which we have found to be present for simply maintaining the weight in its fixed position, he must expend a muscular energy of thirty foot-pounds in order to raise the weight to its new position.

Similarly, if we increase the current in a coil, we must add electrical energy sufficient to increase the energy stored in the magnetic field to its new value. This amount of energy must be added over and above any incidentally used up in I^2R losses during the process.

Prob. 16–8. Assuming the inductance constant, what energy is stored in the field coil of a generator where the flux linking the coil is 1.4×10^6 maxwells, the number of turns 800 and the field-coil current 11 amperes?

Prob. 17–8. The energy of a coil carrying 0.3 ampere is 2.3×10^{-7} joule. What is the energy in foot-pounds and what is the inductance in henries?

84. Energy in a Magnetic Field. We have seen that when a current of I amperes flows in a coil of inductance L henries, the total energy storage in the field is

$$W = \frac{LI^2}{2} \text{ joules.} \tag{46}$$

If I is in abamperes and L is in abhenries, that is, if we are dealing with c.g.s. electromagnetic units, the energy will be given in ergs; that is,

$$W = \frac{LI^2}{2} \text{ ergs.} \tag{47}$$

It should be noted carefully that in deriving these formulas, the value of L has been assumed constant, that is, we assume that L does not vary with the amount of the current. These expressions for the energy in a magnetic field hence hold strictly only for cases where the reluctance of the magnetic circuit is constant, that is, when B is proportional to H. The formulas consequently apply only to coils in air, the magnetic field about a transmission line and so on. In cases where there is iron present of varying permeability, these formulas must be modified. We may sum this up by saying that the energy stored in a magnetic field is as given above provided there is no material of variable permeability present.

It will be convenient to put this formula into a somewhat different form. Still dealing with constant permeability, we know that the inductance is given by the flux linkages per abampere in the c.g.s. system; that is,

$$L = \frac{N\phi}{I} \text{ abhenries.} \tag{48}$$

If A is the cross-sectional area of the circuit, assumed constant, then this may be written

$$L = \frac{NAB}{I} \text{ abhenries.} \tag{49}$$

Inserting this value in expression (47), we obtain

$$W = \frac{NABI}{2} \text{ ergs.} \tag{50}$$

Since from Ohm's law for the magnetic circuit we know that

$$B = \frac{4\pi NI\mu}{l} \text{ gausses,} \tag{51}$$

which may be written in the form

$$NI = \frac{Bl}{4\pi\mu} \text{ abampere-turns,} \tag{52}$$

we may insert this expression in (50) and obtain

$$W = \frac{B^2lA}{8\pi\mu} \text{ ergs.} \tag{53}$$

However, the length times the cross-section of the magnetic circuit is equal to its volume; that is,

$$V = lA. \tag{54}$$

For the energy per cubic centimeter of volume of the magnetic field, therefore, we finally obtain the expression

$$W = \frac{B^2}{8\pi\mu} \text{ ergs per cubic centimeter.} \tag{55}$$

This means that if we establish a magnetic field, and the flux density is in gausses, then there is an energy storage per cubic centimeter of this magnetic field which is proportional to the square of the flux density. Where the field is in air, μ is equal to unity, and we may write

$$W = \frac{B^2}{8\pi} \text{ ergs per cubic centimeter.} \tag{56}$$

As noted, our expression for the energy storage per cubic centimeter holds only where the permeability is constant. We shall see in the next section that the energy stored per cubic centimeter of iron is always somewhat less than would

be given by this formula. For an air gap in a magnetic circuit, however, the formula is strictly true. This expression we shall find convenient in dealing with electromagnets, solenoids and so on.

Prob. 18–8. The air-gap density of a generator is 60,000 lines to the square inch and the volume of the space containing this flux is 13.3 cubic inches. What is the amount of energy in ergs? Joules? Horse-power-hours?

Prob. 19–8. If the energy of the field in Prob. 18–8 were dissipated in 0.001 second, what would be the energy dissipated per second? Express the power in ergs per second, joules per second, horse power.

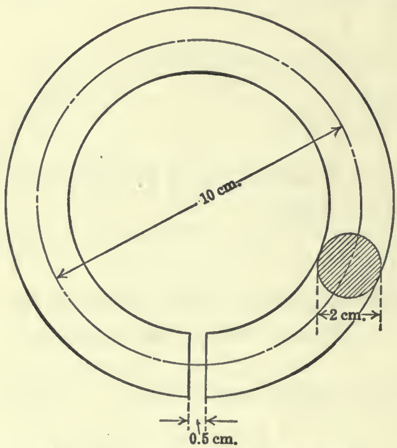

FIG. 139. A cast-steel ring with 0.5 centimeter opening.

Prob. 20–8. The cast-steel ring of Fig. 139 has a permeability of 1000 and a flux density of 10,000 gausses. Make no allowance for fringing in the air gap and compute the total energy in the air gap.

85. Magnetic Pull. We have seen above that magnetic flux lines always tend to shorten in length. When flux lines pass between two iron surfaces, they accordingly tend to pull these surfaces together. More exactly, the flux lines passing completely around a magnetic circuit and tending to shorten exert a force in the direction tending to compress the material of the entire magnetic circuit. If the circuit is in two parts, with an air gap between, then the force tends to draw these two parts together. We are all familiar with this effect from the tendency of electromagnets or permanent magnets to pull pieces of iron to themselves. From the

work of the preceding section we are now in a position to calculate exactly the amount of this force.

In Fig. 140 is shown a magnetic circuit consisting of two pieces of iron with air gaps between of length h. We will assume that the cross-sectional area of each gap is constant and of value A square centimeters. The magnetizing coil, we will assume, forces an amount of flux through the circuit which gives a uniform flux density of B gausses everywhere. Let us see what the force is in the air gaps tending to pull the two pieces of iron together.

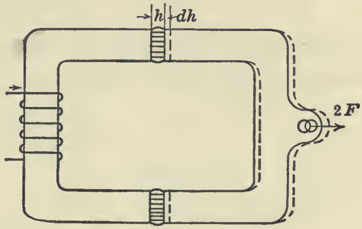

FIG. 140. The opening h is increased by an amount dh, by means of the force $2F$.

The air gap, we will assume, is short compared to the cross-sectional area, so that there is negligible fringing. This means that the lines pass straight across the air gap and the flux density is equal everywhere to B in the air gap.

The energy storage in each air gap will therefore be

$$W = \frac{B^2}{8\pi} V \text{ ergs,} \tag{57}$$

which by inserting the volume of the air gap becomes

$$W = \frac{B^2 A}{8\pi} h \text{ ergs.} \tag{58}$$

Let us now assume that a force is applied which pulls the two pieces of iron apart a certain additional small distance which we will call dh. This small increment of air gap will not appreciably affect the flux density. There will, however, be an additional volume in the air gap and hence an additional energy storage of amount

$$dw = \frac{B^2 A}{8\pi} dh \text{ ergs.} \tag{59}$$

Call the force necessary to effect this separation $2F$, that is, the force per air gap F. Then the mechanical work done in separating the pieces of iron will be the force times the distance moved, or

$$Fdh \text{ ergs.}$$

This work done must be equal to the additional energy now stored in the field; that is,

$$Fdh = \frac{B^2A}{8\pi}\,dh \text{ ergs.} \tag{60}$$

Dividing by dh we obtain

$$F = \frac{B^2A}{8\pi} \text{ dynes.} \tag{61}$$

The magnetic pull in an air gap is hence proportional to the area and to the square of the flux density. If B is in gausses and A is in square centimeters, the force will be given in dynes, since the dyne is the c.g.s. unit of force. Converting this expression to the practical system, we obtain

$$F = 0.014\,B^2A \text{ pounds,} \tag{62}$$

where

F is the force in pounds,
A is the cross-sectional area in square inches,
B is the flux density in kilolines per square inch.

It should be carefully noted that the pull is proportional to the square of the flux density. For a given amount of flux, the greatest pull will therefore be obtained when this flux is confined to a small cross-sectional area. An example will make this clear.

In Fig. 141 is shown a simple form of lifting magnet. Let us assume that the pole face P_1 has a cross-sectional area of ten square inches, while P_2 has a cross-sectional area of five square inches. Assume that the magnetizing coil forces

a total amount of flux of 500,000 lines completely around the magnetic circuit. Computing such a lifting magnet, we must be careful to allow for an air gap even when the magnet comes closely in contact with the sheet or other iron object to be lifted, unless the surfaces are very carefully brought together. If there is the slightest film of scale or even oil on the surfaces where they touch, the resulting air gap may exert a very appreciable effect on the amount of flux forced through the circuit. We will assume, however, that the coil supplies sufficient magnetomotive force to pass this total amount of flux through the circuit.

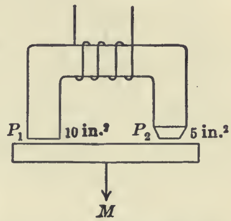

Fig. 141. A simple lifting magnet.

The flux density at pole face P_1 will be 50,000 lines per square inch, while at pole face P_2 it will be 100,000 lines per square inch.

The total pull at pole P_1 will therefore be

$$F_1 = 0.014 \times 50^2 \times 10 = 350 \text{ pounds,} \tag{63}$$

while at pole face P_2 the pull will be

$$F = 0.014 \times 100^2 \times 5 = 700 \text{ pounds.} \tag{64}$$

We thus obtain twice the amount of pull at the pole face of small cross-sectional area, even although the same total flux passes across the air gap. We have assumed, of course, that there is no fringing of the flux. In a practical example, the above conclusion would be usually somewhat modified by the effect of this fringing. For accurate results, however, the effect of the fringing can if desired be taken into account.

This effect, noted in the example above, can be illustrated practically in a striking manner. If a magnet is arranged as in Fig. 141 and a cord or chain is applied to the lower portion of the circuit at a fixed point M, it will be found that the circuit will always open at the face P_1 before it will let go at

P_2. In fact, the point of attachment would need to be moved toward P_2 until its distances from the two pole faces were approximately at the ratio of two to one before we could apply the maximum allowable force without separating the two parts of the magnetic circuit.

Advantage is taken of this effect in lifting magnets and similar devices, by rounding pole faces where they come in contact with the material to be lifted. If this rounding is carried too far, the reluctance of the magnetic circle will, of course, be increased to the extent where the resulting decrease in flux will give a disadvantage.

Prob. 21–8. If the core of a transformer has a cross-section of 4.2 square inches and a transverse gap of 0.032 inch is cut so as to be at right angles to the flux lines, and the flux density of the gap is 9000 gausses, what force in dynes acts to close the gap?

Prob. 22–8. A simple "U"-shaped magnet is made of a round cast-steel bar 2 inches in diameter. The mean length of the bent bar is 22.5 inches. Across the ends of the magnet is placed a cast-steel plate 2 inches wide, $1\frac{3}{4}$ inches thick and 10 inches long. Interposed between this plate and each of the ends of the magnet there is a disk of brass 0.021 inch thick. How many ampere-turns will be needed to support 200 pounds by means of this magnet when the "U" portion of it is held in suspension and the weight is secured to the cast-steel plate?

86. Mutual Induction. When a current is varying in a coil, there is a voltage induced in the coil which we call the voltage of self-induction. It is equal to the coefficient of self-induction, L, times the rate of change of the current. This coefficient of self-induction we define as the number of linkages produced in the coil by unit current.

In the same manner, we may speak of the mutual induction of one coil upon another. The coefficient of mutual induction we will call M. The mutual inductance of coil 1 upon coil 2 (that is, M_{1-2}) is the number of linkages of flux with the turns of coil 2 produced when there is unit current

passing in coil 1. Similarly, the mutual inductance of coil 2 upon coil 1 (that is, M_{2-1}) is equal to the number of flux linkages with coil 1 produced by unit current flowing in coil 2. These two coefficients may be shown to be equal; that is, the mutual inductance of one coil upon a second is equal to the mutual inductance of the second upon the first.

When the current is varying in one coil, we have shown in Art. 79, page 250 that there is a voltage produced in a neighboring coil through which the flux links. This voltage will be proportional to the rate of change of flux linkages. The voltage produced in the second coil is thus equal to the coefficient of mutual inductance times the rate of change of current in the first coil; that is,

$$E = M\frac{di}{dt}. \tag{65}$$

If M is in henries and the current is in amperes, the voltage produced will be measured in volts. Similarly, if M is in c.g.s. abhenries and I in abamperes, E will be in abvolts.

When two coils are situated close together, or upon the same iron core, so that much of the flux produced by one will pass through the other, we say that the coils are closely coupled. As a measure of the closeness of coupling, we define a coefficient of coupling as follows. The coefficient of coupling between two coils is the coefficient of mutual inductance between them, divided by the mean self-inductance of the coils. We use the geometric mean of the self-inductance in this definition. If K is the coefficient of coupling, we define K thus:

$$K = \frac{M}{\sqrt{L_1 L_2}}. \tag{66}$$

When all of the flux produced by one coil will pass through the second, that is, when there is no leakage flux, the coefficient of coupling is unity. Consider two identical coils with

leakage. Their coefficients of self-inductance will then be equal, or

$$L_1 = L_2. \tag{67}$$

Hence

$$K = \frac{M}{L_1} \quad \text{or} \quad \frac{M}{L_2}. \tag{68}$$

If there is no leakage flux, however, the self-inductance of a coil will be equal to its mutual inductance upon the other coil, for the same number of linkages will be produced by one ampere in each case. In this case the coefficient of coupling will therefore be unity. Suppose that the coils have now different numbers of turns; that is, assume that coil 1 has N_1 turns and coil 2 has N_2 turns. Write the reluctance of the magnetic circuit connecting them as R. The flux passing through coil 1 is then

$$\phi_1 = \frac{4\pi N_1 I_1}{\mathcal{R}},$$

so that the self-inductance of the first will then be

$$\begin{aligned} L_1 = N_1\left(\frac{\phi_1}{I}\right) &= \frac{N_1 4\pi N_1 I}{\mathcal{R} I} \\ &= \frac{4\pi N_1^2}{\mathcal{R}}, \end{aligned} \tag{69}$$

and of the second, similarly,

$$L_2 = \frac{4\pi N_2^2}{\mathcal{R}}. \tag{70}$$

In the same way, the coefficient of mutual inductance between the coils will be

$$M = \frac{4\pi N_1 N_2}{\mathcal{R}}. \tag{71}$$

Comparing these three expressions we see immediately that

$$K = \frac{M}{\sqrt{L_1 L_2}} = 1. \tag{72}$$

When there is no leakage flux, then, the coefficient of coupling is unity.

In Fig. 142 are shown two coils so arranged as to have mutual inductance. Flux lines are drawn for the condition that coil 1 is carrying a current and coil 2 is not. It will be noted that some of the flux lines due to coil 1 link coil 2. This shows that there is coupling between the two coils. Let the inductances be L_1 and L_2 and M. Suppose now that the two wires at A are connected, and that without moving the coils we examine the inductance of the combination as measured by the two remaining free ends. Assume that the windings on the two coils are in the aiding direction, that is, so as to force the flux in the same direction through each.

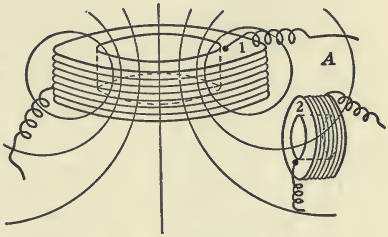

FIG. 142. Flux lines set up by coil 1 link coil 2.

The resulting self-inductance of the combination will be the total linkage per ampere of the combination. A unit current flowing in coil 1 causes L_1 linkages with its own turns and M linkages with the turns of coil 2. Similarly, this same current flowing in coil 2 causes L_2 linkages with its own turns and M linkages with the turns of coil 1. The resultant self-inductance, that is, the total flux linkages per ampere when the coils are connected in series, will be

$$L = L_1 + L_2 + 2M. \tag{73}$$

If the connection to one of the coils is now reversed, that is, if the coils are connected bucking, the flux linkages due to mutual inductance will be in the opposite direction to those produced by self-inductance. In this case the effect of mutual inductance will therefore subtract, and we shall have for the net inductance

$$L = L_1 + L_2 - 2M. \tag{74}$$

This principle is made use of in an instrument called a variometer. This is simply a variable self-inductance. It is used in radio telegraph and telephone circuits, for measurement work and for purposes where a small variable inductance is needed. The voltage regulator used on alternating-current circuits is a somewhat similar arrangement. It consists of two coils, as shown in Fig. 143, the outer one fixed and the inner one rotatable about a vertical shaft as shown. The two coils are connected in series. By rotating the inner coil, we can vary the mutual inductance between the coils from a maximum, with the coils aiding, to zero, and to a maximum with the coils bucking. The self-inductance of the combination will then be variable between the limits given by the expressions (73) and (74). The range of the device evidently depends upon the ratio of M to L_1 and L_2, that is, upon the coefficient of coupling between the two coils. In order to obtain a large range for the instrument, this coefficient of coupling is kept as large as possible. The coils are for this purpose placed with little clearance between them when in the same plane, in order that there may be little leakage of flux. Turning the inner coil 180° varies the inductance of the combination from its minimum to its maximum value. When the coils are at right angles as shown in the figure, there is no mutual induction between them, and the self-inductance of the combination is simply the sum of the self-inductances of the two coils alone.

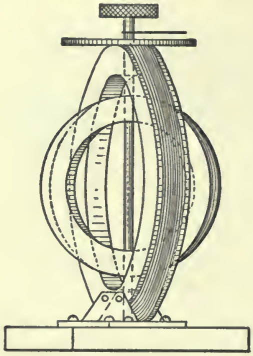

Fig. 143. A variometer.

When studying coefficients of self-induction, we noted that the self-inductance of a coil was a constant only when the permeability of the magnetic circuit in question was also constant, that is, when there was no iron present. With iron

present, the self-inductance depends upon the value of the current at which it is measured. Similarly in the case of mutual induction, the coefficient of mutual induction between two coils is a fixed value only when the permeability of the magnetic circuit connecting them is a fixed value. When iron is present, the coefficient of mutual induction is dependent upon the amount of current in the coil at the time that the coefficient is measured, and the coefficient no longer has a definite meaning.

Prob. 23–8. A concentrated coil of 200 turns of small wire (small to cut down leakage) has an inductance of 0.010 henry. The number of turns is increased to 400. What will be the approximate change in inductance?

Prob. 24–8. Two coils have inductances of 3.20 henries and 2.10 henries respectively and the mutual inductance is 2.00 henries. What is their coefficient of coupling? Sketch the coils in position for maximum and for minimum mutual inductance and show the direction of current in each for the two conditions.

Prob. 25–8. Two inductances are connected in series in such a way that they have a total inductance of 0.20 henry. The coils have self-inductances of 0.05 and 0.09 henry respectively. What is the mutual inductance and what would be the inductance of the two coils in series if the leads of one were reversed and their position in respect to each other unaltered?

Prob. 26–8. It is often necessary to construct an inductance in such a way that its value may be calculated. (For details see "Construction and Calculations of Standards of Inductance," Bulletin, U. S. Bureau of Standards, Vol. 2, pp. 87–143, 1906.) We know that for a maximum inductance a given amount of wire should be wound in a channel of square cross-section and the mean radius of the coil should be 1.85 times the length of a side of the channel. The self-inductance of such a coil is expressed by

$$L = 19.347\, an^2\, 10^{-9} \text{ henries,}$$

where

a = the mean radius of the coil in centimeters,
n = the number of turns.

Design a coil for use as a standard, of inductance 10 millihenries. This inductance coil is to be used for low frequencies, consequently it may be made of No. 18 d.c.c. (double cotton covered) wire, diameter over cotton 0.048 inch.

SUMMARY OF CHAPTER VIII

LENZ'S LAW STATES THAT: —

(*a*) Whenever there is a change in the amount of magnetic flux linking an electric circuit, a voltage is set up tending to produce a current in such a direction as to oppose this change in flux;

(*b*) The voltage thus set up is directly proportional to the rate of change of flux linkages.

As an equation this is expressed as follows: —

$$E = N \frac{d\phi}{dt} 10^{-8} \text{ volts.}$$

IF THE FLUX VARIES HARMONICALLY with the time, then

$$B = B_{max} \sin 2\pi ft,$$

where

f = the frequency in cycles per second,
t = the time in seconds.

The voltage will then be

$$E = 2\pi NAB_{max} f \cos 2\pi ft.$$

THE COEFFICIENT OF SELF-INDUCTION is the number of linkages set up by a coil with its own turns when the coil is carrying unit current.

THE VOLTAGE OF SELF-INDUCTION IS

$$E = -L \frac{dI}{dt},$$

where

L = the coefficient of self-induction.

THE GENERAL LAW CONCERNING THE DISTRIBUTION OF VOLTAGES in an electric circuit containing resistance and inductance is as follows.

At any instant the sum of the resistance drops and induced voltages equals the impressed voltage at that instant. The general form of the equation is

$$iR + L\frac{di}{dt} = E.$$

WHEN THE SOURCE OF THE ELECTROMOTIVE FORCE IS SHORT CIRCUITED, this equation becomes

$$iR + L\frac{di}{dt} = 0.$$

IF THE IMPRESSED VOLTAGE IS SUDDENLY DOUBLED or if it is reduced one-half, the equation becomes respectively

$$iR + L\frac{di}{dt} = 2E$$

and

$$iR + L\frac{di}{dt} = \frac{E}{2}.$$

THE CURRENT IN A CIRCUIT CONTAINING RESISTANCE R AND INDUCTANCE L GROWS according to the equation

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right).$$

THE CURRENT DECREASES IN THE SAME CIRCUIT when the e.m.f. is removed according to the equation

$$i = I\epsilon^{-\frac{Rt}{L}}.$$

THE TIME CONSTANT of such a circuit is the time in which the current will decrease to $1/\epsilon$ of the original value when the e.m.f. is removed; or it is the time in which the current would reach zero if it continued to decrease at its initial rate.

MAGNETIC ENERGY is analogous to the energy of moving bodies.

$$\text{Kinetic energy} = \frac{MV^2}{2} \text{ foot-pounds.}$$

$$\text{Magnetic energy} = \frac{LI^2}{2} \text{ ergs.}$$

THE ENERGY STORED IN A MAGNETIC FIELD (constant permeability) equals

$$W = \frac{B^2}{8\pi\mu} \text{ ergs per cubic centimeter.}$$

THE MAGNETIC PULL can be found from the equation

$$F = \frac{B^2 A}{8\pi} \text{ dynes,}$$

where

B = the flux density in the air gap in gausses,
A = the area of the air gap in square centimeters.

THE COEFFICIENT OF MUTUAL INDUCTION is the number of changes in flux linkages made in Coil 2 when Coil 1 changes its current value one ampere.

$$M = \frac{4\pi N_1 N_2}{\mathcal{R}}.$$

COILS ARE SAID TO BE CLOSELY COUPLED when nearly all the flux which threads one coil also threads the other. The measure of this coupling is expressed by the equation

$$K = \frac{M}{\sqrt{L_1 L_2}},$$

where

K = the coefficient of coupling,
L_1 = the self-inductance of Coil 1,
L_2 = the self-inductance of Coil 2,
M = the mutual inductance of the coils.

FOR TWO COILS CONNECTED IN SERIES with their magnetomotive forces in the same direction, the self-inductance of the combination may be expressed by the equation

$$L = L_1 + L_2 + 2M.$$

If the magnetomotive forces of the coils are opposed, the equation for self-inductance of the combination becomes

$$L = L_1 + L_2 - 2M.$$

PROBLEMS ON CHAPTER VIII

Prob. 27–8. A series circuit consisting of a 15-volt battery with an internal resistance of 1.6 ohms, a resistance coil (non-inductively wound) of 10 ohms and an inductance of 0.10 henry is carrying its normal current $\frac{E}{R} = \frac{15}{11.6} = 1.293$ amperes. Half of the cells of the battery are suddenly short circuited. Show on a plot the original constant current and the transient current caused by the short circuiting of the cells.

Prob. 28–8. Four pancake inductance coils are to be connected in series and placed in a square box so as to make a standard of inductance. How would you place these coils in the box so that their mutual inductance would be a minimum? Could you reduce the effect of mutual inductance to zero?

Prob. 29–8. Loading coils are used in telephone lines to enable the speech to be transmitted over long distances. (See "Telephonic Transmission," by J. G. Hill.) These coils are iron-cored and toroidal in form. The ratio of R to L is approximately as follows:

For the "loading" of aerial lines $\frac{R}{L} = 25.$

For the "loading" of underground lines $\frac{R}{L} = 50.$

This ratio can be held nearly constant and is determined by measuring the inductance when a current of 0.8 milliampere flows in the coil. For aerial loading, the inductance "L" is usually about 260 millihenries to the coil and the spacing is such that there is one coil about every 8 miles. Taking a soft-steel core of circular section (see Fig. 112) where $r_2 - r_1 =$ 0.75 inch and $r = 1.5$ inches, design a winding on the coil for an inductance of 260 millihenries when the current I_1 in the winding is 0.8 milliampere.

Prob. 30–8. A General Radio Company variometer (Type 190–650–59) has the following constants:

Stator turns 1174,
Rotor turns 1174,
Total inductance with the two coils in series aiding (fields in the same direction and coils in the same plane) 626.5 millihenries,
Total inductance of the two coils in series opposing (fields in opposite directions and the coils in the same plane) 106.5 millihenries.

What is the mutual inductance? The energy stored when $i = 0.2$ ampere with coils aiding? With coils opposing?

Prob. 31–8. An artificial transmission line in the Electrical Engineering Research Laboratory, Massachusetts Institute of Technology, is made up of twenty-six sections in series. Each section has an inductance of 0.0567 henry and a resistance of 3.359 ohms. If a voltage of 110 is applied to the line (the resistance and the inductance of the generator being negligible), one lead connected to each end of the line, what will happen if, after the current has reached a constant value, 50 ohms is suddenly inserted in series with the line and potential source? Plot the current from its original value to the value it has with the additional resistance in series. What is the total magnetic energy of the line before inserting the resistance? After?

Prob. 32–8. A variable inductor (variometer) such as the type 107, General Radio Company, has with its coils in series aiding and opposing inductances of 0.6 and 0.12 millihenry respectively. What is the coefficient of coupling "k"? L is same for both coils.

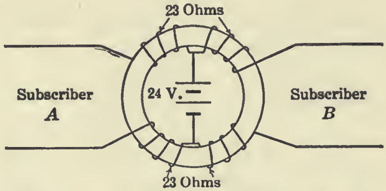

FIG. 144. A toroidal repeating coil used in telephone circuits.

Prob. 33–8. A toroidal repeating coil (see Fig. 144) used in ordinary telephone-cord circuits of central energy systems (Pender, "American Handbook," p. 1542, Fig. 29) was measured for inductance. One of the four windings of the coil had an inductance of 240 millihenries when measured by a standard inductance bridge. The same winding when measured with a current of one ampere flowing had an inductance of 0.523 millihenry. Explain the discrepancy.

Prob. 34–8. Telephone loading coils are placed one on top of another and concentrically in iron "pots." In this way they are placed where they are to be used on the line. When the coils are put into place in the pots, they are separated by thin sheet-iron disks. What is the reason for taking this precaution?

Prob. 35–8. In using a ballistic galvanometer as a flux meter for measuring fluxes of large values, why, if you wanted to reduce the galvanometer deflection, would you put a resistance in series with the galvanometer and not in parallel?

Prob. 36–8. The energy stored in an inductive circuit is equal to $\frac{1}{2} Li^2$, and the rate of energy dissipation, i^2r. It is sometimes desired to keep the ratio of $\frac{1}{2} Li^2$ to i^2r large. In the case of the toroidal repeating coil, Fig. 144, the total inductance of all the coils in series aiding is 960 millihenries, the total resistance 92 ohms. What is the energy storage if the rate of energy dissipation is 5×10^{-5} watts? (Such a value of energy storage though small is characteristic of telephone circuits.)

Prob. 37–8. The battery current is sent to the telephone instruments of the subscribers by means of the repeating coil connections as shown in Fig. 144. What effect has this flow of current on the inductance of the repeating coil if the lines to the subscribers are both of the same resistance?

Prob. 38–8. Christie gives the following design of a 250-kilowatt, 250-volt, 8-pole, 400-revolutions-per-minute direct-current generator ("Electrical Engineering," Christie): flux per pole 7.08×10^6 lines, number of turns in field coil 960, field current 7.5 amperes, resistance of each field coil 3.35 ohms. Assuming that all the flux links all the turns and that the inductance remains constant, how long after a voltage has been applied to the coil would an ammeter read 63% of the final constant current value $\frac{E}{R}$? With the above assumptions, if the field circuit were opened with the eight pole windings in series, what total energy would be dissipated in the resistance of the winding and the spark at the switch points?

Prob. 39–8. In a Cutler circuit-breaker (Cutler-Hammer Company circuit-breaker, Type W), Fig. 145, rated at 100 amperes and 200 volts, the weight of A–A is 0.4 pound. Find the flux density in the gap C'–C necessary to lift the armature A–A. Assume that half the weight of A–A is sup-

ported by the hinge and consider that the air gap is of the same area as the shoe C' and that the magnetic circuit is of soft iron.

Prob. 40–8. If, in Prob. 39–8, 52% of the flux due to a coil of 9 turns wound on the core B_1-B_1 fails to pass through the gap C'–C, what must be the value of the current in the coil to lift the latch A–A, Fig. 145? (The reluctance of the iron path is very small compared with that of the air gap.)

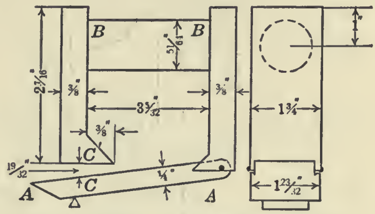

FIG. 145. A magnetic circuit-breaker.

Prob. 41–8. A 10-horsepower, 220-volt motor has a starting box whose contact arm is equipped with a no-voltage release. The coil of the release is in series with the motor field, and is wound on the portion B–B of the soft iron magnetic path shown in Fig. 146. If the spring attached to the starting arm exerts a pull of 0.4 pound on the center of the armature A perpendicular to axis of B–B, at what value of voltage will the device operate? The normal field current is 1.6 amperes and the release coil has 100 turns of wire.

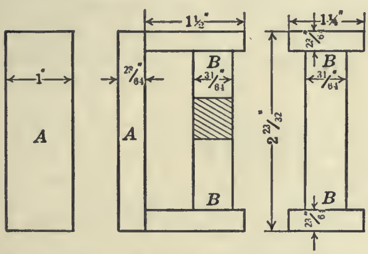

FIG. 146. A spring attached to the center of A pulls it away from the magnet when the flux density drops below a certain value.

Prob. 42–8. A series contactor (General Electric Review, Vol. 15, p. 261, 1912) is made entirely of soft iron. The core is round and is wound with 300 turns of No. 18 d.c.c. copper wire. With a current of 3 amperes, what pull will be produced? See Fig. 147.

Prob. 43–8. The total flux per pole from the permanent magnets alone of a standard bipolar telephone receiver is 311 maxwells. The area of each pole face is 0.199 by 1.14 centimeters. With this flux, what is the pull on the receiver diaphragm, assuming that all this flux passes from the pole faces into the diaphragm perpendicularly? If a voice current in-

creases the total flux by 5 maxwells, what is the change in pull on the diaphragm? If the effect of the permanent magnets were removed, and with the same pole faces and diaphragm, what would be the pull exerted on the diaphragm by 5 maxwells?

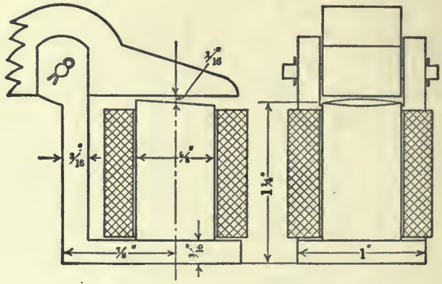

FIG. 147. A series contactor.

Prob. 44–8. Design the winding for the magnetic chuck of Fig. 148 so that it will hold to its face in intimate contact with a total force of 1200 pounds a cast-iron billet or bar of the dimensions shown. The material used throughout the construction of the chuck is cast steel. Allowable watts per

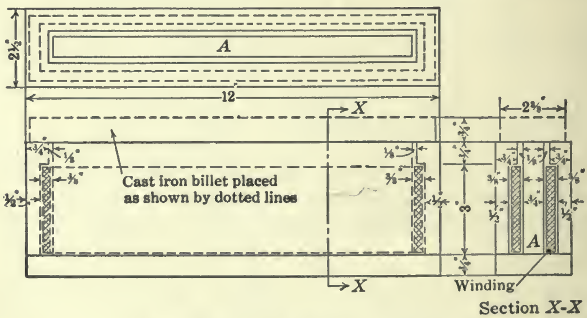

FIG. 148. A magnetic chuck.

square inch of coil (curved) radiating surface 0.5. The impressed voltage is 230. Neglect leakage in the air gap. Select the size of wire, determine the total length by finding the mean length of one turn and multiplying by the number of turns (see "American Machinist," 1915, article by Fish).

Prob. 45–8. The annular chuck of Fig. 149 is to be operated on 110 volts to hold to its surface with an average force of 50 pounds to the square inch a disk of cast steel of the same diameter as the chuck and 0.75 inch thick. The factor of safety shall be one and one-half. Allowable watts radiation per square inch of radiating surface (surface not in contact with the disk) 0.5 watt. Design the winding. (References: "American Machinist," 1915, article by Clewell; "Electrical World," 1919, article by Kenyon.)

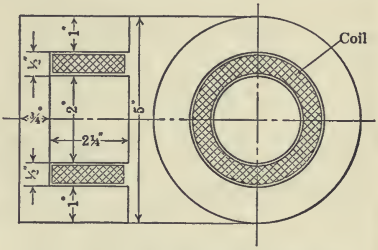

Fig. 149. An annular magnetic chuck.

Prob. 46–8. An inductance coil having 1 ohm resistance and 2 henries inductance is connected in series with a 4-ohm non-inductive resistance. If at zero time a voltage of 50 volts is impressed upon this circuit and 0.1 second later this voltage is suddenly increased to 100 volts, how long will it take for the current to reach 95 per cent of its final value?

Prob. 47–8. A telegraph relay having a resistance of 10 ohms and an inductance of 1 henry is operated by a 24-volt storage battery. The iron is worked at such a low flux density that the permeability may be considered constant with only slight error. To the armature of this relay is attached a contact which connects a non-inductive resistance of 1 ohm across the terminals of the relay when the armature of the relay picks up, and disconnects this shunt resistance when the armature drops. The relay is permanently connected to the battery through a resistance of 8 ohms. The armature is so light that inertia effects may be neglected. The armature picks up when the current reaches a value of 1 ampere and drops when the current falls to 0.5 ampere. Will the relay vibrate? If so, what time is required for the relay to go through a complete cycle?

Prob. 48–8. A circuit containing inductance and resistance has 10 volts impressed upon it and the resulting current read by an ammeter is 0.10 ampere. The growth of this current in the circuit is shown in Fig. 150, which is an oscillogram taken

at M.I.T. The time is given by the 60-cycle wave at the bottom. What is the inductance of the circuit?

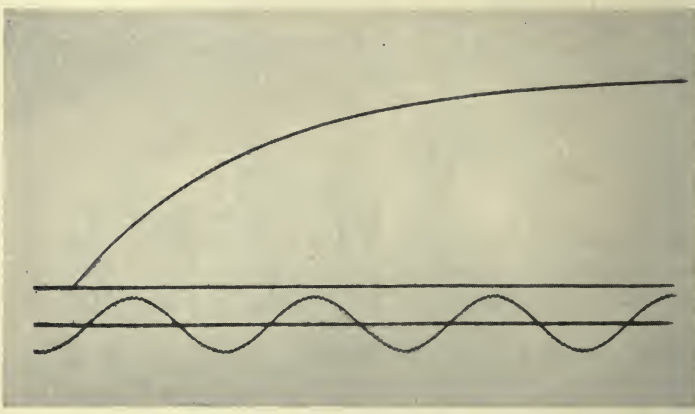

FIG. 150. An oscillogram of the growth of a current in a circuit containing inductance and resistance. *Taken by Prof. F. S. Dellenbaugh, Research Division, Electrical Engineering Dept., M.I.T.*

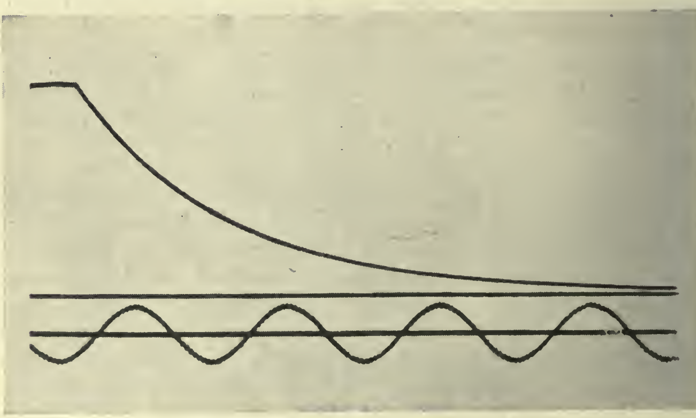

FIG. 151. An oscillogram of the decay of a current in the circuit of Fig. 150. *Taken by Prof. F. S. Dellenbaugh, M.I.T.*

Prob. 49–8. Fig. 151 is an oscillogram taken at M.I.T. of the decay of current in the same circuit as Fig. 150. The coil was short circuited and the power taken off, the constants of

the circuit and current scale during the decay of current being the same as during the growth investigated in the last problem. Show by plotting the synthesized curve from the equation that this curve checks equation 23, page 263.

Prob. 50–8. In Fig. 151a the inductance coil (L) has an inductance of 50 millihenries and a resistance of 2 ohms; the resistance (R) is five ohms. The battery has an e.m.f. of 20 volts and an internal resistance of 0.25 ohm.

(*a*) If the key (K) is suddenly closed, determine the values of the currents I_R, I_E and I_L at any time (t). (*b*) What is the current in each at 0.0005 second? at 0.005 and at 0.05 second? After conditions have become steady, the key is opened. (*c*) If it were possible to open this switch with absolutely no spark, what would be each current the instant after opening and at the end of 0.002 second? (*d*) What would be the voltage across L immediately after opening K?

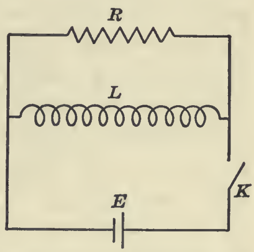

FIG. 151*a*. A circuit containg resistance and inductance.

Prob. 51–8. A transmission line is made up of two parallel conductors of 750,000 circular mils cross-section, separated by a distance of 10 feet. The line is 170 miles long. Compute the magnetic energy stored in the space surrounding the line, when it is carrying a current of 700 amperes. If the circuit is interrupted when carrying this current, what must become of this energy?

CHAPTER IX

THE MAGNETIC PROPERTIES OF IRON AND STEEL

Iron and steel are practically the only materials available for the construction of magnetic circuits. No other material approaches these in permeability. We shall consider in this chapter the magnetic properties of iron and steel.

87. Magnetic Retentivity and Hysteresis. We have already seen that the permeability of iron depends upon the extent to which it is magnetized. The permeability of ordinary iron varies from one hundred or so for very low values of the magnetizing force to three or four thousand for moderate degrees of magnetization and then decreases, so that at very high flux density the permeability becomes very low indeed. This is strikingly shown in Fig. 152, in which are curves drawn from data taken on sheet steel by Dr. Miles Walker, Manchester, England. Note that for a flux density of 25,000 gausses, a magnetizing force of 2600 ampere-turns per centimeter or 3260 gilberts per centimeter is required. The permeability of the steel at this density is only 25,000/3260 or 7.7.

If the magnetizing force is sufficiently increased, the permeability will approach unity; that is, the effect of the presence of iron is almost entirely lost, and the additional flux which is forced through by very high magnetizing forces encounters a reluctance almost as great as would be present were the magnetic circuit of air alone. We say that the iron is then saturated.

Not only is the value of the permeability dependent upon the flux density but there is a certain further effect present. The flux density which will be set up by a given magnetizing force depends upon how that force is applied; that is, whether

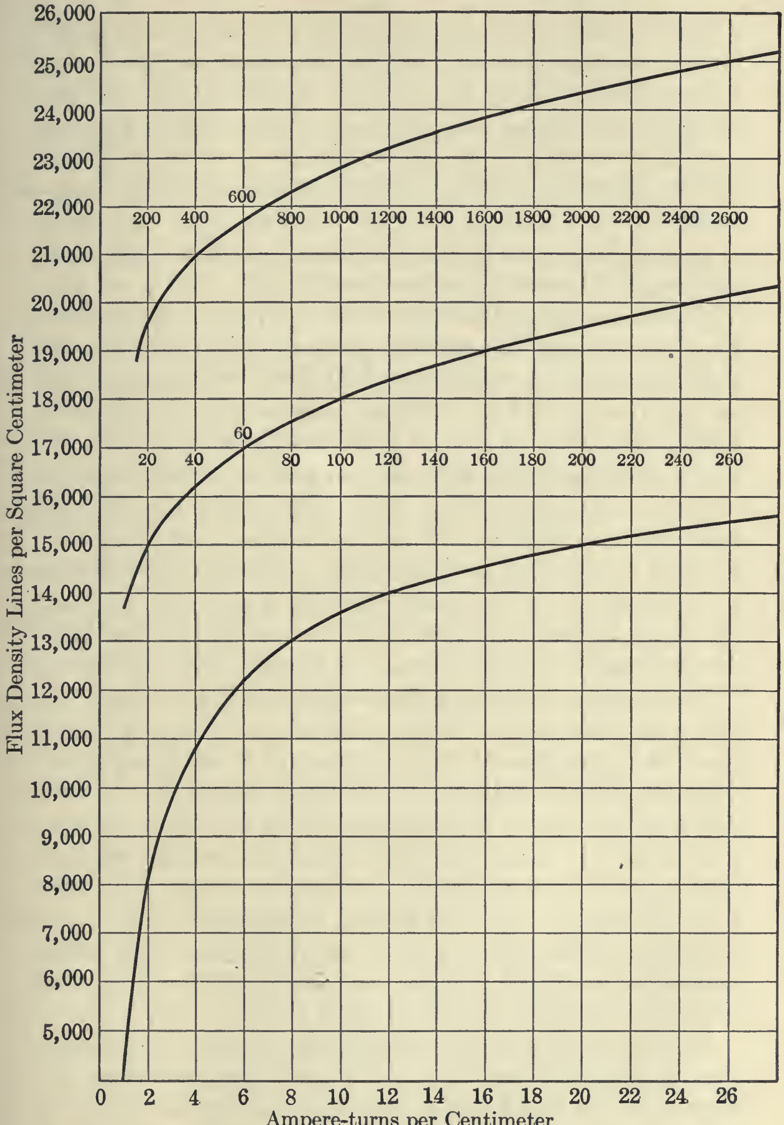

FIG. 152. Magnetization curve of sheet steel at unusually high flux densities. *Courtesy of Dr. Miles Walker, Manchester, England.*

it is increased to its final value or has previously been raised higher and then decreased. The iron tends to retain magnetism when it is once set up in it. In fact, if the magnetizing force is applied to a specimen of iron and the force is then removed, the iron will be found to retain a certain fraction of its magnetism. This is called the **residual magnetism** of the specimen. We are familiar with this effect in the preparation of permanent magnets. It is much more marked with hard steel than with soft iron. A piece of soft iron, strongly magnetized by a coil and then removed from the vicinity of the magnetizing force, will be found to retain but little permanent magnetism. A piece of hardened steel, on the other hand, may retain as much as fifty to eighty per cent of the maximum flux set up in it. The residual magnetism of a sample of iron or steel is the flux density which it retains after being magnetized and having the magnetizing force slowly removed. In order to remove this residual magnetism it is necessary to apply a magnetizing force in the opposite direction. The magnetizing necessary in order to bring the specimen back to a state where it is entirely demagnetized is called the **coercive force**. See Fig. 153.

The residual magnetism (flux density) of a specimen after being **saturated** is called the **retentivity** of the iron. The residual magnetism of a given kind of iron or steel will depend upon the maximum flux density used in the test and also upon the shape of the specimen. A ring-shaped specimen uniformly magnetized will retain much more residual magnetism than a straight bar of the same material. The retentivity of a material should be measured on a ring specimen. The reason that a bar will retain less magnetism is on account of the demagnetizing effect of its poles. This matter will be considered again later.

We have seen that when a magnetic circuit is carrying a flux of constant magnitude, there is no heating of the iron. However, it is found that if the flux is varying in amount or in direction, the iron becomes heated, thus showing that

there is a loss occurring. This loss is due to the fact that the flux density does not strictly follow the magnetizing force but lags behind it in value. A certain amount of energy which appears as heat is thus used up in the specimen. This effect is known as **hysteresis** and the loss which appears as heat is known as hysteresis loss when it is due to this cause.

The reason for the appearance of hysteresis loss can be made clear by means of an analogy. Suppose that an automobile is coasting down a hill with its gears enmeshed and its clutch engaged, so that the engine is turning over. Suppose that the throttle is entirely closed. In an actual machine it is, of course, not possible to close the inlet entirely without special adjustment. If there were no mechanical friction in the engine and the transmission, the rotation of the engine would then offer no resistance to the progress of the car. Air would be compressed in the cylinders during the compression stroke and would expand during the stroke which would normally be a working stroke. If the valves do not leak and there is no other leakage passage open, the amount of work done upon the gas in compressing it is exactly returned when the gas expands. However, if one of the valves leaks a little, the pressure on the expansion stroke is less than at the corresponding point on the compression stroke. As a result, not all of the energy of compression is returned on the expansion stroke, and the engine takes power to drive it and hence slows down the car.

The same thing is true in a magnetic circuit. If the flux density exactly follows the magnetizing force at every point, all of the work stored in the magnetic field as the flux increases will be returned to the circuit when the flux decreases and no loss will occur. Due to the retentiveness of the iron, however, the flux will not exactly follow the magnetizing force applied and a loss will occur whenever the flux density varies. If the flux density varies through a complete cycle of changes and returns to its original value, a certain net

amount of loss will be converted into heat, called the hysteresis loss per cycle.

In a transformer we have seen that the flux varies periodically between certain maximum limits. In the iron of the transformer core there is hence a core loss due to hysteresis. There is also an additional core loss due to eddy currents which we will study in the next chapter. The total hysteresis power loss will be equal to the hysteresis loss per cycle times the number of cycles per second. If the constants for hysteresis loss apply to one cubic centimeter of material, we must then multiply the result by the volume of the core in cubic centimeters.

An electric generator or motor consists of an iron armature which rotates in a magnetic field. The flux in such an armature is thus varying continuously in direction relative to the iron. It has been found experimentally that the same loss occurs due to hysteresis, when the direction of a flux changes continuously in this manner, as will occur when the direction of the flux is unchanged and the magnitude of the flux is varying between the same limits and with the same frequency. A dynamo armature has thus also a hysteresis loss which can be computed as above.

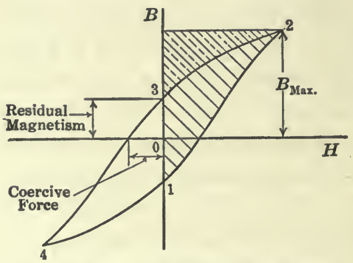

FIG. 153. A hysteresis loop.

88. Energy of a Magnetic Field in Iron. In the last chapter we saw that when a magnetic field was set up in a substance of constant permeability, the energy storage per cubic centimeter could be written as

$$w = \frac{B^2}{8\pi\mu} \text{ ergs per cubic centimeter.} \tag{1}$$

This expression held, by reason of its derivation, only where

the material was of constant permeability. We may use it, however, to find the additional energy which must be put into the material to change the flux density by a small amount, for when we consider a sufficiently small change, the variation of the permeability during such change will not be great enough to affect the results. Differentiating the above expression, we obtain

$$dw = \frac{1}{4\pi}\frac{B}{\mu}dB, \tag{2}$$

which gives the increment of energy to be added in order to change the flux density by the differential amount dB.

In this expression, we may insert the value

$$H = \frac{B}{\mu}, \tag{3}$$

giving

$$dw = \frac{1}{4\pi} HdB. \tag{4}$$

This expression does not contain μ and hence may be used in cases where μ is not a constant and where the expression above would be in error, such for instance as that of a field in iron. In order to obtain the total amount of energy per cubic centimeter of the field, we must integrate this expression, obtaining

$$w = \frac{1}{4\pi}\int HdB \text{ ergs per cubic centimeter,} \tag{5}$$

where, of course, H is a variable depending upon B and not a constant. The relation between H and B is given by the B–H curves for the material; that is, the magnetization curves of the material plotted with gilberts per centimeter as abscissas and gausses as ordinates.

Applying this equation to the magnetization curves shown in Fig. 154, we may find the energy per cubic centimeter necessary to raise the flux density in the iron to any given value.

Thus to establish a flux density B_1 in a sample originally unmagnetized, we must evaluate the integral where B varies from zero to B_1. The value of this integral is evidently the area which is shaded in the figure. The energy per cubic centimeter stored in the iron when the flux density of the latter is raised to the value B_1 will therefore be equal to $\frac{1}{4\pi}$ times the shaded area, the result being in ergs per cubic centimeter.

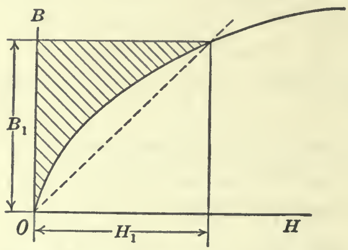

Fig. 154. The energy put into the magnetic field in magnetizing a specimen from O to B is represented by the shaded area.

We may note in passing that if the permeability had been constant, the magnetization curve would have taken the form shown by the dotted line. The energy storage in this case would have been $\frac{1}{4\pi}$ times the area of the triangle instead of the area that is shaded, or

$$\frac{1}{4\pi} \times \tfrac{1}{2}BH = \frac{BH}{8\pi}$$
$$= \frac{B^2}{8\pi\mu},$$

where μ is the ratio of final B to final H. On account of the curvature of the magnetization curve, the energy per cubic centimeter stored in the iron will thus always be less than this value.

In a magnetic circuit which is made largely of iron and contains a small air gap, the energy storage will usually be almost entirely in the air gap, on account of the large value of μ which reduces the expression above to a low amount.

89. Hysteresis Loops. When a magnetizing force is applied to iron which has been previously entirely demagnetized, and then removed, the iron will not return to its former magnetic state, but will retain a certain amount of residual

magnetism. In fact, the magnetic condition of the iron depends not only upon the magnetizing force applied but also upon the immediate previous history of the iron. In electrical machinery, however, we deal almost always with flux that is alternating, so that the magnetizing force is applied first in one direction and then in the other alternately. After a few reversals of the magnetizing force in this manner, the iron will come to a stable condition in which it will repeat a certain series of values of flux densities. If we then plot the magnetizing force against the flux density, we shall obtain what is known as a **hysteresis loop** for the material. Such a curve is shown in Fig. 153. As the magnetizing force is increased to a maximum first in one direction and then in the other, the flux density will also alternate, but the flux density when H is decreasing will be larger than for corresponding values of H when it is increasing. On the diagram is shown, as we have already pointed out, the value of the residual magnetism for the material, which is the flux density remaining when H is entirely removed, and also the value of the coercive force, which is the value of H in the opposite direction necessary to entirely remove the residual magnetism.

We have seen that when iron is magnetized to a flux density B, an amount of energy is required of value

$$W = \frac{1}{4\pi}\int HdB. \tag{6}$$

Therefore, when the value of H is increased from zero to its maximum value, the flux density in the meanwhile changing from the negative value (0–1) to its maximum value $B_{\max}$, an amount of energy will be supplied to the iron which is equal to $\frac{1}{4\pi}$ times the area included between the curve 1–2 and the B axis. This area is lightly cross-hatched. When the magnetizing force is again decreased to zero, the flux density meanwhile decreasing from its maximum value to the value of the residual magnetism 0–3, energy will be returned from

the magnetic field to the circuit of amount equal to $\frac{1}{4\pi}$ times the area between the curve 2–3 and the B axis. This area is cross-hatched in addition with dotted lines. The net amount of work done on the iron, that is, the excess of the energy put in over that returned, is the difference between these two areas multiplied by $\frac{1}{4\pi}$. This difference of area is the area 1–2–3 of half the hysteresis loop. Exactly the same procedure occurs when H increases to its maximum value in the opposite direction and again decreases to zero. For a complete cycle in which the flux density varies completely from a maximum in one direction to a maximum in the opposite direction and back again, the net amount of work supplied is thus equal to $\frac{1}{4\pi}$ times the area of the hysteresis loop. This energy is converted into heat.

The hysteresis loss per cycle in a sample of iron subjected to an alternating magnetizing force is equal to $\frac{1}{4\pi}$ times the area of the hysteresis loop. If H is plotted in gilberts per centimeter and B in gausses, the energy thus found will be in ergs per cubic centimeter of the material per cycle. If other scales are used, correction must be made accordingly. These corrections will appear from the examples given below.

FIG. 155. The hysteresis loop for soft iron. A narrow loop means a small loss.

The area of the hysteresis loop is thus a measure of the amount of the hysteresis loss. When this loss is small, the area will be long and narrow, as for instance in Fig. 155, which shows the hysteresis loop for a sample of carefully

annealed soft iron. Fig. 156 shows the other extreme, a hysteresis loop for a sample of hard steel.

The hysteresis loop for a material can be found experimentally by applying successive values of magnetizing force, measuring the corresponding values of flux density and plotting the results. In making these measurements it is, of course, necessary to be sure that *H*, and hence the magnetizing current, increases steadily and without decreasing until it reaches its maximum value, and then decreases steadily to zero. For such work a permeameter is especially useful. This instrument consists simply of an arrangement whereby a measuring coil may be made to surround a sample of iron to be tested, and may suddenly be removed by opening the magnetic circuit and pulling the coil out. It is certain in this way that the flux through the coil decreases to zero, so that a ballistic galvanometer connected with it shows a deflection corresponding to the flux at the instant the coil is removed.

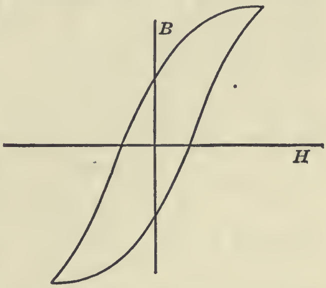

FIG. 156. The hysteresis loop for hard steel. The wide loop means a large loss.

To find the value of *B* corresponding to a given value of *H* on the hysteresis loop, we consequently proceed as follows. Place the measuring coil in position. Reverse the magnetizing current several times to be sure that the flux is accurately following the hysteresis loop in value. Then, moving always in the correct direction, increase the magnetizing current until the desired value of *H* is present. Snap the measuring coil out of position and note the deflection of the ballistic galvanometer. From this, calculate the value of *B*. Plot these values of *B* and *H* as a point on the hysteresis loop. By proceeding in this manner the entire hysteresis loop can be traced.

Example 1. Let us examine an example of the application of these principles. A certain transformer has a core containing in all 500 cubic inches of transformer iron. This is placed on a circuit and magnetized by an alternating current so that its maximum flux density is 80,000 lines per square inch. The frequency of the magnetizing current is 60 cycles per second; that is, the iron is magnetized in each direction sixty times every second, a complete cycle thus occupying one-sixtieth of a second. A sample of this iron is tested and its hysteresis loop plotted. This loop is plotted to a scale such that one space in a vertical direction is equal to 1,000 lines per square inch, and one space in a horizontal direction equal to one ampere-turn per inch of length of the core. The area of this hysteresis loop is measured by a planimeter and is found to be 400 square spaces. We wish to compute the watts loss in this core due to hysteresis.

In the first place, one square space is equal to the product of 1000 lines per square inch by one ampere-turn per inch. Changing these values to gausses and gilberts per centimeter, we have

$$1 \text{ square space} = \frac{1000}{6.45} \text{ gausses} \times \frac{0.4\pi}{2.54} \text{ gilberts per centimeter.}$$

The loss is equal to $\frac{1}{4\pi}$ times the area of the loop when converted to terms of gausses and gilberts per centimeter. The energy loss per cubic centimeter of the material for each reversal of the magnetizing current is therefore

$$\frac{1}{4\pi} \times 400 \times \frac{1000}{6.45} \times \frac{0.4\pi}{2.54} \text{ ergs per cubic centimeter per cycle.}$$

Multiplying this by 10^{-7} to convert to joules, we have

$$\frac{4 \times 10^4}{\overline{2.54}^3} \times 10^{-7} \text{ joules per cycle per cubic centimeter.}$$

Let us multiply this energy by the number of cubic centimeters in the core and by the number of cycles per second. We shall then find that the total watts loss is

$$\frac{4 \times 10^4}{\overline{2.54}^3} \times 10^{-7} \times \overline{2.54}^3 \times 500 \times 60 \text{ watts,}$$

or the total loss is

$$120 \text{ watts.}$$

The hysteresis loss for a sample of iron in an electric machine is evidently proportional to the frequency. If the frequency is doubled, the hysteresis loss will also be doubled, other factors remaining the same. Hysteresis loss is for this reason a matter of much greater moment in high-speed machinery than in low-speed machinery. In transformer design also, it is of greater importance with high frequency. This is particularly true where very high frequencies are employed. In alternating-current generators for very high frequencies, such as the Alexanderson alternator for the generation of frequencies up to 100,000 cycles per second for radio telegraph purposes, the prevention of very high hysteresis loss is a matter of great moment. This is accomplished by using an excellent grade of iron which has a low area of the hysteresis loop, and also by utilizing a very small volume indeed. These alternators are so arranged that the iron part in general carries a constant flux with the exception of a small volume which carries an alternating flux. The hysteresis loss per cubic centimeter of this material is high, but since a very small volume is used, the total losses are kept within reason.

In considering core losses in electrical machinery where the flux density is varying, we must consider not only the hysteresis loss as dealt with in this chapter, but also **eddy-current loss** as treated in the next chapter. Together they are grouped as **core loss**. Wherever the flux density in electrical machinery is varying and iron is present, these losses must be taken into consideration. Not only will they decrease the efficiency of the apparatus in question, but it is necessary to make provision for properly getting rid of the heat evolved to prevent an undue temperature rise of any part of the machinery.

Prob. 1–9. A hysteresis loop for a given specimen of iron (a rod 0.236 inch in diameter and 8.42 inches long) has an area of 8.36 square inches. The loop is plotted on cross-section paper so that an ordinate of one inch is equivalent to 4000 gaus-

ses, an abscissa of one inch to 20 gilberts per centimeter. What energy loss in ergs per cubic centimeter per cycle is represented by the area of the loop? Express this energy in joules per cubic inch per cycle.

Prob. 2–9. If the hysteresis loop of the specimen of iron in Prob. 1–9 were taken with the magnetizing coil connected to a 25-cycle source, what would be the energy in joules per second or watts dissipated in the iron?

90. Mean Magnetization Curves. Froelich's Equation. When an alternating magnetizing force is applied to a sample of iron, we obtain a hysteresis loop which shows the variation in flux density corresponding to the variation in magnetizing force. If the maximum value of the magnetizing force is adjusted to different values, we obtain a series of such hysteresis loops corresponding to the different values of the maximum flux density of each. Such a series of loops is shown in Fig. 157. The dotted curve which connects the peaks of these hysteresis loops is usually plotted as the mean magnetization curve of the specimen. The curves between ampere-turns and flux densities which are shown in Fig. 61 of Chapter VI are obtained in this manner. This mean magnetization curve is usually the curve in which we are interested in determining the relation between B and H, for we usually deal with periodically varying magnetizing forces.

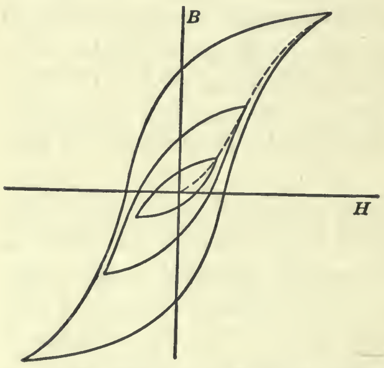

Fig. 157. Hysteresis loops for different maximum flux densities.

It was determined by Froelich that the magnetization curve of irons and steels may be approximately represented

by an equation of the form

$$B = \frac{aH}{b + H}. \tag{7}$$

This is the equation of a hyperbola. It is an empirical equation since it is derived from the study of a mass of data. It does not apply to the extreme lower end of the curves, but fortunately is a very close approximation for that part of the magnetization curve which is in greatest use.

The value of the constants in Froelich's equation for any particular magnetization curve may be derived as follows.

If

$$B = \frac{aH}{b + H}, \tag{8}$$

then

$$\frac{H}{B} = \frac{1}{a}(b + H). \tag{9}$$

Let

$$b = J \tag{10}$$

and

$$\frac{1}{a} = K. \tag{11}$$

The equation becomes

$$\frac{H}{B} = K(J + H). \tag{12}$$

This is the equation for a straight line with intercept J and slope K if $\frac{H}{B}$ is considered one of the variables. We have only to determine sufficient values of $\frac{H}{B}$ (which is the reluctivity of the material) and plot these values against H. The result is a straight line as in Fig. 158 if the curve follows Froelich's equation.

The straight line in Fig. 158 is the plot between the values of $\frac{H}{B}$ and H taken from the magnetization curve for annealed sheet steel which is drawn on the same sheet. The equation for this straight line is

$$\frac{H}{B} = K(J + H),$$

when J is the intercept on the H axis and K is the slope of the curve.

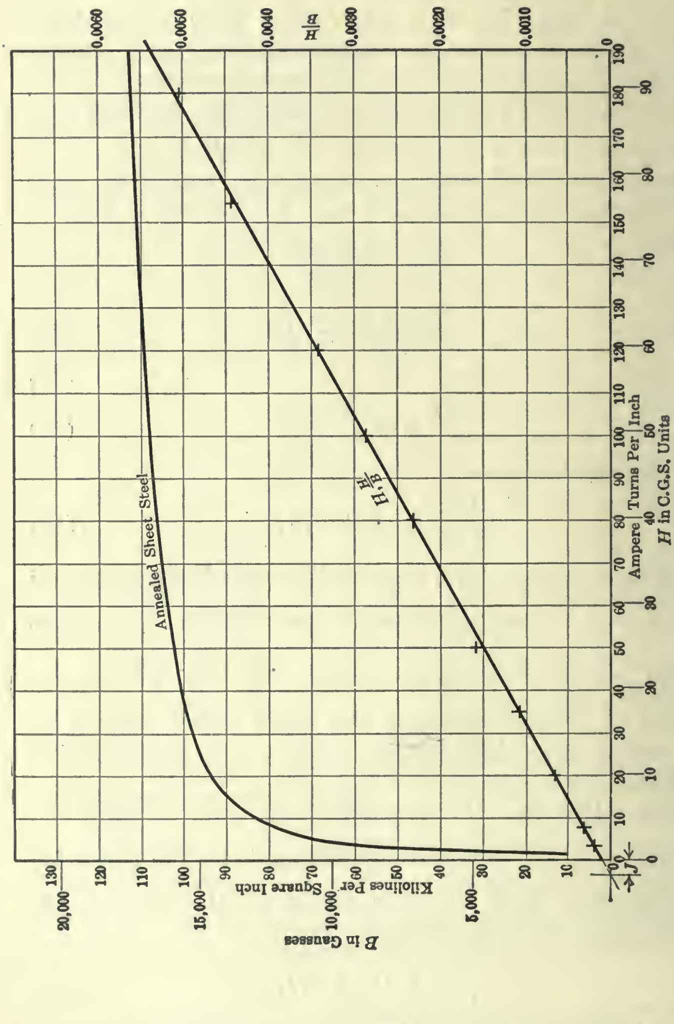

FIG. 158. The straight line is a plot between the values of H and $\frac{H}{B}$ obtained from the magnetization curve shown here.

But from equations (10) and (11)

$$K = \frac{1}{a}$$

and

$$J = b.$$

The value for a in Froelich's equation is therefore the reciprocal of the slope of the curve between $\frac{H}{B}$ and H, and the value for b is the intercept on the axis of H.

Note that in plotting the curve between $\frac{H}{B}$ and H the extreme low values of H should not be used.*

Prob. 3–9. Determine Froelich's equation for the magnetization curve of annealed sheet steel as shown in Fig. 158.

Prob. 4–9. What part of the curve, if any, for cast iron as given in Fig. 61 satisfactorily follows Froelich's equation? Determine the constants a and b for this part of the curve.

91. Methods of Demagnetizing Steel. If we start out with an entirely unmagnetized piece of iron and gradually increase the magnetizing force, a variation of flux density will be obtained which will give a curve very similar to the dotted curve of Fig. 157. For most purposes the curve thus obtained and that of Fig. 158 are identical. The mean curve obtained as the locus of the peaks of the successive hysteresis loops is, however, the one generally employed in practice. Of course it must be realized that in most of the iron with which we deal in electrical apparatus, the hysteresis loop will be long and narrow, more of the appearance of Fig. 155 rather than the exaggerated form which we have shown in Fig. 157 for convenience. In other words, the flux density on rising and decreasing magnetizations will not differ as much as here shown, when we are dealing with steel which is of proper quality to be used in electrical apparatus where alternating magnetizing forces are employed.

* For precise methods of obtaining values of J and K from curves, see Lipka's "Graphical and Mechanical Computation."

When the magnetizing force is removed from a sample of iron, there will remain a certain flux density, which we have said is called the residual magnetism. In order to completely demagnetize the iron it is necessary to employ a coercive force, which is a magnetizing force in the opposite direction, and of sufficient amount to neutralize this residual magnetism. There is another and more convenient method, however, of demagnetizing a sample of iron. If the magnetizing force is made alternating and then is gradually decreased to zero, the flux density will vary in accordance with a hysteresis loop of constantly decreasing size, which will finally wind its way into a very small loop about the origin. By thus varying the magnetizing force and at the same time decreasing it to zero, the flux can be removed almost entirely from even a sample of hard steel. Thus to demagnetize the hair-spring of a watch we may place it near an open-core solenoid, the winding of which is carrying an alternating current, and with the current still turned on in the solenoid gradually remove the watch from the field. An almost equal effect can be obtained by bringing the watch close to a permanent magnet, rotating it rapidly, and gradually withdrawing it from the field. In either of these cases the magnetizing force applied to the spring is alternated and gradually decreased to zero.

92. The Steinmetz Equation. We have seen that the hysteresis loss which is produced when the magnetizing force applied to a given sample of iron is alternated is proportional to the volume of the iron and also to the frequency of alternation. This means that hysteresis loss per cubic centimeter per cycle is a constant for a given maximum flux density. It is of great practical interest to know how this loss varies with the maximum flux density employed.

There is no theoretical law governing this variation, but Steinmetz, as the result of an extended series of tests, has derived an empirical equation which fits the facts sufficiently closely for engineering purposes if used over a moderate

range. This equation states that the hysteresis loss varies as the 1.6th power of the maximum flux density. That is,

$$w = \eta f B_{\max}{}^{1.6} \text{ ergs per cubic centimeter per second,} \quad (13)$$

where

w = loss in ergs per second per cubic centimeter of the material,

$B_{\max}$ = maximum flux density,

f = number of cycles per second of the magnetizing force.

The flux density used is the maximum flux density obtained during the cycle and is expressed in gausses. To obtain the loss in watts per cubic centimeter, we simply multiply the value obtained from the above equation by 10^{-7}. Eta (η) is a coefficient, called the **coefficient of hysteresis loss,** which depends upon the kind of iron being used. For good silicon steel, the value of η is 0.001.* The value for soft iron is 0.002 to 0.004. For hard cast steel it is as high as 0.025, and in an extreme case for tungsten steel it may run as high as 0.058.

It should be emphasized that this formula is an empirical formula only and simply sums up the results of experience. It applies fairly well for the ordinary flux densities used in practice, that is, from 1500 to 12,000 gausses. If the flux density is higher than this, the loss will be found to increase faster than is indicated by the formula. For very low values of flux density, also, the formula will be found to be greatly in error.

As an example of the use of this formula, suppose that the hysteresis loss in a certain transformer is 400 watts. It is proposed to double the voltage applied to this transformer, which will result in practically a doubling of the maximum

*Note: Lloyd, "Magnetic Hysteresis," Journal of the Franklin Institute, July, 1910.

flux density. If the frequency is maintained as before, what will be the new hysteresis loss?

Since the loss varies as the 1.6th power of the flux density, doubling the flux density will make the new loss

$$400 \times 2^{1.6} \text{ watts,}$$

or

$$1210 \text{ watts.}$$

The Steinmetz formula is based on the hysteresis loop. It states in effect that the area of this hysteresis loop varies as the 1.6th power of its height. The formula applies only for normal hysteresis loops; that is, in cases where the maximum flux density attained in one direction is equal to the maximum flux density obtained in the opposite direction. For unsymmetrical loops, such as would be obtained where an alternating and a constant magnetizing force are applied simultaneously, the formula will not hold. It will not hold either for cases where the magnetizing force is not increased continuously to its maximum value. Suppose, for instance, that we apply to a magnetizing coil a current which varies in accordance with the curve shown in Fig. 159. It will be noted that the current increases for a while, then decreases slightly and finally increases to its maximum value, then decreases to zero, and then repeats its performance in the opposite direction. The result will be a hysteresis loop with a re-entrant loop as shown in Fig. 160. The small twist in the curve is caused

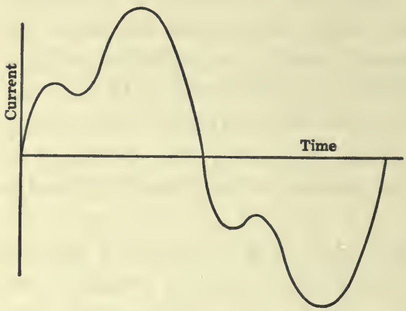

FIG. 159. A non-sinusoidal exciting current of this form produces a hysteresis loop of the form shown in Fig. 160.

by the decrease in the current before it has arrived at its maximum value.

For such cases of distorted wave forms as this, the formula will naturally not apply even approximately. Such cases can be analysed only by actually plotting the hysteresis loop itself and measuring the area. In measuring this area with a planimeter, we pass around the curve as it is actually traced. This process will add in the area of the small loops in addition to the area of the large loop. The result obtained for the area will be the actual hysteresis loss under the conditions which produce such a distorted curve.

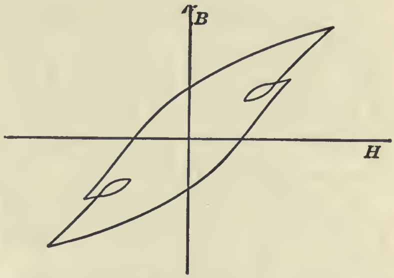

FIG. 160. The form of hysteresis loop produced by a current of the form in Fig. 159.

Prob. 5–9. The sheet-steel core of a transformer has a mean length of 32 inches and a cross-section of 2 square inches. When 110 volts, 60 cycles, is impressed on the winding, the maximum flux density is 40,200 lines per square inch. For this particular steel $\eta = 0.0013$. Find the hysteresis loss in watts per cubic inch. Find it in watts per pound, assuming the volume of iron to be 0.9 times the volume of the core, owing to scale on the laminations of which the core is made up. What is the total hysteresis loss in the core?

Prob. 6–9. A transformer core with a volume of 8.56 cubic inches has a hysteresis loss at 25 cycles of 2.14 watts. The maximum flux density in the material is 60,000 lines to the square inch. Find η. If the frequency is raised to 60 cycles and the flux density reduced one-half, what will be the new loss in watts per cubic centimeter?

93. Effect of the Composition of Steel. It has been noted above that the chemical composition of a steel has a very great effect indeed upon its hysteresis loss. A hard

tungsten steel may have fifty times as much hysteresis loss for a given flux density as a silicon steel of proper composition.

The hysteresis coefficient and the permeability depend not only upon the chemical composition of the steel but also upon its heat treatment. Carefully annealing a sample of iron may reduce its hysteresis loss by as much as fifty percent. The properties also vary with the temperature. Both iron and steel, in fact, lose all their magnetic qualities at about 750 degrees centigrade and have a permeability of practically unity above that temperature. This is called the **recalescence** point of the material.

If ordinary steels are subjected to a temperature of about 100 degrees centigrade for a long period, the hysteresis coefficient will gradually increase. This process is known as aging. Modern silicon steels, such as are used in transformer construction, are almost entirely free from this aging. This and a fairly low hysteresis coefficient are the reasons for the use of such steel in cases where alternating magnetizing forces are employed. These steels usually contain from three to four percent silicon. It is necessary that the carbon content be low, less than 0.1 percent. A small percentage of either sulphur or phosphorus will render the steel extremely poor electrically. The sheets, after rolling, are annealed at about the recalescence point and cooled very slowly.

In the study of eddy-current loss, we shall see later that high resistivity is desirable in transformer steels. This property is also obtained in silicon sheet steel.

For low retentivity, a Norway iron is usually employed. This material has a very high permeability but a low value of residual magnetism, and a low coercive force. It is accordingly used for the armatures of electromagnets or relays in places where an attraction due to residual magnetism is undesirable. Ordinary wrought iron has a high permeability but its hysteresis loss is fairly large. Cast iron has a relatively low permeability, some samples having a

maximum of only 150, as compared with values of 3000 for wrought iron. Malleable cast iron may have a permeability as high as 700.

There has recently been produced an electrolytic iron melted in a vacuum which has extraordinary properties.* A maximum permeability of 19,000 has been obtained at a density of 9500 gausses. The hysteresis loss was half that found for the best grade of commercial transformer iron. Unfortunately, however, at the present time such iron is very expensive and is simply a laboratory product.

94. Permanent Magnets. In electrical apparatus where only a moderate field strength is required, a permanent magnet is often employed. A familiar example is the magnet employed to supply flux to a magneto used for gas-engine ignition. For dynamos of large output, an electromagnet is always used, but where the output is very small, as in the above case, a permanent magnet can sometimes be employed. In electric meters also permanent magnets are used, and in this case they must be strictly constant in field strength over long periods of time in order that the meters may not show large errors. Large numbers of permanent magnets are also used on integrating wattmeters, such as we find on house circuits for measuring the kilowatt-hours used on alternating-current lighting circuits.

Such magnets are usually constructed of tungsten steel, containing about seven percent of tungsten. This steel is hardened carefully by heating it to the recalescence point and quenching. This must be done with great care. If the temperature is incorrect by even fifteen or twenty degrees, a considerable reduction in the retentivity will be found. It is also necessary to quench very carefully in order to avoid cracking the steel.

The magnets are then magnetized by surrounding them with a coil through which can be passed a heavy current, usually from a storage battery. The magnetizing force

* Proceedings A. I. E. E., February, 1915, p. 236.

should be such as to saturate the steel, magnetizing forces of 100 ampere-turns per centimeter or so being employed. Horseshoe magnets are usually magnetized in pairs in order to give them a complete magnetic circuit. Magnets for magnetos generally have iron keepers placed across the ends at the time when they are magnetized. In very careful magneto construction, the keeper is never removed completely until the magnet is placed in position, when the iron parts of the magneto take its place. A certain increased magnetization will usually result from jarring a magnet somewhat while the magnetizing force is applied. This apparently allows the molecules to move more freely into the magnetic position. By using care with good steel, a permanent magnet may be constructed which will give in an air gap of small length a flux density of about 6000 gausses.

If a freshly charged magnet is violently jarred, it will lose some of its magnetism. Snapping the keeper into place is especially damaging. In the preparation of magnets which must maintain an extremely constant flux, they are subjected to artificial jarring after being magnetized to remove such parts of the magnetism as are likely to disappear in time.

Permanent magnets, when left standing, will gradually lose a portion of their magnetism, and after a time settle down to constant values, unless they are abused in some way. Meter magnets are often artificially aged by maintaining them at a temperature of 100 degrees centigrade for several days in order to arrive quickly at this final value of strength. They then become very permanent and will not change their strength appreciably in a long time.

SUMMARY OF CHAPTER IX

THE RESIDUAL MAGNETISM is the magnetism remaining in a material after the magnetizing force has been removed.

MAGNETIC HYSTERESIS LOSS is the heat loss in magnetic materials due to the LAGGING of the values of flux density behind those of the magnetizing force.

A HYSTERESIS LOOP is formed if the values of flux density are plotted against the corresponding values for magnetizing force throughout a complete cycle of positive and negative values of magnetizing force. The area of this loop is 4π times the hysteresis loss per cycle per cubic centimeter of the specimen.

THE ENERGY OF A MAGNETIC FIELD IN IRON is expressed by the equation

$$w = \frac{1}{4\pi}\int H dB \text{ ergs per cubic centimeter,}$$

where H is a variable depending upon the value of B.

The STEINMETZ EQUATION for hysteresis loss is

$$w = \eta f B_{max}^{1.6} \text{ ergs per cubic centimeter per sec.,}$$

where

η is the hysteresis coefficient depending upon the material,
f is the number of cycles per second of the magnetizing force.

FROELICH'S EQUATION applies approximately to all but the lower and upper ends of the MEAN magnetization curve of magnetic materials.

$$B = \frac{aH}{b + H},$$

where a and b are constants depending upon the material.

To determine the values of these constants, plot H against H/B and call J the intercept on the x axis and K the slope of the curve, which will be a straight line throughout the region where the magnetization curve follows Froelich's equation. Then

$$a = \frac{1}{K}$$

and

$$b = J.$$

TO DEMAGNETIZE A MAGNETIC MATERIAL, place it in an alternating field and gradually decrease the strength of the field.

THE COMPOSITION AND HEAT TREATMENT of magnetic materials have great effect upon the hysteresis coefficient of the materials.

THE BEST PERMANENT MAGNETS are made of tungsten steel. When properly aged and carefully handled, such a magnet will retain a practically constant flux density for years. Heat and rough handling will materially decrease its strength.

PROBLEMS ON CHAPTER IX

Prob. 7–9. The following data were taken in the Technical Electrical Measurements Laboratory, Mass. Inst. of Tech., on a bar of iron by means of a "Koepsel Permeameter," the readings being taken successively in this order:

H gilberts/centimeter	*B gausses*
+ 0	+ 0
+ 20	+ 5000
+ 4.5	+ 4000
+ 0	+ 3400
− 6.0	+ 1750
− 10.0	− 100
− 14.5	− 2500
− 18.5	− 4200
− 20.5	− 5000
− 5.5	− 4000
− 2.0	− 3500
+ 0	− 3200
+ 7	− 750
+ 10.5	+ 1600
+ 15.0	+ 3600
+ 20	+ 5000

Plot the values and determine from the hysteresis loop the coercive force and the relation between the residual magnetism and the maximum flux density. Note that this is not the retentivity since the maximum flux density does not represent saturation. The loop formed may be assumed to represent with sufficient accuracy conditions after many flux reversals.

Prob. 8–9. With the data of Prob. 7–9 find the energy in ergs per cycle represented by the hysteresis loop.

Prob. 9–9. The same specimen of iron as in Prob. 7–9 is tested to a higher flux density by the same method with the following result:

H gilberts/centimeter	*B gausses*
+ 66.5	+ 12000
+ 41.0	+ 11250
+ 19.0	+ 10000
+ 9.0	+ 9000
+ 5.5	+ 8600
+ 3.0	+ 8200
0	+ 7600
− 2	+ 7000
− 4	+ 6500
− 7	+ 5250
− 10	+ 3600
− 15	− 250
− 19	− 2800
− 20.5	− 4000
− 27.5	− 6900
− 34	− 8500
− 44	− 10250
− 66	− 12000
− 32	− 11100
− 22.5	− 10500
− 10.0	− 9200
0	− 7800
+ 10	− 3200
+ 15	+ 100
+ 19	+ 3100
+ 25	+ 5800
+ 35	+ 8400
+ 46	+ 10000
+ 66.5	+ 12000

From a plot of these data to the same scale as that used in Prob. 7–8, determine the coercive force and the residual magnetism. Superimpose this plot on a plot of the data of Prob. 7–8, and sketch the mean magnetization curve as determined by the points of the two hysteresis loops.

Prob. 10–9. The coefficient of hysteresis loss, η, determined from the above data is 0.00192 and the exponent of B_{max}, 1.56, which agrees experimentally with the exponent 1.6 as determined by Steinmetz. What would be the watts loss per pound for this specimen at 60 cycles?

Prob. 11–9. The data for a magnetization curve are as follows in c. g. s. units:

H	B
5.5	550
12	1776
19	4500
25	7100
40	9700
66	12075
109	13875

Plot a curve such as Fig. 154 and determine the energy in ergs per cubic centimeter stored in the iron when the flux density is 77,000 lines per square inch. What is the percentage difference between this energy and the amount of energy which would have been stored had the permeability had the same constant value that it had at the density of 77,000 lines to the square inch? (Assume that Simpson's rule will apply to the curve.)

Prob. 12–9. The following design constants for a 20-kilowatt, 220–2200-volt transformer are representative of American practice:

Mean length of core, 43 inches,
Cross-section of core, 4.6 $\times$ 4.6 inches,
Maximum flux density, 68 kilolines to the square inch,
Material of core, silicon steel, $\eta = 0.0011$,
Net volume of iron, 0.92 times the gross volume of core as determined from the dimensions given.

Determine the hysteresis loss in the core, expressing it in joules per cycle. At a frequency of 60 cycles, what is the total hysteresis loss in watts?

Prob. 13–9. The following magnetic-circuit dimensions are taken from a Weston 5–15–150 voltmeter (Fig. 185):

Mean length of permanent magnet, 12.10 inches,
Cross-section (uniform), 1.25 $\times$ 0.312 inch,
Cross-section of air gap (very nearly), 1.32 square inches,
Length of air gap (taken radially), 0.15 inch.

It was proposed by Hookham (Philosophical Magazine, 1889) that if:

A = area of cross-section of air space,
L = distance between pole pieces (if two air gaps, L = two times the length of one),

a = cross-section of magnet,
l = length of magnet (mean length of lines of force in the permanent magnet),

then for permanency, the ratio

$$\frac{A}{L} \times \frac{l}{a}$$

should be greater than 100. This ratio is known as the permanency factor. Determine the permanency factor for the magnet above; also the flux density in the air gap and in the iron if the flux crossing the gap is 8.51×10^3 maxwells.

Prob. 14–9. A direct-current relay has a resistance of 4 ohms and is operated by a 6-volt storage battery. The armature of this relay starts to pick up for a flux of 20,000 maxwells. Since the magnetic circuit is made of iron, the flux is not proportional to the current. An empirical relation between flux and current which is reasonably accurate is as follows:

$$\phi = \frac{80\ Ni}{1 + 0.002\,Ni},$$

where N is the number of turns and in this case is equal to 500. Assume that every flux line links all of the turns. What time after the relay is connected to the battery will elapse before the armature starts to pick up? The internal e.m.f. of the battery is 6 volts, the internal resistance 0.01 ohm and the resistance of the leads from relay to battery is 0.19 ohm.

Prob. 15–9. In 60-cycle transformers with silicon-steel cores, containing 3 to 4 percent silicon, the hysteresis loss is from 0.54 to 0.82 watt per pound of core.* Using the value of η in the Steinmetz equation as 0.0007, compute the maximum flux density at which the steel is worked when the loss is 0.54 watt per pound. The specific gravity of steel is 7.5.

Prob. 16–9. Compute the limiting values of η in the Steinmetz equation for ordinary annealed steels which have hysteresis losses of from 1.0 to 2.0 watts per pound at a flux density of 64,500 lines per square inch and at a frequency of 60 cycles.*

Prob. 17–9. A 220-volt relay has a straight cylindrical iron core, the length and diameter of which are respectively $2\frac{1}{2}$ inches and $\frac{5}{8}$ inches. It is wound with fifteen hundred turns,

* Trans. A. I. E. E., vol. XXVIII, page 465.

and the resistance of the winding is 200 ohms. It closes when the flux density in the core is 20,000 lines per square inch. Use the permeability obtained by taking the slope of the line drawn through the origin and the point of maximum flux density on the magnetization curve, and determine the time required for the relay to act after the closing of the controlling key.

Prob. 18–9. The inductance of a circuit may be defined as the rate of change of flux linkages with current. Bearing this in mind, devise a graphical method for the solution of the Prob. 17–9 taking into consideration the non-linear magnetization curve.

Prob. 19–9. The following magnetization data were taken of a $3\frac{1}{2}$-h-p. motor at M.I.T. Between what limits of flux density does Froelich's equation hold for this motor? Compute the values of a and b in Froelich's equation for this part of the curve.

Field Current in Amperes	Generated Voltage $E = kB$	Field Current in Amperes	Generated Voltage $E = kB$
0.10	15.0	1.20	167.0
0.20	34.0	1.40	179.0
0.30	52.3	1.60	190.0
0.50	89.0	1.80	199.0
0.70	119.5	2.00	205.5
0.80	131.0	2.20	210.2
0.90	143.0	2.40	214.3
1.00	152.0	2.60	218.3
		2.80	221.0

CHAPTER X

GENERATED VOLTAGES

We have seen how by means of a change in the magnetic linkages an electromotive force is induced in the turns of a stationary coil or in the conductors of a transmission line. In this chapter will be shown how this change in magnetic linkages, in direct- and alternating-current generators, is utilized to produce the standard voltages for lighting and power.

95. Change of Linkages. Wherever the magnetic flux linkages through a coil change in number, there is voltage induced. This voltage is proportional to the rate of change of linkages, that is,

$$e = N\frac{d\phi}{dt}10^{-8}\text{ volts}, \tag{1}$$

where

N = number of turns,
ϕ = flux in maxwells,
e = electromotive force in volts.

In all the cases we have so far considered, the coil has been stationary and the variation of flux through it has been due to an increase or a decrease in the amount of flux in the magnetic circuit. There are other ways of changing the amount of flux in a coil, however. We may move the coil and the magnetic circuit with respect to each other. It makes no difference whether we move the coil or move the flux as long as the motion causes a change in the number of flux lines threading the coil. In either case there will be a change of linkages and hence a voltage of the above amount produced.

Fig. 161 shows a coil situated in an air gap between two heavy pole faces. The plane of the coil is perpendicular to the flux lines. If the area of the coil is A and the flux density B, there is the total number of linkages NBA, where N is the number of turns in the coil (shown as *one* in the figure for convenience).

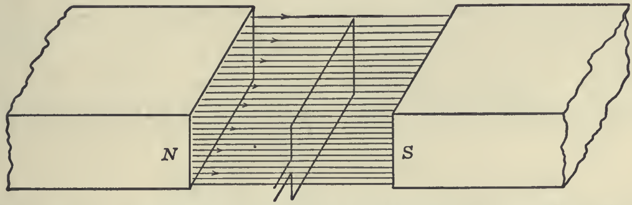

FIG. 161. The plane of the coil is perpendicular to the direction of the magnetic flux.

Suppose the coil is now turned to the position of Fig. 162, that is, so that its plane is parallel to the flux lines. There is now no flux linking the coil. If T seconds are

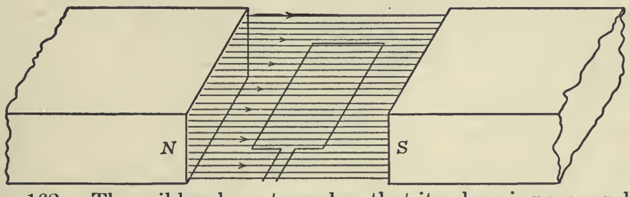

FIG. 162. The coil has been turned so that its plane is now parallel to the magnetic flux.

used in turning the coil from one position to the other, then the average rate of change of linkages is

$$\frac{1}{T} N\phi = \frac{1}{T}(NBA) \tag{2}$$

and the average voltage produced during the process is

$$e_{av} = \frac{NBA}{T} 10^{-8} \text{ volts.} \tag{3}$$

An arrangement in which a coil of wire is moved relative to a magnetic field in order to produce a voltage is called an electric generator.

There is another way of looking at generation of voltage in this manner which is more convenient in the study of certain problems. In Fig. 163 is shown the pole of a magnet with the flux lines issuing from it perpendicularly. Horizontally in this field is located a U-shaped piece of wire, with another short piece of wire laid across to complete a circuit. Suppose that this short piece of wire is maintained in contact with the U, and moved bodily toward the

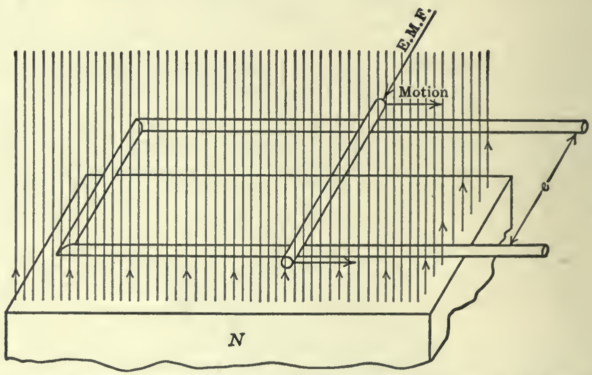

FIG. 163. The slider moves perpendicular to the lines of force and cuts them.

right. The number of lines of force passing through the closed electric circuit is then increasing, and accordingly there will be a voltage set up. This voltage will be in such a direction as to tend to pass a current around the loop in a direction opposing the increase of flux, that is, clockwise about the loop as viewed from above. The direction of the voltage produced is thus from back to front in the short piece of wire.

It is obvious that in the arrangement shown in Fig. 163, the voltage, which is produced by the motion of the short

cross piece of wire, must be produced in this cross piece only; for the remainder of the apparatus is stationary and the flux is constant. It is customary to say that the cross piece has a voltage generated in it by reason of the fact that it is " cutting " across the lines of force.

If the flux density produced by the pole is B and the length of the cross piece is l, and if the cross piece is moving to the right at a velocity v, the area of the loop will be increasing at a rate

$$lv \text{ square centimeters per second.}$$

The flux through the loop is thus increasing at a rate

$$Blv \text{ maxwells per second.}$$

Since the number of turns in the loop is unity, the voltage produced by the motion of the cross piece will accordingly be

$$e = Blv \text{ abvolts.} \tag{4}$$

Considering the fact that this voltage is all generated in the short wire, we note that a wire of length l moving perpendicular to a field of flux density B and moving at a velocity v, will have generated in it a voltage equal to Blv. If all quantities are expressed in units of the c.g.s. system, the voltage will be in abvolts. Converting to volts we have

$$e = Blv\, 10^{-8} \text{ volts.} \tag{5}$$

Note that it is velocity perpendicular to the field which counts. The wire must be actually cutting across the flux lines. A conductor moving parallel to a field will have no voltage whatever generated in it. A conductor moving in a direction oblique to the direction of the field will have in it a voltage proportional to the component of its velocity perpendicular to the direction of the field.

The relation between the direction of the flux, the motion and the voltage produced may be expressed conveniently by means of Fleming's right-hand rule. Referring to Fig. 164, this rule may be expressed as follows. Place the right

hand with the thumb, forefinger and center finger all at right angles. Let the thumb point in the direction in which the wire is moving. Point the forefinger in the direction of the flux lines. The center finger will then point in the direction in which the voltage is produced.

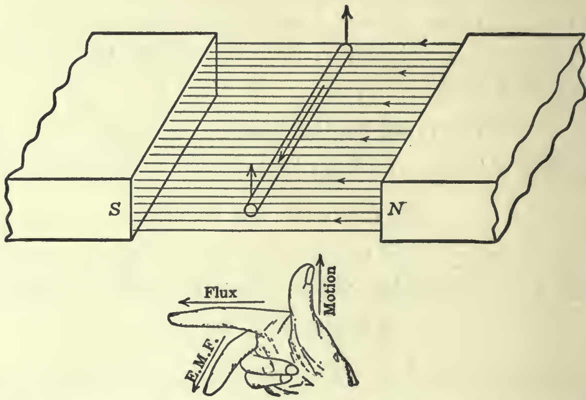

FIG. 164. The thumb points in direction of the motion of the wire, the forefinger in the direction of the flux and the middle finger shows the direction of the induced electromotive force.

Applying this rule to Fig. 163, in which we found that the voltage was from back to front of the moving wire, we see that the rule is correctly expressed.

Prob. 1–10. When the coil of one turn shown in Fig. 161 occupies a position with its plane at right angles to the flux lines, a total flux of 3×10^4 maxwells links the coil. The coil is then turned at a uniform speed to a position parallel with the flux lines in a time of 0.005 second. What average voltage is generated in the coil? What is the instantaneous voltage at the beginning and at the end of this time interval?

Prob. 2–10. A square coil of 10 closely packed turns and of 10 square inches mean area is rotated about a central axis parallel to the sides of the coil through an angle of 90° in 0.0012 second. An average voltage of 8 volts is generated in the coil during this interval. The coil is situated in a uniform field

and is initially parallel to the flux lines. What is the flux density?

Prob. 3–10. The wire in Fig. 164 is moved through the gap at right angles to the field at a velocity of 60 feet a second and the voltage induced as it cuts the flux is 0.85. The pole faces are 3 centimeters wide and 5 centimeters long (the length being taken in the direction parallel with the conductor). Assuming no fringing of flux, what is the air-gap flux?

96. Elementary Alternators. Fig. 165 shows a cross-section of the arrangement shown in Fig. 161. Suppose that the coil is mounted on a shaft perpendicular to the paper and caused to revolve in this uniform field. Applying the right-hand rule, at the instant shown, we see that in the conductor on the left there is induced a voltage towards the reader, and in the other conductor a voltage away from the reader. The amount of this voltage is proportional to the component of the velocity in a direction perpendicular to the field. This component of the velocity is proportional to the sine of θ, which is the angle between the plane of the coil and a plane perpendicular to the flux lines. If the coil is revolving at a uniform velocity, θ is varying uniformly with the time, and this component of the velocity is accordingly a sine function of the time. The voltage produced is then

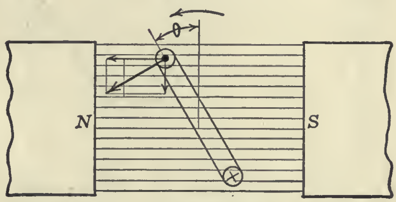

Fig. 165. A cross-section of the arrangement shown in Fig. 161.

$$e = E_{\max} \sin \omega t, \tag{6}$$

where ω = the angular speed of the coil in radians per second,

$E_{\max}$ = maximum voltage generated.

The maximum rate of cutting of flux and the maximum voltage will occur when the plane of the coil is parallel to the

flux lines. If v is the peripheral speed of the coil we shall have at this instant a voltage generated

$$E_{\max} = \frac{Blv}{10^8} \text{ volts,} \tag{7}$$

where l is the length of total conductor in the coil which is active. The length l thus includes only the length of the wire which lies in the direction perpendicular to the paper. The end connections of the coil which lie in planes parallel to the paper have no voltage generated in them since they do not cut across the lines of force.

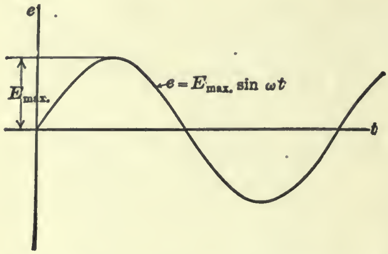

Fig. 166. The form of voltage curve induced in the coil of Fig. 161 and 165.

This arrangement is called an alternator, or an alternating-current generator. It produces a voltage which varies with the time in the manner shown in Fig. 166. The frequency of this voltage is, in cycles per second, the number of revolutions per second of the coil in the arrangement shown.

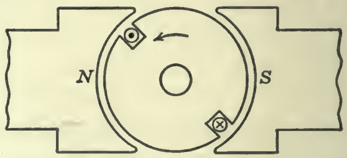

Fig. 167. To decrease the reluctance of the magnetic path, the coil is wound on an iron core.

Obviously, if such an arrangement as Fig. 165 were used, the reluctance of the air gap would be large and a large magnetizing coil would be necessary in order to force the necessary constant working flux across the long gap. To obviate this difficulty the arrangement used in practice is more as shown in Fig. 167.

The revolving coil is situated in slots in an iron armature

which revolves as a whole. The air gap between the armature and the pole pieces is small and hence the reluctance of the magnetic circuit is kept down. The field coils, which are the magnetizing coils for forcing the flux through the circuit, will therefore require only a small current in order to maintain the working flux. Placing the conductors in slots in this manner greatly decreases the necessary air gap and does not affect the average voltage produced with a given speed and total flux. The flux will, of course, be distorted and not uniform. The average voltage in a conductor is, however, unchanged, for the total change of linkages per half revolution remains the same.

Prob. 4–10. An elementary alternator such as the one of Fig. 165 has a uniform air-gap flux density of 1000 gausses. If the coil revolves 1200 revolutions per minute, what length of the conductor is necessary in the coil in order to generate 12 volts at the maximum of the voltage wave? The wire used is small and when wound occupies a space $\frac{1}{4}$ inch square in cross-section. The formed coil is 3 inches square inside dimensions and $3\frac{1}{2}$ outside. Is it allowable to use averages in the computations?

97. The Direct-Current Generator. The alternator furnishes a potential which reverses in direction many times a second. For certain purposes, a generator is needed which will furnish a potential which does not reverse, that is, a continuous potential. Such a direct-current generator employs a rectifying device called a commutator which causes the potential delivered by the machine to be unidirectional.

A direct-current generator is shown diagrammatically in Fig. 168. A slotted armature, built up of steel laminations, is mounted on a shaft and rotated between pole pieces which furnish a flux exactly as in the alternator. In the figure, for convenience, are shown only six armature slots with twelve conductors or two conductors per slot. In actual machines, a much larger number of slots than this is used, but a simple arrangement is shown in the diagram for the sake of clearness.

These armature conductors are connected as follows. The circuit passes up through conductor 1, down through 2, up through 3, down through 4, up through 5, down through 6 and so on. The end connections between the conductors are shown by light lines, those on the near end of the arma-

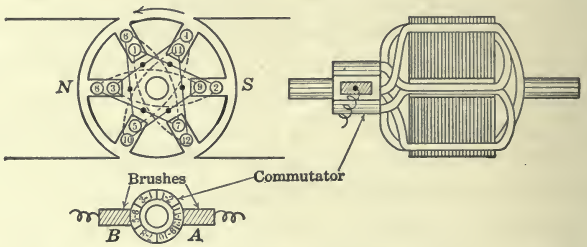

FIG. 168. Diagram of a simple 6-coil direct-current generator.

ture by full lines and those on the far end by dotted lines. It will be noted that the circuit passes up through 11, down through 12 and then up through 1, and so on, so that the circuit is entirely continuous.

On the same shaft with the armature, as shown in the side view, is mounted a commutator. It consists in this case of six copper segments, insulated from the shaft and also insulated from one another by mica separating sheets. The cylindrical surface of this commutator is turned off to a smooth bearing surface. The commutator bars are then connected with the winding as follows: each segment or commutator bar is connected to one of the end connections passing between two conductors. Thus the segment labeled 1–2 is connected to the end connections between conductors 1 and 2. The positions on the end connections where the commutator bars are connected are shown by dots in the figure. The commutator is shown below instead of in its correct position in front of the armature for convenience in drawing.

Resting on the cylindrical surface of the commutator are two brushes as shown. These are carbon blocks held in

brush holders stationary on the frame of the machine. They rest upon the surface of the commutator and make electrical contact with it. To these brushes are connected the leads of the machine.

Let us now trace the path of the current through the armature from one brush to the other. Beginning at brush *A*, we pass immediately to segment 11–12. This is connected to an end connection at the point shown by the dot. The path then divides and we have a choice of two parallel paths through the armature. Choosing one of these paths, we pass down through conductor 11, up through 10, down through 9, up through 8, down through 7, up through 6, then to the segment marked 5–6 and the brush *B*, that is, to the other lead of the machine.

Consider, however, the direction in which the voltage is generating in the conductors due to the revolution of the armature. If the armature revolves in the direction shown by the arrow, we shall have a voltage generated down through conductors 11, 9 and 7, and up through 10, 8 and 6. In the path from one brush to the other we thus have six voltages generated which are all so directed as to force current through the circuit in the direction from brush *A* to brush *B*. The voltage between *A* and *B* is thus the sum of the six voltages generated in these separate conductors.

If we consider the other path through the armature, we shall pass from brush *A* to the segment 11–12, then, taking the new path, down through conductor 12, up through 1, down through 2 and so on. Examining the voltages in this path, we find that they are in the same direction and of the same magnitude as the voltages which were encountered in the other path. We thus have between the brushes *A* and *B* two parallel paths through the armature in each one of which an equal voltage is generated in a direction from brush *A* to brush *B*. Brush *B* will hence be positive and brush *A* negative.

Let us now consider what happens to the connections as

the armature revolves. Consider the instant when the armature has turned through one-sixth of a revolution beyond the position shown in the figure. Brush *A* will then rest on segment 9–10 and brush *B* on segment 3–4. That is, the connections to the armature are changed by the commutator. In passing from brush *A* to brush *B*, however, we find upon examination that we go by exactly the same paths as before, if we consider simply the positions of the conductors. The conductors have all moved around one position, but new conductors have taken their places. In going from *A* to *B*, we again, however, pass down through a conductor under a south pole, and so on. This is exactly what we did before, and so the voltage generated is exactly the same as before and in the same direction. After another one-sixth revolution the connections will again change, but we shall have as before exactly the same voltage between brushes. The potential developed by the machine accordingly does not change in direction as the armature revolves; that is, it is a direct-current generator.

In the machine shown in the figure, with only six slots, the voltage will vary somewhat as the armature revolves, due to the variation in position of the armature conductors during the time that the brush is passing over a single segment. The voltage delivered by the machine will hence have somewhat the appearance shown in Fig. 169. It will be unidirectional, that is, it will not reverse, but it will have a very considerable ripple. In an actual machine made with perhaps 100 slots, this ripple will be almost entirely removed, and the voltage will appear as shown in Fig. 170, for the variation

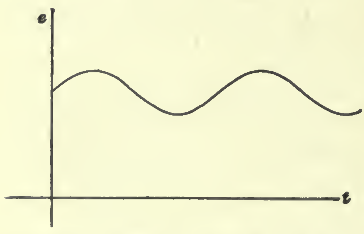

FIG. 169. The curve of voltage delivered to the brushes of the generator in Fig. 168.

in position of the conductors under the poles during the time that the brush is passing over a single segment is almost entirely negligible.

The machine shown in diagram in Fig. 168 is a two-pole machine. A much more common form in practice is a multipolar machine, having four or six poles or even more. The simplest form of drum winding is also shown on the armature in the diagram. There are many forms of armature winding, all of which are designed to give the same result as the winding shown in the figure, that is, to connect between brushes a series of conductors in such a manner that the voltages always add. Fig. 171 shows the armature of a small

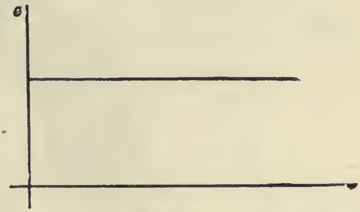

Fig. 170. The terminal voltage of a direct-current generator having a large number of commutator segments.

Fig. 171. The armature of a direct-current generator. *General Electric Co.*

commercial generator, the poles and frame of which are shown in Fig. 172.

The construction, winding and performance of direct-current machinery are too large subjects to be taken up here. We can consider at this time only the basic principles underlying all such machinery.

Prob. 5–10. On the armature of a certain 8-pole direct-current generator, there are 146 active conductors in series between the brushes. Each pole has 1.2×10^6 lines of flux. The speed is 1800 revolutions per minute. What is the average generated voltage between brushes?

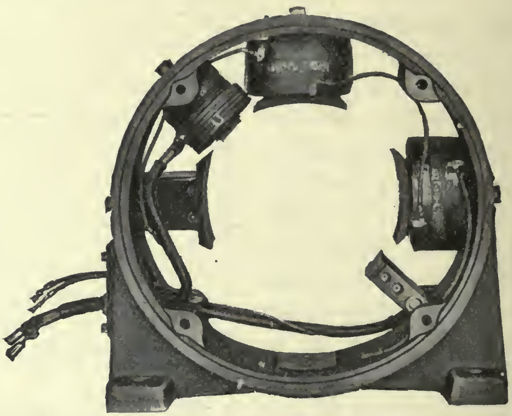

FIG. 172. Partly assembled frame of a four-pole direct-current generator. *General Electric Co.*

Prob. 6–10. It is desired to design a generator which will generate an average electromotive force of 230 volts. An armature is to be used which can give 180 active conductors between brushes. How many poles must be used, each with 3×10^6 lines of flux, in order to allow the machine to run at about 480 revolutions per minute?

Prob. 7–10. At a flux density at the pole surface of 65,000 lines per square inch, how large must the poles of a 6-pole generator be if it is to generate an average electromotive force of 115 volts? There are 208 conductors between brushes and the speed is 900 revolutions per minute.

98. Conductor in a Moving Field. In order to generate a voltage we must move a conductor relative to a magnetic field. The voltage then produced will be proportional to the length of the conductor, the strength of the field and the component of the velocity of the conductor perpendicular to the field. It does not matter, however, whether we move

the conductor and keep the magnetic field stationary, or whether we keep the conductor stationary and move the magnetic field. It is only relative motion that counts. The same voltage will be generated in either case provided the relative velocity is the same.

This principle is made use of in the revolving-field alternator. In this machine, which is the usual form of practical commercial alternator, the armature windings are stationary and the field structure revolves.

A simple form of revolving-field alternator is shown diagrammatically in Fig. 173. A coil, AA, is placed in slots in a stationary iron yoke called a stator. This stator is mounted rigidly on the frame. Inside of the stator revolves a large electromagnet called the revolving field. In the diagram a two-pole field is shown. This magnet receives its exciting current, that is, its magnetizing current, by means of two slip-rings which are simply insulated metal rings mounted on the shaft, to which current is conducted by brushes sliding on their surface.

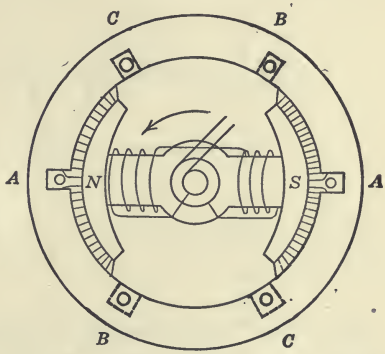

FIG. 173. The diagram of a three-phase alternator. The armature stands still, while the field revolves.

The field magnet as it revolves carries its flux with it. This flux is thus caused to move across the coil A. A voltage will then be set up in this coil, and its magnitude may be computed from the voltage formula above. This voltage will be alternating, for since the direction of the flux passing the conductor changes as the second pole comes under the conductor, the direction of the voltage generated will also change.

With a concentrated coil mounted in a single slot as shown, the voltage generated at any instant will be proportional to the flux density at that time passing the coil. The voltage in the coil will accordingly vary in exactly the same manner that the flux density varies across the face of the field pole. The voltage generated may hence have somewhat the form shown in Fig. 174. If the air gap is made

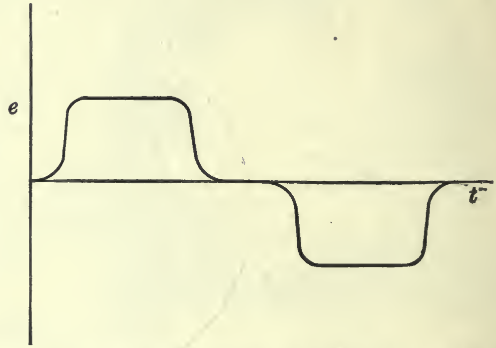

FIG. 174. Curve of voltage generated in one coil of the armature of Fig. 173.

larger near the pole tip than at the center, then the shape of the curve of flux density across the pole face may be made nearly sinusoidal. In this case the delivered voltage will also be nearly sinusoidal.

Suppose that we make more slots in the stator and place coils as shown at *BB* and *CC*. These coils will also have voltages generated in them, and voltages which will alternate in exactly the same manner as does the voltage in coil *A*. The voltage in *B* will, however, come to its maximum value a certain definite time later than the voltage in *A*. Similarly coil *C* will attain its maximum at a still later time. Accordingly, if we assume that the flux distribution across the pole face is sinusoidal, the voltages delivered by these three coils may be plotted on the same diagram as shown in Fig. 175 and will have the relations shown.

Such a machine is called a three-phase generator. It is really three generators combined into one. Each one delivers an alternating potential, but they are not in step. They come to a maximum successively, pass through zero successively and so on. Most commercial alternators are of the three-phase type.

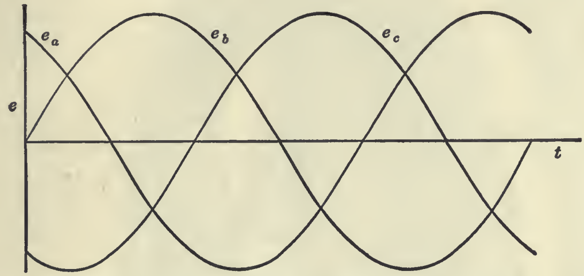

Fig. 175. Curves of the voltages generated in the three coils of a generator like that shown in Fig. 173. The flux distribution has been changed from that of Fig. 174 so that sine-wave forms are generated.

In the figure we have shown each coil side entirely situated in a single slot. Such an arrangement is called a concentrated coil winding. It is, however, more usual to distribute the coils through several adjacent slots. Such a distributed coil winding will give much better wave form than the concentrated form shown.

The two-pole arrangement shown in Fig. 173 is almost never used, except in high-speed turbo-alternators. Low-speed alternators are always multipolar in type. Engine-driven alternators may contain a large number of poles, as high as eighty. Fig. 176 shows the rotor of a vertical type of water-wheel-driven alternator having forty-eight poles. Fig. 177 shows the stator or armature of the same machine.

The revolving-field type of alternator is almost always used today, and principally for the following reasons. It will be noted that the coils in which the voltage is generated may be connected directly to the external circuit, since they are stationary. No collecting-rings are necessary to com-

plete the circuit. This is advantageous in machines where a high voltage is generated. Large alternators today usu-

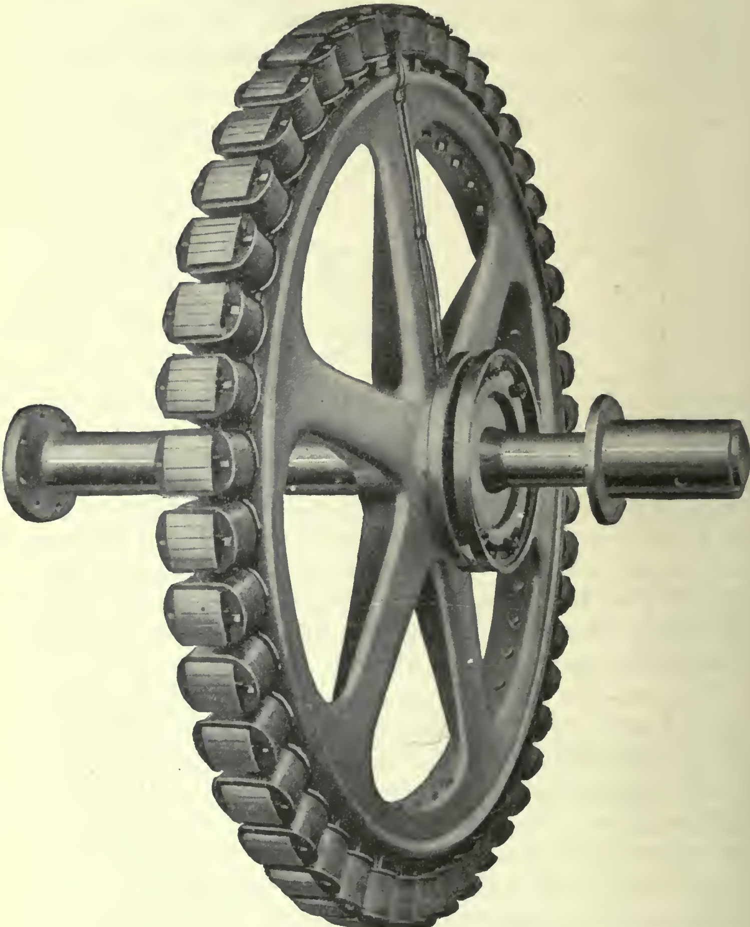

FIG. 176. The rotating field of a large water-wheel-driven alternator.

ally generate directly voltages of from 2300 to 13,000 volts. It is not at all convenient to handle such voltages on collecting-rings. In addition the large current delivered by

modern high-output alternators will not have to be carried by collecting-rings. Alternators having outputs as high as 45,000 kilovolt-amperes on a single machine are built, and this means very large currents indeed. The revolving-field type also has certain advantages as regards strength of construction.

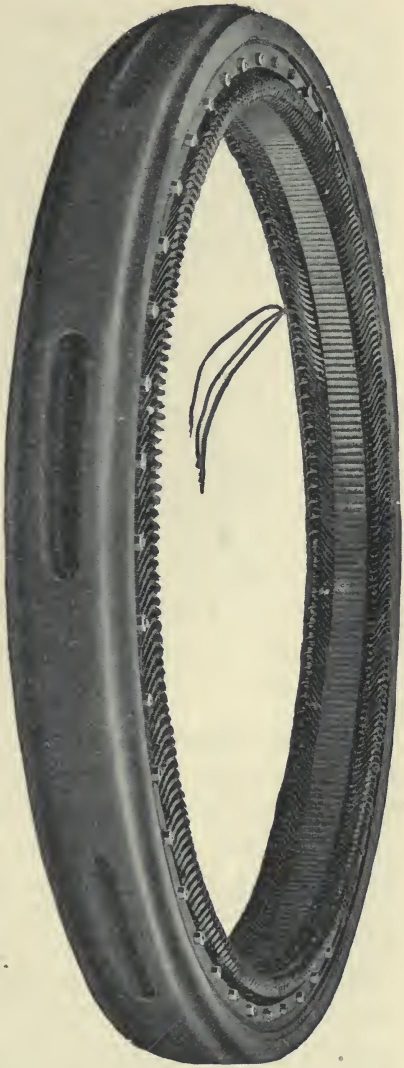

FIG. 177. The stationary armature for the rotating field shown in Fig. 176.

Prob. 8–10. The revolving-field alternator of Fig. 173 has 8 conductors in each of the slots "C." The conductors are connected in series so that the generated voltages are added. The effective length of the slot is 8.5 inches and the speed of the rotor is 1200 revolutions per minute. The radius from the center of the shaft to the center of the bundle of conductors in the slot is 6.1 inches. What maximum flux density is necessary in the gap to generate 25 volts maximum?

99. The Homopolar Generator. It is entirely possible to con-

struct a direct-current generator without using a commutator. Such machines may be made by causing a conductor to move continuously through a field of constant intensity and always in the same direction. They are called homopolar generators. They are not much used except where very high currents are desired at low voltages, and are manufactured almost exclusively for this purpose.

Such a homopolar generator is shown diagrammatically in Fig. 178*a* and *b*. The machine is shown in section. It is made

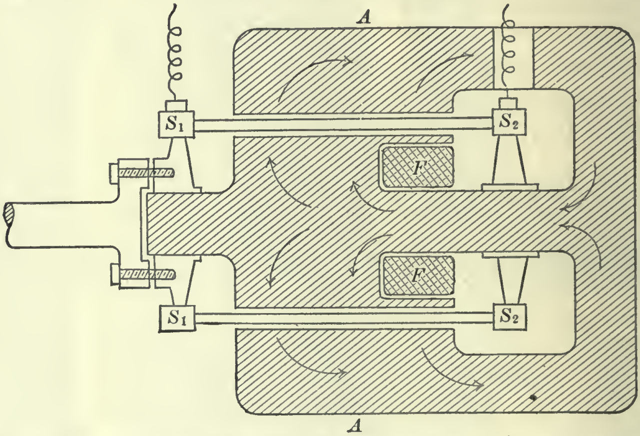

FIG. 178*a*. Diagram of a homopolar generator. No commutator is needed.

of an iron piece having a cylindrical air gap. Field coils shown in section as F_1 and F_2 are stationary and carry a current which forces a flux through the stationary iron structure as shown by arrows. This flux thus passes out radially across the air gap.

Metal rings shown at S_1 and S_2 are mounted on insulating spiders so that they may be revolved. Between these rings is connected a heavy conductor which thus revolves with them around through the cylindrical air gap. The current is collected by means of brushes resting on the slip-rings. These brushes are held in stationary brush holders. Ac-

cess may be had to the brushes by means of hand-holes made in the iron structure. It will be noted that the conductor is always passing through a field of constant intensity, and always perpendicular to this field. It accordingly has a constant potential generated in it.

By using a large magnetic flux and a high speed, a voltage as high as 40 volts may be obtained from a single conductor. Several conductors are generally used, however, placed at intervals around the air gap. Each one is connected to two entirely separate collecting-rings, so that there are twice as many collecting-rings as there are conductors. By external connections with the brushes, these conductors are then placed in series as far as the external circuit is concerned. Their voltages thus add and the voltage delivered by the machine is the sum of the voltages of the separate conductors.

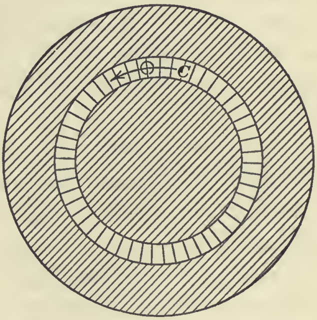

FIG. 178*b*. Section *A-A* of the view shown in Fig. 178*a*.

The machine requires no commutator and delivers a constant direct voltage. It has, however, certain obvious structural difficulties which have prevented its adoption for any purposes except those noted above.

Prob. 9–10. The effective conductor length in Fig. 178*a* and *b* is 9.3 inches. The conductors are at a radius of 4 inches from the center line of the shaft. Eight conductors are used connected in series by means of slip-rings. At a speed of 3600 revolutions per minute and a total air-gap flux of 8×10^6 maxwells, what voltage is generated in the machine?

100. Eddy Currents. Whenever the amount of flux linking a circuit changes, there is a voltage set up in the circuit, and if the circuit is closed, a current flows. All materials

have resistance, at least at ordinary temperatures. Accordingly whenever a current flows in a conductor there is a loss which appears as a heating of the conductor. Any variation of flux therefore always entails losses in circuits which the flux links.

It is not necessary that the circuit be a wire and that the flux pass entirely through it. If a solid block of metal is traversed by a varying flux, metallic circuits in the block itself which are linked by the flux will carry current. If a magnetic circuit is made up of iron and if the flux in the circuit is variable, there will be current set up by induction in the iron of the circuit itself. All of these stray currents entail losses, and such losses are known as eddy-current losses.

Electrical machinery usually involves varying fluxes. In alternating-current work, the transformer depends for its action upon the variation of flux in a core. In direct-current machinery as well as alternating-current machinery, we have an armature built of iron which is either revolved in a permanent field, or through which a revolving field passes. In either case there will be losses in the iron due to hysteresis and also losses due to eddy currents. The toothed-core armature of a direct-current machine causes tufts of flux to sweep across the pole faces, thereby setting up eddy currents in the iron of the poles. In a dynamo, in fact, there will be eddy-current losses in the conductors themselves if they are of large size. In addition to the main voltage which is set up in the conductors, there will be locally generated voltages due to variation in the density of flux which will cause local current to circulate in the material of the conductors without passing through the external circuit.

Eddy currents always tend to flow in planes perpendicular to the flux, since they are set up by the variation of this flux through a circuit. If the material of which the magnetic circuit is made is subdivided into thin sheets, that is, if the

material is laminated, and these sheets are placed parallel to the flux and insulated from each other, the eddy-current loss will be reduced. The subdividing of the iron into insulated sheets in this manner breaks up the paths in which eddy currents flow, and greatly reduces the magnitude of the eddy currents.

If a direct-current generator were made with an armature constructed of a solid piece of soft iron, the eddy-current loss would be enormous. In fact, some early generators thus constructed required several horse power to drive them while the useful output of the machine was only a small fraction of a kilowatt. Almost all of the input of the machine was lost in the stray currents produced in the armature by the variation of the flux density at various points as the armature revolved. In order to avoid this large loss, armatures are made up of sheet-steel laminations, usually between 0.014 and 0.030 inch thick, assembled on a shaft, so that the armature is subdivided into sheets which are parallel to the working flux of the machine. Eddy currents which tend to flow perpendicular to this flux are thus reduced to a small value by the high resistance between the sheets.

In ordinary direct-current generators, no special precaution is taken to insulate these sheets from one another, for the scale which is naturally on the sheet steel after it is rolled prevents good electrical contact from taking place. In constructing transformers, the sheets are often given a coat of shellac or varnish in order to insulate them. Care must be taken, of course, that in bolting the sheets together they are not electrically connected to one another. Also filing the edges of a core, or the slot of an armature, will greatly increase eddy-current loss by bringing the successive laminations into electrical contact. In high-frequency machines or transformers, great care indeed must be taken to keep down the eddy-current losses. In generators delivering frequencies of 50,000 cycles per second, iron of a thickness of approximately 0.0015 inch is used, and

is carefully japanned in order to prevent contact between the sheets.

The following analysis will show the magnitude of the eddy-current loss to be expected, as well as the effect of subdividing the material into thin sheets, the effect of resistivity and so on.

Let us consider first the eddy-current loss in a ring which completely surrounds a magnetic circuit. Such an arrangement is shown in Fig. 179 and a long, thin copper ring is shown because of its bearing on the losses in laminations to be considered below. The iron piece which threads this ring we will consider as carrying a flux ϕ. This flux is varying, and due to its variation there is a voltage generated which causes current to flow around the short-circuited copper ring. The amount of loss, that is, the I^2R in the ring, caused by this current is to be computed.

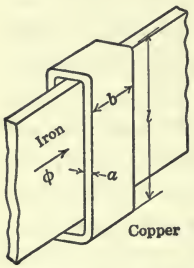

FIG. 179. A thin copper strip surrounding an iron bar in which the magnetic flux is changing.

We will consider the ring to be long and narrow, so that the resistance of its ends can be neglected in comparison with the resistance of the sides. The resistance then is

$$R = \rho \frac{2l}{ab} \text{ ohms}, \tag{8}$$

where ρ is the resistivity of the material of which the ring is constructed. If l, a and b are in centimeters, then ρ must be in ohms per centimeter cube of the material.

When the flux is changing, there will be a voltage generated of value

$$e = \frac{d\phi}{dt} 10^{-8} \text{ volts}, \tag{9}$$

and, by Ohm's law, this will cause a current to flow as given by

$$i = \frac{\dfrac{d\phi}{dt}}{\rho \dfrac{2l}{ab}} 10^{-8} \text{ amperes.} \qquad (10)$$

The watts loss at any instant due to this current is the product of the square of the current and the resistance, that is,

$$p = i^2R = \frac{\left(\dfrac{d\phi}{dt}\right)^2}{\rho \dfrac{2l}{ab}} 10^{-16} \text{ watts.} \qquad (11)$$

The electrical machinery in which we are usually interested has fluxes that vary sinusoidally with the time. That is,

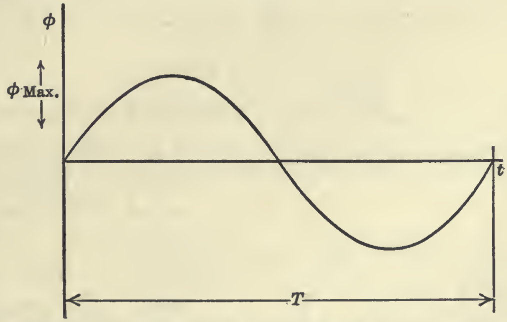

FIG. 180. Curve showing how the flux in the iron bar of Fig. 179 varies.

we are interested in a flux which varies in such a manner that its value at any instant is given by the equation

$$\phi = \phi_{\text{max}} \sin 2\pi ft \text{ maxwells.} \qquad (12)$$

This manner of variation is shown in Fig. 180. f is the frequency of the flux; that is, it is the number of times the flux changes through a complete cycle per second. Ex-

amining this figure, we see that when $t = 0$, the flux is zero and is increasing. When $t = 1/f$, the flux is again zero and is again increasing. The quantity $2\pi ft$ is always to be taken in radians, and the sine of 2π radians is zero. T or $1/f$ is therefore called the period of the alternating flux, since it is the time in which the flux makes a complete change.

When a flux is varying in this manner, its rate of change may be found for any instant by differentiating the above expression, giving

$$\frac{d\phi}{dt} = 2\pi f \phi_{\max} \cos 2\pi ft \text{ maxwells per second.} \qquad (13)$$

We may insert this value for the rate of change of flux in the expression for the i^2R loss at any instant, obtaining

$$p = i^2R = \frac{4\pi^2 f^2 \phi_{\max}{}^2 \times 10^{-16}}{\rho \dfrac{2l}{ab}} \cos^2 2\pi ft \text{ watts} \qquad (14)$$

$$= P_{\max} \cos^2 2\pi ft \text{ watts,}$$

where $P_{\max}$ is an abbreviation introduced for convenience to denote the maximum value to which the instantaneous loss attains. The loss thus varies from instant to instant in accordance with a cosine-squared term. The manner in which the loss varies is obtained by plotting this expression as is done in Fig. 181.

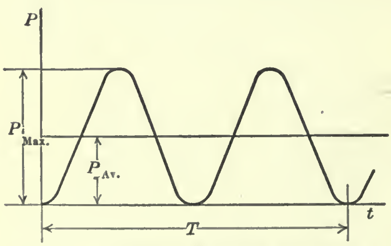

FIG. 181. Curve of power loss due to eddy currents when flux varies sinusoidally.

In order to find the loss in the ring, we are, of course, interested in the average value of the i^2R loss, that is, in the average value of the above expression. We can obtain this average

value by integrating the expression over a complete period and dividing by the length of the period. That is,

$$P_{av} = \frac{1}{T}\int_0^T p\,dt. \tag{15}$$

Inserting into this expression the value of p, remembering that $T = \frac{1}{f}$,

$$P_{av} = f\int_0^{\frac{1}{f}} P_{max} \cos^2 (2\pi ft)\,dt.$$

Dividing f by $2\pi f$, and multiplying the factor under the integration sign by $2\pi f$ in order not to change the value of the expression, we obtain

$$P_{av} = \frac{1}{2\pi} P_{max} \int_0^{\frac{1}{f}} \cos^2\,(2\pi ft)\;(2\pi f)\,dt. \tag{16}$$

To evaluate this expression, we may make use of the known integration

$$\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4}. \tag{17}$$

In our expression, x is replaced by $2\pi ft$. Inserting this value, we obtain

$$P_{av} = \frac{P_{max}}{2\pi}\left(\pi ft + \frac{\sin 4\pi ft}{4}\right)\Bigg]_0^{\frac{1}{f}}. \tag{18}$$

Upon substituting the limits, the second term of this expression becomes zero for each limit. The entire expression thus reduces to

$$P_{av} = \frac{1}{2} P_{max}.$$

This means that the average height of the curve in Fig. 181 is just one-half of its maximum height. The average i^2R loss in this ring is half of the maximum value to which it attains during the cycle.

Insert the value of P_{max} in this expression and it becomes

$$P_{av} = \frac{\pi^2 ab}{\rho l} f^2 \phi_{max}{}^2 10^{-16} \text{ watts.} \qquad (20)$$

This expression gives the average watts loss in the ring due to the variation of flux through it. Several interesting points are to be noted immediately. The watts loss varies as the square of the frequency. Thus if the flux is varying at the rate of fifty cycles per second, there will be four times as much loss as if the same flux were varying at the rate of twenty-five cycles per second. The loss also varies as the square of the maximum flux. If the flux density is doubled, the loss in the ring will be made four times as great. The loss also varies inversely as the resistivity of the material. The lower the resistivity, the higher the loss will be. This may at first sight seem peculiar, but when it is remembered that decreasing the resistance increases the current, and that the loss goes up as the square of the current, the reason will be clear. There will thus be much more loss in a short-circuited ring of copper around a magnetic circuit than, for instance, in a short-circuited ring of brass. We must remember, of course, that for the maximum flux density we are to take the value of the flux density which actually passes through the magnetic circuit. The amount of this flux density will be affected not only by the magnetomotive force acting on the circuit but also by the current flowing in the short-circuited ring itself. The effect of this current is to tend to cause less flux to pass through. A heavy, short-circuited ring of copper will therefore often have less loss for the reason that the heavy current flowing will reduce the net amount of flux passing through the circuit. If the same flux actually passes, however, the copper ring will have a higher loss than a ring of greater resistivity.

Rings of copper forming a short-circuited path are used occasionally in alternating-current apparatus. For instance, in alternating-current contactors such rings are used

to keep the contact from chattering. We are more interested, however, in an eddy-current loss which occurs when the conducting path does not surround the magnetic circuit, but rather forms a part of the magnetic circuit itself; that is, we are more interested in the losses which occur in the sheet-steel laminations of which the magnetic circuit is built.

It has been found experimentally that the same loss occurs when a lamination is revolved in a constant field of a certain flux density as will occur if an alternating flux of this maximum value is passed through the sheet. We will accordingly analyse only the latter case. Our conclusions will, however, apply not only to alternating-current transformers, but also to the armatures of dynamos.

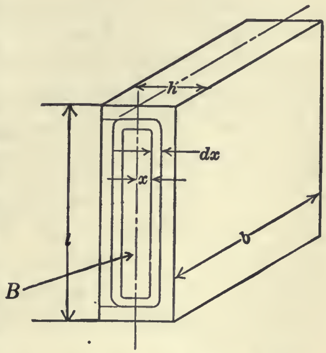

Fig. 182. Diagram of eddy-current circuits in a thin sheet of iron.

A piece of iron lamination is shown in Fig. 182. The flux passes in the direction of the arrow and parallel to the plane of the sheet, and is, we will consider, of maximum flux density B_{max}. Eddy currents will tend to flow around through the sheet in paths similar to the one drawn on the face of the sheet. If the thickness of the lamination is small compared to its width, these paths may be considered as parallel to the surface of the sheet without serious error. Moreover, in computing the resistances, we can with sufficient precision neglect the resistance of the small end portion of the path. Our analysis is hence only approximate. As we have noted above, however, various practical matters, such as the manner in which the sheets are bolted, greatly affect the eddy-current loss. Since these accidental vari-

ations may amount to fifty percent or even more, it is useless for us to make a too careful theoretical analysis. Our assumptions then are justified, and we can expect to obtain as a result of our analysis an indication of the way in which the loss will vary, rather than an accurate formula for computing its value.

Consider the path at a distance x from the center line of the sheet. Let the thickness of the path be dx. There will be a voltage generated around this path due to the amount of flux

$$\phi_x = 2xlB \text{ maxwells,} \tag{21}$$

which links through the path.

Conditions for this path are now exactly the same as for the ring shown in Fig. 179, if we consider simply the portion of the flux which passes inside of the chosen path. The loss in this portion of the sheet will therefore be given by formula (20) which we have already derived, and the fraction of the total loss which occurs in this path is

$$dP_{av} = \frac{\pi^2 b dx}{\rho l} f^2 (2xlB_{max})^2 \, 10^{-16} \tag{22}$$

or

$$dP_{av} = \frac{4\pi^2 f^2 bl}{\rho} B_{max}{}^2 \, 10^{-16} \, x^2 dx \text{ watts.} \tag{23}$$

The total loss in the laminations will be obtained by adding the losses for all the possible paths similar to the above. In order to do this, we integrate the partial loss where x varies from zero to half the width of the sheet, which gives

$$P_{av} = \frac{4\pi^2 f^2 bl}{\rho} B_{max}{}^2 \, 10^{-16} \left. \frac{x^3}{3} \right]_0^{\frac{h}{2}} \tag{24}$$

$$= \frac{\pi^2}{6\rho} blh(h^2 f^2 B_{max}{}^2) 10^{-16} \text{ watts.}$$

However, the volume of the laminations is given by

$$v = blh \text{ cubic centimeters.} \tag{25}$$

The loss per cubic centimeter of the material is obtained by dividing the total loss by the volume and hence becomes

$$P_{av} = \frac{\pi^2}{6\rho} h^2 f^2 B_{max}{}^2\, 10^{-16} \text{ watts per cubic centimeter.} \quad (26)$$

This formula gives the watts loss per cubic centimeter of a transformer core or a dynamo armature when

h = the thickness of the laminations,
ρ = the resistivity of the laminations,
f = the frequency of the flux,
B_{max} = the maximum flux density.

All the quantities in this formula are to be expressed, of course, in c.g.s. units.

In practical cases this formula will usually give results which are too low, owing to imperfect insulation between the sheets. It is often written in the form

$$P_{av} = \epsilon\, (hfB_{max})^2\, 10^{-16} \text{ watts per cubic centimeter} \quad (27)$$

and the constant ϵ is obtained from experience. If all quantities are in c.g.s. units, a reasonable value of the coefficient ϵ for good transformer steel is about 65,000. Values of this coefficient will be found in all electrical handbooks.

Note that eddy-current loss in the laminated sheets varies as the square of the thickness of the sheets. That is, if we have a transformer built of 0.024 sheet steel, and we construct it with the same volume of iron but of 0.012 sheet steel of the same grade, then the eddy current will be one quarter of what it was before. If the sheets are made too thin, a great amount of expense is naturally involved in handling them, and the space factor is poor. By the space factor is meant the volume of actual iron divided by the total volume of the core. Increasing the insulation between laminations also reduces the space factor. The choice of a

proper thickness of steel for constructing a core is usually a balance between the losses to be expected and the cost of construction.

The loss, it will be seen, varies as before with the square of the frequency and with the square of the maximum flux density.

The loss also varies inversely as the resistivity of the material. Good silicon steel containing several per cent of silicon will have a resistivity about three times as great as ordinary transformer steel. This is one of the principal reasons for using silicon steel in the construction of transformer cores and similar apparatus.

The eddy currents set up in the aluminum disk of the meter shown in Fig. 182*a* are used to regulate the speed at which the disk rotates.

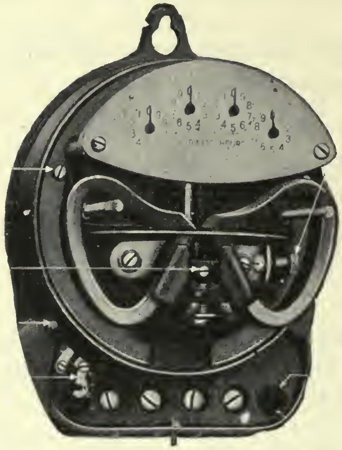

FIG. 182*a*. A watt-hour meter. The eddy currents set up in the disk as it revolves between the poles of permanent magnets regulate the speed of the meter. *The General Electric Co.*

Prob. 10–10. If in Fig. 179 the bar of iron carries a sinusoidally varying flux $\phi_{\max} = 5600$ lines, $f = 50$ cycles per second, what will be the power loss in a copper ring where $l = 3$ inches, $a = \frac{1}{8}$ inch and $b = \frac{3}{4}$ inch, at a temperature of 20° C.? If the temperature of the ring rises to 150° C., what will be the loss at this new temperature? How many watts per square inch of external surface of the ring in each case?

Prob. 11–10. A transformer core has a maximum flux density of 62,000 lines per square inch and a lamination thickness of 0.010 inch (No. 32 gauge). With the same net volume of iron, if the laminations were made 0.016 inch thick (No. 28 gauge), what change would have to be made in the flux density in order that the eddy-current loss might remain the same?

101. Total Core Loss. When the flux varies in a magnetic circuit, there is a loss due to eddy currents, and also, as we have seen, a loss due to hysteresis. The sum of these two losses is called the core loss of the material.

We have seen that the hysteresis loss varies directly as the frequency, but that the eddy-current loss varies as the square of the frequency. We have also seen that the hysteresis loss varies as the 1.6th power of the maximum flux density, while the eddy-current loss varies as the square of the maximum flux density. These facts may be summed up in the statement

$$P_{h+e} = k_1 f\, B_{\max}{}^{1.6} + k_2 f^2\, B_{\max}{}^2 \text{ watts,} \qquad (28)$$

where k_1 and k_2 are constants.

These facts enable us to predict how the total core loss in a transformer or machine will vary with the frequency if it is changed, or with a change of flux density. They also enable us to separate the two losses and find the value of each by methods which measure the total loss. The total core loss in a machine can be measured by finding the power necessary to drive the machine at no load when only these losses are present. The core loss in a transformer may be measured by somewhat similar means. An example will show how the losses can be thus separated.

Suppose that the total core loss of a machine at 1000 revolutions per minute is measured and found to be 1800 watts. Keeping the flux density constant, the machine is then run at 1500 revolutions per minute, and the total core loss found to be 3000 watts. It is required to find the hysteresis and eddy-current losses separately at 1000 revolutions per minute. Since the flux density is constant throughout, we may include it with the constant and write our expression

$$P_{h+e} = k_3 f + k_4 f^2 \text{ watts,} \qquad (29)$$

and inserting our values, we obtain

$$\begin{aligned} 1800 &= k_3 1000 + k_4 \overline{1000}^2, \\ 3000 &= k_3 1500 + k_4 \overline{1500}^2. \end{aligned} \tag{30}$$

Since we are dealing with a variation of speed only, it makes no difference whether our frequencies are expressed in cycles per second or as in this case in revolutions per minute.

Solving these two simultaneous equations gives us

$$\begin{aligned} k_3 &= 1.4, \\ k_4 &= 0.0004, \end{aligned} \tag{31}$$

and inserting these in the expression for the loss at 1000 revolutions per minute we obtain

$$1800_{h+e} = 1400_h + 400e \text{ watts,} \tag{32}$$

showing that the total loss of 1800 watts at this speed is divided into 1400 watts of hysteresis loss and 400 watts of eddy-current loss. If we substitute the values in the second expression, we obtain the losses at 1500 revolutions, which gives us

$$3000_{h+e} = 2100_h + 900e \text{ watts,} \tag{33}$$

showing that at the higher speed, the hysteresis loss is 2100 watts and the eddy-current loss 900 watts.

Suppose it is required to find the total core loss in this machine if the flux density is now increased by thirty percent. We will assume that the speed is held at 1000 revolutions per minute.

The hysteresis loss at the new flux density will be

$$P_h = 1400 \times 1.3^{1.6} = 2132 \text{ watts,} \tag{34}$$

since the flux density is 1.3 times as great as before, and the hysteresis loss varies as the 1.6th power of the flux density. The eddy-current loss, similarly, is

$$P_e = 400 \times 1.3^2 = 676 \text{ watts,} \tag{35}$$

and hence we obtain for the total core loss at 1000 revolutions per minute and thirty percent increase in flux density

$$P_{h+e} = 2132 + 676 = 2808 \text{ watts.} \tag{36}$$

Prob. 12–10. A transformer has a total core loss of 585 watts at 60 cycles per second. The flux is reduced one-half and the frequency is reduced to 25 cycles, and the core loss becomes 56.1 watts.

(*a*) What is the eddy-current loss at each frequency?

(*b*) What is the hysteresis loss at each frequency?

SUMMARY OF CHAPTER X

A CHANGE IN THE NUMBER OF FLUX LINKAGES with an electric circuit produces a voltage in the circuit

$$e = N \frac{d\Phi}{dt} 10^{-8} \text{ volts.}$$

WHEN MAGNETIC LINES OF FORCE ARE CUT BY A CONDUCTOR an electromotive force is set up in the conductor. This is only another way of stating the above fact. The same equation applies to both ways of regarding the phenomenon. In the latter case, $\frac{d\Phi}{dt}$ means the rate of cutting the lines of force. **IF THE THUMB, FOREFINGER AND MIDDLE FINGER** of the right hand are extended at right angles to one another, the thumb pointing in the direction of the motion of the conductor and the forefinger in the direction of the flux, the middle finger will point in the direction of the generated voltage. **AN ALTERNATOR MAKES USE OF THE PRINCIPLE** of changes in flux linkages or the cutting of lines of force to set up an alternating electromotive force. **BY CAUSING THE RATE OF CUTTING OF LINES OR CHANGING IN FLUX LINKAGES TO VARY SINUSOIDALLY**, an alternating electromotive force having a sine-wave form is produced.

$$e = E_{max} \sin \omega t,$$

where

e = the instantaneous voltage,
ω = the angular speed of the coil in electrical radians per second,
t = the time elapsed in seconds.

BY ATTACHING A SWITCHING DEVICE CALLED A COMMUTATOR to the revolving element of an alternator, the alternating electromotive force produced in the armature may be delivered to the brushes as a direct electromotive force. **IN A HOMOPOLAR GENERATOR**, the armature moves through a field of constant intensity and always in the same

direction, thereby inducing a direct electromotive force. No commutator is therefore needed.

EDDY CURRENTS are local electric currents which are set up in the core and the pole faces of machines or in any solid conductor, when magnetic lines are cut by the conducting materials composing these parts.

WHEN THE CUTTING OF FLUX OR THE VARYING OF FLUX LINKAGES IS SINUSOIDAL, the eddy-current loss may be expressed by the equation

$$P_{av} = \frac{\pi^2}{6\rho} h^2 f^2 B_{max}^2 \times 10^{-16} \text{ watts per cubic centimeter,}$$

where

ρ = the resistivity of the sheets in ohm-centimeters,
h = the thickness of the sheets in centimeters,
f = the frequency in cycles per second,
B_{max} = the maximum flux density in gausses.

The energy of these currents is dissipated as heat in the machine. LAMINATING THE CORES decreases the eddy currents and the accompanying loss. THE TOTAL CORE LOSS is the sum of the hysteresis and the eddy-current losses.

PROBLEMS ON CHAPTER X

Prob. 13–10. At what speed must the rotor of Fig. 176 operate to produce a 60-cycle frequency?

Prob. 14–10. Assume the following data for a 48-pole alternator with concentrated armature windings.

Maximum flux density in conductor slot, 72,000 lines per square inch;
Frequency, 25 cycles;
Number of armature circuits, 1;
Number of conductors in the armature circuit, 192;
Active length of each conductor, 30 inches;
Maximum voltage desired per phase, 3250 volts.

What diameter must the rotor have?

Prob. 15–10. What diameter would the rotor of Prob. 14–10 have if the frequency were 60 cycles?

Prob. 16–10. The area of a certain hysteresis loop is 6.21 square inches. The ordinate scale is 5000 gausses per inch, the abscissa scale is 5 gilberts per centimeter per inch. The maximum value of the flux density is 16,000 gausses. What is Steinmetz' hysteresis coefficient for the sample? Frequency, 60 cycles.

Prob. 17–10. The volume of annealed sheet steel in a 60-cycle, 220–2200-volt, 20-kw. transformer is 910 cubic inches at a maximum flux density of 68,000 lines per square inch. The losses are found to be as follows: hysteresis loss 208 watts, eddy-current loss 96 watts. If the frequency is changed to 25 cycles and the flux density to 72,000 lines per square inch, what would be the new total loss and what would be separately the eddy-current loss and the hysteresis loss?

Prob. 18–10. An axle of an ordinary railroad coach has a length of 8 feet. The flux density of the earth's field is approximately 0.6 gauss and the flux lines make an angle of 70° with the horizontal. The locomotive speed is 70 miles per hour due east. What voltage is generated in the axle? Does it make any difference whether the train is running north or east?

Prob. 19–10. A certain 8-pole generator has a rated speed of 800 revolutions per minute. The equivalent maximum flux density is 65,000 lines per square inch. The armature

core is made up of " regular dynamo " steel sheets 0.014 inch thick and weighs 1000 pounds. (Specific gravity is 7.79.) The hysteresis coefficient is 0.0013 and the eddy-current coefficient is 0.0021. What is the total core loss?

Prob. 20–10. To what value would the core loss of Prob. 19–10 be reduced if silicon steel sheets were substituted for the " regular dynamo " sheets? Specific gravity, 7.5; hysteresis coefficient, 0.00071; eddy-current coefficient, 0.000067.

Prob. 21–10. A solenoid 40 centimeters long and 2 centimeters in diameter is wound with 1200 turns of copper wire. The current in the wire at a certain instant is varying at the rate of 400 amperes per second. What eddy-current loss is produced at this instant in a round graphite rod of 0.10% conductivity, 1 centimeter in diameter and 20 centimeters long placed centrally in the coil, with its axis coinciding with the axis of the coil?

Prob. 22–10. What average eddy-current loss will take place in the rod of Prob. 21–10 when the coil carries an alternating current varying sinusoidally, 60 cycles per second, with a maximum value of 1.24 amperes?

Prob. 23–10. A coil containing 700 turns is wound on a closed iron core of 5 square inches sectional area and 15 inches effective length. Assuming that the flux in the iron follows the magnetization curve for annealed sheet steel: —

(*a*) Determine the constants of Froelich's equation applied to this magnetization curve.

(*b*) Determine the equation showing the relation between flux and time, when the current in the winding follows the law

$$i = 2 \sin 377t;$$

(*c*) Bearing in mind that the e.m.f. induced in the coil is given by $n\frac{d\phi}{dt}$, determine the equation for the voltage induced in the coil by this flux.

(*d*) Plot a half wave for parts (*b*) and (*c*) by choosing 7 convenient values of $377 \times t$.

Prob. 24–10. The coil in Prob. 23–10 has applied to its terminals a sinusoidal voltage of 283 volts maximum value.

(*a*) Plot to a suitable scale a half wave of this voltage.

(*b*) Plot on the same set of axes the flux through the coil corresponding to this voltage. The flux equation may be determined by an integration.

(*c*) To each value of flux there corresponds a definite magnetizing current. Plot the curve of this current. This may be done by choosing values of flux and obtaining the corresponding values of current from the magnetization curve.

(If the hysteresis loop of the iron were used, instead of the magnetization curves, the resulting current curve would be distorted, but more nearly correct.)

(*d*) Compare the results of Prob. 23–10 and 24–10. Why does a sinusoidal current correspond to a nonsinusoidal voltage, and vice versa?

CHAPTER XI

FORCE ON A CONDUCTOR

It is an axiom that every electric generator is reversible; that is, that if electric power in the proper form is supplied to a machine used as an electric generator, it will tend to run as an electric motor. In order that such a machine may operate satisfactorily as a motor, it is generally necessary to make certain adjustments of brushes, rheostats etc., and to add auxiliaries such as starting motors and speed-limiting devices, but the fact remains that the tendency is present in the machine to operate as a motor. All that is necessary is the proper control and regulation of this tendency.

102. Force on a Conductor Carrying a Current. We have seen that when a conductor is moved sideways in a magnetic field, there is a voltage generated in the conductor. This principle gives rise to the electric generator. There is another principle which is somewhat similar and which makes possible the electric motor. Whenever a conductor is situated in a magnetic field and not parallel to the flux and is carrying an electric current, there is a force upon the conductor which tends to move it sideways through the field.

Thus in Fig. 183, the conductor shown situated in a uniform field and carrying a current which flows from back to front is acted upon by a force which tends to push it up. In order to find the direction of a force on a conductor, we can use the left-hand rule. It will be remembered that the right-hand rule was used for a generated voltage. The **left-hand rule** is similar and is used for a motor. If the thumb and first two fingers of the **left hand** are held perpendicular to one another, the forefinger pointing in the

direction of the flux, the middle finger pointing in the direction in which the current flows in the conductor, then the thumb will point in the direction in which the force tends to move the conductor in the field.

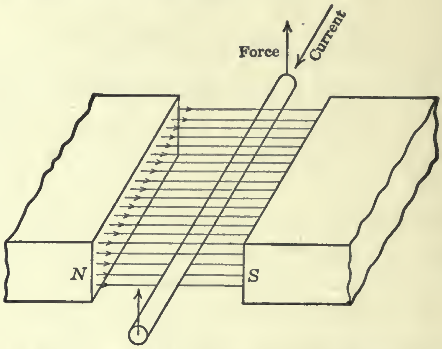

FIG. 183. A wire carrying a current and situated in a magnetic field is acted on by a force.

The amount of this force has been found experimentally to be proportional to the flux density, proportional to the current flowing in the conductor and proportional to the projection of the conductor perpendicular to the field. If the conductor is perpendicular to the field as is shown in Fig. 183, this last factor becomes simply the length of the conductor in the field. If the conductor makes an angle θ with the flux lines, then the length to be taken is the length of the conductor multiplied by the sine of θ. In the following discussion, unless it is stated otherwise, we will assume that the conductor is perpendicular to the field.

If we write everything in c.g.s. units, the force on the conductor in dynes is equal to the product of the flux density in gausses, the current in abamperes and the length of the conductor in centimeters; that is,

$$F = BlI \text{ dynes.} \tag{1}$$

The reason that the proportionality factor is unity in this case may readily be shown.

Assume that the above conductor is without resistance, so that there are no losses involved in the flow of current through it. Assume that it is moving through the field at a velocity v. Then there will be generated in it a voltage of amount

$$e = Blv \text{ abvolts.} \tag{2}$$

By the **right-hand rule,** referring to Fig. 183, the direction of this voltage will be opposite to the direction in which there is flowing the current which produces the force to cause the conductor to move with this velocity. That is, the voltage is a back voltage. There is hence an input in the conductor equal to

$$eI = BlvI \text{ ergs per second.} \tag{3}$$

This power input is used up in maintaining the conductor at a velocity v. However, a force F on a conductor which is moving at a velocity v in the direction of the force is doing work at the rate

$$Fv \text{ ergs per second.}$$

If, as we assumed, there is no resistance loss in the conductor, and if in addition we assume that the velocity is constant, the entire electrical input must appear as this mechanical power output, in accordance with the law of conservation of energy. Therefore

$$Fv = BlvI \text{ ergs per second,} \tag{4}$$

which shows that

$$F = BlI \text{ dynes.} \tag{1}$$

We thus see that the force on the conductor is equal to the product of B, l and I, all in c.g.s. units.

If I is in amperes, then

$$F = Bl\frac{I}{10} \text{ dynes.} \tag{5}$$

If we wish to convert this equation to practical units, we obtain

$$F = \frac{BlI}{10} \times \frac{1}{980 \times 454} \text{ pounds}$$

$$= 2.25 \times 10^{-7} \, BlI \text{ pounds,} \tag{6}$$

where

I is in amperes,
B is in gausses,
l is in centimeters.

If a coil of a single turn, as shown in section in Fig. 184, is situated in a uniform magnetic field, there will be a torque acting upon the coil which will be proportional to the current. We will compute the torque on the assumption that the current carried is I abamperes, that the plane of the coil makes an angle θ with the plane of the flux, that the length of the coil in the direction perpendicular to the flux is l and that the width of the coil is $2r$.

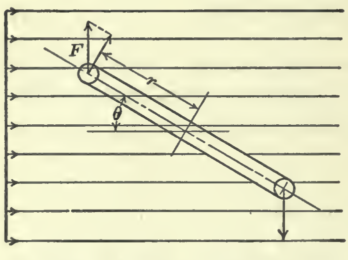

FIG. 184. A coil carrying a current and situated in a parallel magnetic field is acted on by a torque proportional to the cosine of the angle θ.

By the left-hand rule, the upper conductor is forced upward and the lower conductor is forced downward. These forces are each of amount

$$F = BlI \text{ dynes.}$$

The torque acting on the coil is the couple produced by these forces, or the sum of their moments about the axis. Each conductor is acted upon by a force of which the component perpendicular to the line joining the conductor to the axis is

$$F \cos \theta = BlI \cos \theta, \tag{7}$$

The total moment, or total turning torque acting upon the coil, is therefore

$$T = 2\,BlIr \cos\theta \text{ dyne-centimeters,} \tag{8}$$

which is expressed in dynes at one-centimeter lever arm if all of the quantities in the equation are in c.g.s. units. This may readily be changed to be in terms of pounds at a lever arm of one foot.

Note that the turning moment is at maximum when the plane of the coil is parallel to the flux lines; also that when the angle θ is ninety degrees the torque becomes zero. If θ is greater than ninety degrees, the torque is reversed and the coil tends to turn backwards. This may be summed up by stating that the coil tends to move into such a position that it will link a maximum amount of flux in a right-handed-screw direction with respect to the current.

Prob. 1–11. The flux density in the air gap of a magnet such as is shown in Fig. 183 is 63,000 lines to the square inch. What force acts on a conductor of length 4 inches, carrying a current of 40 amperes and placed in this field at an angle of 30° with the flux lines?

Prob. 2–11. What torque in gram-centimeters acts on a coil of a single turn situated with respect to a magnetic field as shown in Fig. 184, if the current flowing is 50 amperes, the flux density is 7000 gausses and r is 2 centimeters? Compute for three values of the angle θ of 30°, 60° and 90°. The length of each side is 12 centimeters.

Prob. 3–11. If the coil of Prob. 2–11 has 4 turns and the flux density is 58,000 lines to the square inch, what will be the torque in pound-inches when $\theta = 0°$ and the wire carries a current of 30 amperes?

103. Meters. Electric meters are of many kinds. Ordinary direct-current meters usually depend for their action, however, simply upon the force exerted upon a conductor carrying current when situated in a magnetic field. This

force is used to deflect a pointer and thus gives a measure of the amount of current flowing.

Portable instruments of this sort are usually constructed as shown in Fig. 185. The cross-section of the core and poles is shown in Fig. 186. M_1 and M_2 are the poles of a per-

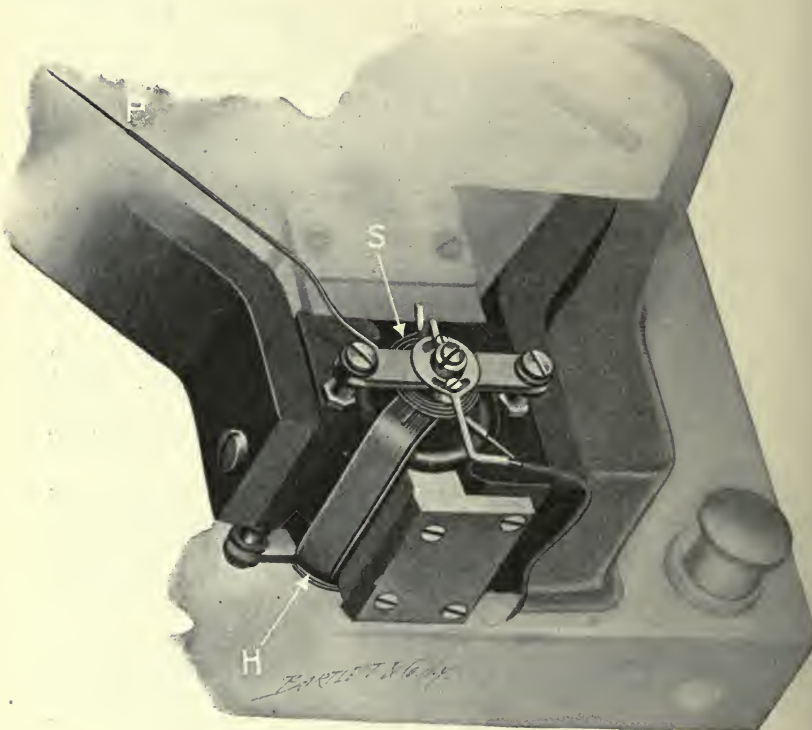

FIG. 185. Cutaway view of a direct-current meter. *Weston Electrical Instrument Co.*

manent magnet which is constructed in such a manner as to have a very constant value of field strength. C is a soft-iron core cylindrical in form and situated between the poles.

The core is stationary. A rectangular coil H as shown in Fig. 185 is pivoted in bearings in such a manner that its sides revolve in the space between the magnet poles and the core. The spindle on which this coil is mounted carries a pointer P which moves over a scale and gives the indication. The pointer is returned to its zero position by means of a hairspring S, through which the current is also usually conducted to the coil. Sometimes two hairsprings are used, one to conduct the current in and the other to conduct the current out of the coil. One of these may be replaced by a flexible lead.

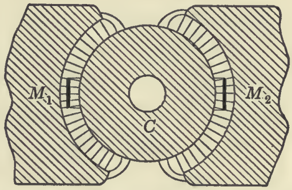

FIG. 186. Cross-section of the core and poles of the meter of Fig. 185. The flux is radial.

As shown in Fig. 186, the influence of the core C is to cause the flux to be radial in the air gap and of very nearly constant flux density. The coil H is therefore always situated in a field of the same strength. The force acting upon the sides of the coil is simply proportional to the current flowing in the coil and is independent of the position of the coil. The restoring force of the hairspring is proportional to the deflection. Thus the deflection of the coil, and hence the movement of the pointer, will be proportional to the current flowing. This gives a meter with evenly divided divisions on the scale.

It should be noted that in this construction we do not have a case of a coil situated in a parallel uniform field. The field is everywhere radial, so that the force acting upon the sides of the coil is always perpendicular to the plane of the coil. The entire force, therefore, and not a component of the force, is effective in producing a turning moment.

As has been noted before, such a meter may be used to measure either voltage or current, depending upon the

resistance to which the coil is wound. In voltmeters, it is usual to supply an external resistance in addition to the resistance of the coil itself. Ammeters are also constructed by supplying an external resistance in parallel with the coil.

If such a meter is used in an alternating-current circuit of the usual frequencies, it will not deflect at all. The current in an alternating-current circuit flows first in one direction and then in the other, reversing its direction many times a second. When the current is flowing in one direction, the coil of the meter tends to turn clockwise, but this is offset by an exactly equal counter-clockwise impulse when the current next reverses. Such a meter, therefore, cannot be used to measure alternating current. In order to make an alternating-current meter, we replace the permanent magnet by an electromagnet, and excite this electromagnet by a coil which is connected in series with the movable coil. The direction of the current in the coil, therefore, as well as the direction of the flux in which it is situated, reverses each half cycle. Since both the flux and the current reverse, it will easily be seen, by utilizing the left-hand rule, that the turning moment on the coil is in the same direction as before. Such instruments will therefore deflect when an alternating current flows through them and may be used to measure alternating current. They are called dynamometer instruments.

A dynamometer instrument will not, however, be an instrument utilizing an evenly divided scale. The flux in which the coil is situated is not constant as in a direct-current instrument, but depends upon the strength of the current, and in a properly constructed instrument is proportional to the strength of the current. The turning moment on the coil, therefore, which is proportional to the product of B and I, is proportional to the square of the current. If the current through such an instrument is doubled, the deflection obtained will be four times as great, and so on. The scale used must be divided accordingly. A dyna-

mometer instrument may also be used, of course, to measure a direct current.

Alternating-current meters are also made which depend upon entirely different principles, such as the heating produced in a wire, the tendency of a piece of soft iron to line up with a field, and so on.

The same construction as is used in the direct-current meter may be used also in constructing a sensitive galvanometer. In this case the coil, instead of being pivoted, is usually suspended on a fine suspension thread as shown in Fig. 187. This suspended coil often carries a small mirror by which its deflection may be noted by reflecting a spot of light from the mirror onto a scale, or by looking at the reflection of a scale in the mirror as seen through a telescope. The principle of operation is exactly the same as that of a direct-current meter. By making a coil of a large number of turns of fine wire, by using a strong magnetic field and by using a very delicate suspension, the galvanometer may be made to deflect when extremely small values of current are used.

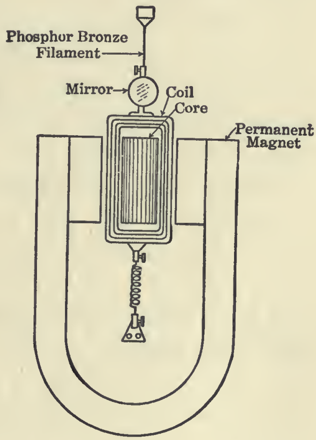

FIG. 187. Diagram of a sensitive galvanometer.

A galvanometer of long period is called a ballistic galvanometer, and is used, as we have seen, for the measurement of flux. The deflection of a ballistic galvanometer is proportional to the total quantity which flows through it; that is, it is proportional to

$$q = \int i dt \text{ abcoulombs,} \tag{9}$$

which, if we assume for the moment that we can neglect the inductance of the galvanometer, is equal to

$$\frac{1}{r}\int edt. \tag{10}$$

We have seen in a previous chapter, however, that the total amount of flux linking a measuring coil can be found from the expression

$$\phi = \int edt \text{ maxwells,} \tag{11}$$

and so the total amount of flux can be found from the deflection of a ballistic galvanometer connected to the measuring coil.

The reason why the deflection of a ballistic galvanometer is proportional to q, that is, to the quantity passing through it, is as follows. The force acting on the coil is proportional to the current, since the coil is situated in a field of constant flux density. The acceleration of the coil is proportional to this force; that is,

$$\frac{dv}{dt} = K_1F = K_2i, \tag{12}$$

where K_1 and K_2 are proportionality factors. The initial velocity acquired by the coil is therefore

$$v = K_2\int idt, \tag{13}$$

which we have seen is proportional to the flux. The deflection of the coil depends simply upon the initial velocity given to it, just as the deflection of a ballistic pendulum used to measure the velocity of a bullet depends simply upon the initial velocity imparted to it. The deflection of the ballistic galvanometer which is proportional to its initial velocity will thus be proportional also to the total number of flux linkages changed in the measuring coil.

In this expression we have neglected the inductance of

the coil itself. It will be seen, however, in the solution of Prob. 4–11, that the quantity of electricity passed through an inductive circuit, when a constant voltage is applied to it for a given interval of time and then removed, is the same as the quantity which would pass if the inductance were not present. To be sure, the current would take a longer time in reaching its maximum value, but it would also persist enough longer to make the total quantity of electricity passed through the circuit exactly the same. The inductance of the coil of the ballistic galvanometer, therefore, does not affect the proportionality of its readings to the flux to be measured. We also note that it makes no difference whether the measuring coil has its voltage induced in it for a short or a long period, provided the total change of flux linkages, that is,

$$\int edt,$$

remains unchanged. This is strictly true only if the measuring coil is removed from the influence of the flux in a time sufficiently short so that the coil of the ballistic galvanometer has not appreciably moved from its position in the meanwhile. In order to make sure that this condition is fulfilled, the coil is made with a large moment of inertia, resulting in a long period.

Prob. 4–11. Prove the statement, "When a constant voltage is applied to an inductive circuit for a given length of time and then removed without opening the circuit, the same quantity of electricity passes as would pass in a non-inductive circuit if the same voltage were applied for the same length of time, the resistances of the two circuits being equal."

104. Motors. A coil of wire carrying current and situated in a magnetic field tends to turn about an axis perpendicular to the field, and so may be used as a motor and caused to perform useful work. Referring to Fig. 188, in which a single-turn coil is shown for convenience, we see by the left-hand rule that the coil tends to turn in a clockwise direction

when a current flows in the coil in the direction indicated by the dot and cross.

When this coil has arrived at a position such that its plane is perpendicular to the flux lines, however, the force acting

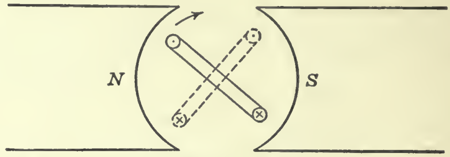

Fig. 188. A current in the direction indicated tends to rotate the coil in the direction of the arrow.

on the wire will be in the plane of the coil and the turning moment about the axis will be zero. In fact, if the coil were turned beyond this position to a position as shown by the dotted lines, its turning moment would be reversed and it would tend to turn backward.

Suppose, however, that just as the coil of Fig. 188 passes through the vertical position, we reverse the direction of the current in the coil. After passing this dead center, it will take the position of Fig. 189, and since the current is

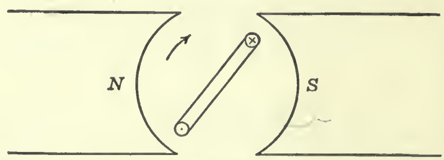

Fig. 189. When the coil of Fig. 188 has reached this position, the current is reversed and it tends to continue rotating in the same direction.

reversed, the turning moment on the coil will be in the same direction as before, so that it will tend to keep on turning. By reversing the current in the coil at the correct intervals, we are thus able to render the turning moment always in the same direction and so construct a motor which will keep turning continuously.

One way in which this can be accomplished is by supplying the coil with an alternating current. If the rotation of the coil then simply keeps step with the alternation of the current supplied, that is, if the armature of the motor runs in synchronism with the supply, the torque will always be in the correct direction and the motor will continue to revolve. Such an alternating-current motor is called a synchronous motor. In Fig. 190 is shown the manner in which the current in the armature, that is, in the revolving coil, reverses with the time, and over various points in the wave are shown for convenience the succeeding positions of the revolving coil.

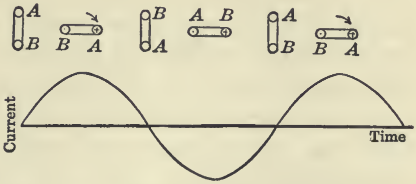

Fig. 190. Showing the relation between the current in the coil and the position of the coil. Note that the current is zero at the instant when a current would produce no torque.

In such a motor, the coil is, of course, actually imbedded in iron, just as in a generator, in order to reduce the reluctance of the flux path. Synchronous motors are usually made multipolar and generally polyphase; this means that there are several coils with different time relations operating simultaneously on one armature. Synchronous motors as well as alternators are also usually built with revolving fields instead of revolving armatures. Since action and reaction are always equal and opposite, it makes no difference in the amount of torque produced whether the armature or the field revolves.

Alternating-current motors are also made in various other forms such as the induction motor or the repulsion motor. The principles of these are too complicated to be discussed at this time.

Instead of reversing the direction of the current supply to a motor as the armature revolves, we may use a commutator

to reverse the connections of the armature conductors to the external circuit and thus construct a direct-current motor. In fact, any direct-current generator will, if supplied with current, run as a direct-current motor.

Examine again the arrangement shown in Fig. 168, which was drawn for a direct-current generator. Suppose that the current enters by brush A and leaves by brush B. By tracing the connections through the winding, we find that the current goes down through each of the conductors under the south pole and comes up through each of the conductors under the north pole. The machine will thus tend to turn in a clockwise direction, that is, in a direction exactly reverse to the direction in which it ran as a generator. Suppose that the armature turns through one-sixth of a revolution. Again examine the path of the current. It is found as before that the current goes down through each of the conductors situated under the south pole and up through each of the conductors on the other side of the armature. Whatever the position of the armature, therefore, the torque remains in the same direction and of approximately the same amount, so long as the current entering through the brush is constant. This machine will hence run as a motor.

The homopolar machine shown in Fig. 178 will also run as a motor if supplied with current. In this case the conductor is always situated in a field of the same strength and direction, no matter what the position of the armature. The torque is consequently constant in magnitude and direction, so long as the current through the conductor remains unchanged. Machines of this sort are not, however, used as motors, for they are subject to strict practical limitations and for most purposes would be far too expensive to be considered.

105. The Back Electromotive Force. The force acting on a conductor in a magnetic field is proportional to the strength of the field and to the current in the conductor. The force remains unchanged when the conductor moves.

It makes no difference whether the conductor is at rest or is traveling at a high velocity, the force is independent of the speed of the conductor and depends simply upon the current and the field strength. In other words, in a motor the torque is proportional to the total flux and to the motor current, and is independent of the speed of the motor.

In the same way, a conductor which is moving through a magnetic field has a voltage generated in it which is proportional to the strength of the field and to the velocity of the conductor. It has the same voltage generated in it whether a current flows in the conductor or not. The terminal voltage across the conductor may depend upon the current flowing, for the terminal voltage is the combination of the generated voltage and the resistance drop. The voltage generated, however, depends simply upon the field strength and the velocity of the conductor. That is, in a dynamo there is a certain generated voltage which is proportional to the speed of the machine and to the total flux, and which in no way depends upon the current flowing.

Suppose that we have a motor which is carrying a current I, and that the terminal voltage of the motor is E. The total power input into the motor is IE. A portion of this input, I^2R, is converted into heat in the motor. In this case R is the resistance of the windings on the armature. The remainder of the input,

$$P = EI - I^2R \text{ watts,} \tag{14}$$

is converted into mechanical work in the motor. The motor is a machine for transforming electrical power into mechanical power. The amount of power thus transformed is, of course, equal to the power input to the machine minus the power losses in the machine itself. Of the amount of power transformed into mechanical form, a portion will naturally be lost in the friction of the bearings and so on, and hence will be unavailable for useful purposes.

This motor, however, must also act as a generator, when

it is running. It has generated in it a voltage which is proportional to the speed at which it runs. This voltage, we have seen, is in a direction opposite to the direction of the applied voltage, that is, it is a back electromotive force. We will denote it by E_B. The current flowing through the motor is determined by Ohm's law; that is, the current is equal to the electromotive force acting divided by the resistance of the motor winding. In applying this law, however, we must take into account all of the voltages acting. There is in addition to the applied voltage a back electromotive force which opposes this voltage. The net voltage acting is therefore

$$E - E_B,$$

and hence the current is

$$I = \frac{E - E_B}{R} \text{ amperes.} \tag{15}$$

We may write this last expression as

$$E = E_B + IR \text{ volts.} \tag{16}$$

If we insert it into the expression above for the amount of power which is transformed into mechanical form, we shall obtain

$$P = (E_B + IR)\,I - I^2R = E_BI \text{ watts.} \tag{17}$$

The amount of power transformed by a motor from electrical to mechanical form is thus equal to the current times the back electromotive force. The output of the motor will be equal to the amount of power transformed minus the frictional losses.

We may define the efficiency of a machine as the output divided by the input; that is,

$$\text{efficiency} = \frac{\text{output}}{\text{input}}, \tag{18}$$

or

$$\text{efficiency} = \frac{\text{input} - \text{losses}}{\text{input}}.$$

In the losses must be included the frictional losses in the machine and the electrical losses in the form of I^2R in the winding.

Prob. 5–11. A motor takes from the line 48 amperes at 110 volts. The I^2R losses are 6% of the input. What is the power transformed into mechanical form? The frictional losses are 8% of the input. What is the output of the motor in horse power?

Prob. 6–11. A motor running at 550 volts takes an armature current of 26 amperes at a certain load. The armature resistance is 1.6 ohms. What is the resistance loss and what is the back electromotive force? How much power is transformed into mechanical form?

Prob. 7–11. What is the efficiency of the motor of Prob. 6–11 if in addition to the I^2R loss in the armature there is allowed 630 watts for friction, windage and all other losses?

106. Speed. The back electromotive force of a motor is proportional to its speed and to the total flux; that is,

$$E_B = KS\phi \text{ volts,} \tag{19}$$

where K is simply a proportionality factor. We have seen, however, that

$$E = E_B + IR \text{ volts,} \tag{20}$$

and if we substitute in this equation for E_B its value above, we obtain

$$E = KS\phi + IR \text{ volts,} \tag{21}$$

which, solved for S, gives

$$S = \frac{1}{K}\,\frac{E - IR}{\phi} \text{ revolutions per minute.} \tag{22}$$

This is a very important equation indeed, since it shows the speed at which a motor will run.

Let us examine first the speed at which a motor will run under very light load. In this case, the output is small, and accordingly the term IR must be small, for the current is small. Neglecting this term, we obtain

$$E = KS\phi \text{ volts.}$$

This, however, is exactly the expression which we obtained for the back electromotive force. In other words, under very light loads a motor will run at such a speed that the back electromotive force approximately equals the applied electromotive force. This must, of course, be true only when a small current, that is, a small input, is to flow into the machine. Suppose that we have a machine which, when run as a generator at 1000 revolutions a minute, delivers a voltage of 220 volts. Let us maintain the flux of this machine strictly constant, and connect it to a supply giving 220 volts, in order to run it as a motor. If it is lightly loaded, it will run at approximately the same speed as before, that is, at a speed slightly less than 1000 revolutions per minute.

When a motor is loaded, the current will increase, for of course its input must increase. In the speed equation, therefore, the term IR increases and the speed accordingly drops. A constant-flux motor will thus slow down slightly as the load is applied. It will not slow down much, for it is necessary to decrease the speed only enough so that the difference between E and E_B is sufficient to force the load current through the small internal resistance of the motor.

Suppose, for instance, that the above motor is loaded until its input is 440 watts, that is, until the current is 2 amperes. If the internal resistance of the machine is 4 ohms, the term IR in the speed equation will be 8 volts. The speed under this load will therefore be

$$S_2 = \frac{1}{K}\,\frac{220 - 8}{\phi},$$

and comparing this with the speed at no load

$$S_0 = \frac{1}{K}\,\frac{220}{\phi},$$

we see that the speed is decreased in the ratio

$$\frac{S_2}{S_0} = \frac{212}{220};$$

this means that the speed decreases about four percent when this load is applied. This assumes that the value of ϕ will remain unchanged when the armature current is increased. In most machines there is a slight change in the value of ϕ from no-load to full-load conditions which introduces a small error in equation (26).

Referring again to the speed equation, we see that the speed at which the motor will run is approximately proportional to the applied voltage so long as the flux remains constant.

Moreover, with a constant applied voltage, the speed is inversely proportional to the flux. This is again an approximate relation. If the flux is doubled, the speed will be halved. In other words, with the increased flux, the motor will need to run only half as fast in order to generate a back electromotive force approximately equal to the applied voltage. Conversely, if the flux is halved in value, the speed of the motor will be approximately doubled. It will be necessary for the motor to run twice as fast as before in order that its back electromotive force may approximately balance the voltage of the line and prevent a very heavy current from flowing.

Suppose that we have a motor in which the flux is obtained by a magnetizing coil connected directly across the line. This form of motor is known as a shunt motor, and the magnetizing coil is known as the field winding. In order to change the speed of the motor, we need simply to adjust the current in the field winding. If we decrease this current, the speed will increase; if we increase the field current, the speed will be lowered.

Prob. 8–11. A 220-volt motor has a no-load speed of 1200 revolutions per minute. When fully loaded, the armature carries 92 amperes and the field flux decreases 6%. The armature resistance is 0.057 ohm. What is the full-load speed?

Prob. 9–11. If an extra winding were put on the fields of the motor of Prob. 8–11 so that the field flux increased 6% from no load to full load, what would be the full-load speed?

SUMMARY OF CHAPTER XI

A CONDUCTOR CARRYING AN ELECTRIC CURRENT and situated in a magnetic field (but not parallel to it) is acted upon by a force according to the equation

$$F = BlI \text{ dynes},$$

where l = length perpendicular to flux.

EXTEND THE THUMB AND FIRST TWO FINGERS OF THE LEFT HAND perpendicular to one another, the middle finger pointing in the direction of the electric current, the forefinger in the direction of the flux, and the thumb will indicate the direction in which the conductor tends to move.

In a **DIRECT-CURRENT METER**, the moving coil is usually situated in a constant, uniform, radial field. The moving force is therefore proportional to the current in the coil. This current can be made proportional either to the current or to the voltage it is desired to measure.

In an **ALTERNATING-CURRENT METER**, the field as well as the current in the coil is proportional to the voltage or to the current to be measured. The force on the coil is therefore proportional to the square of these quantities.

A BALLISTIC GALVANOMETER can be made to deflect proportionally to the quantity of electricity passing through it.

A MOTOR OWES ITS TORQUE to the force on a conductor carrying a current in a magnetic field.

WHEN THE ARMATURE OF A MOTOR ROTATES, an electromotive force is induced in the winding which opposes the flow of the current driving the motor. This electromotive force is therefore called a **BACK** electromotive force.

THE CURRENT TAKEN BY THE ARMATURE OF A MOTOR can be found from

$$I = \frac{E - E_B}{R_a}.$$

THE SPEED OF A MOTOR IS DIRECTLY PROPORTIONAL to the back electromotive force and inversely proportional to the field flux.

$$S = \frac{1}{K}\frac{E - IR_a}{\phi}.$$

PROBLEMS ON CHAPTER XI

Prob. 10–11. A Weston 5–15–150-volt voltmeter has a movable coil of the dimensions shown in Fig. 191. The winding consists of a single layer of 75 turns of wire. For a half-scale deflection, the current in the coil is 0.009 ampere. What torque is exerted in this coil if the flux density in the air gap at all points is 1000 gausses? (The flux lines are radial: note figure.)

Prob. 11–11. Another type of Weston meter has the same coil dimensions as in Fig. 191. The gap flux density is 1000 gausses and the coil has a single layer of 36 turns. A current of 0.010 ampere gives a movement of 3 inches on a scale 3.37 inches from the axis of the moving coil. What is the torque and what work is done in moving the coil this distance?

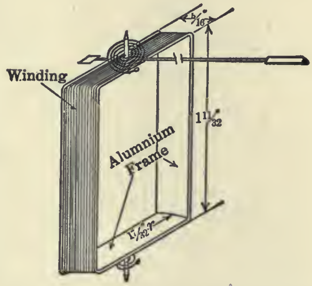

FIG. 191. The movable coil of a Weston voltmeter.

Prob. 12–11. A single turn of wire as in Fig. 184 carries a current of 75 amperes. The radius of the coil is 10 centimeters, the length is 8 centimeters and the flux density in the gap 50 kilolines to the square inch. What work in joules is done in moving the coil from $\theta = 0°$ to $\theta = 90°$? If the coil moves through this distance in 0.005 second, what is the average power?

Prob. 13–11. What work will be done if the coil of Prob. 12–11 is rotated by hand through 360°?

Prob. 14–11. In Prob. 12–11, what force will be exerted tangent to the surface of an 8-inch pulley secured to the axis of the coil if $\theta = 0°$?

Prob. 15–11. Develop a formula for the magnetic force in pounds between the two wires (line and return) of a transmission line. Let the current be in amperes, the length of the line in feet and the distance between wires in inches.

Prob. 16–11. A two-wire transmission line (line and return) spaced 18 inches is strung on poles 50 feet apart. (*a*) In what direction is the magnetic force exerted by one wire on the other? (*b*) How great is the force on a 50-foot span if each conductor carries 400 amperes?

Prob. 17–11. In one place, the transmission line of Prob. 16–11 goes under a bridge. Here it is cleated to the wall every 5 feet. The conductors are spaced 6 inches apart. (*a*) What is the magnetic force on each cleat when the line carries the normal load current of 400 amperes? (*b*) What would be the momentary force on each cleat if the line were accidentally short-circuited and there were a momentary rush of current of 15,000 amperes?

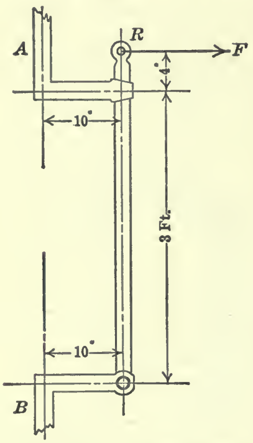

Fig. 192. A disconnecting switch on a high-tension line.

Prob. 18–11. Fig. 192 represents a disconnecting switch on a high-tension line. It requires a pull of 5 pounds at *R* to open the switch. *A* and *B* may be considered to be of indefinite length. The other conductors are too remote to affect the switch. How many amperes must flow in a short circuit which forces the switch open?

Prob. 19–11. The current in the armature of a 220-volt motor at full load is 90 amperes. The resistance of the armature is 0.140 ohm. The speed at full load is 1100 revolutions per minute. Assume that the flux decreases 4% as the load increases from no load to full load and determine the no-load speed. Armature current at no load, 5.0 amperes.

Prob. 20–11. What will be the speed of the motor in Prob. 19–11,

(*a*) At $\frac{1}{2}$ full-load current?
(*b*) At $1\frac{1}{4}$ full-load current?

Prob. 21–11. The value of ϕ at no load in the motor of Prob. 19–11 is 1,500,000 maxwells. How many active conductors are there in each path between the positive and the negative brushes?

Prob. 22–11. At what speed must the motor in Prob. 19–11 be run to operate as a generator and deliver 90 amperes at 230 volts? Assume that the flux at no load will be 103% of the no-load flux of Prob. 19–11, and will decrease 4% as the load rises from zero to 90 amperes.

Prob. 23–11. In Fig. 193 are shown two conducting "rails" and two conducting "sliders," A and B.

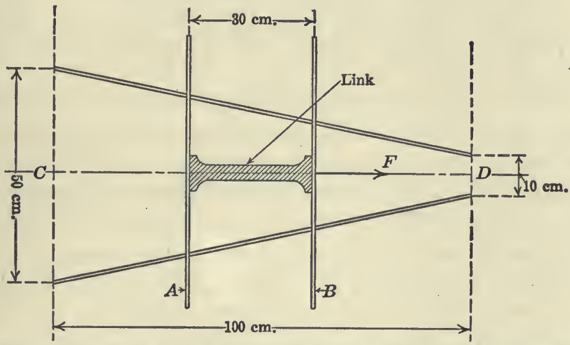

FIG. 193. Sliders A and B cut lines of magnetic flux as they move in the direction of F.

The sliders move in frictionless guides always perpendicular to the center line between the rails. They are connected by an insulated link as shown. The density of the magnetic field perpendicular to the plane formed by the rails and sliders is everywhere 1200 maxwells per square centimeter.

Each rail has a resistance of 20 microhms per centimeter.

Each slider has a resistance of 50 microhms per centimeter.

What force in dynes will be required at F in the direction shown to move the sliders in that direction with a velocity of 80 centimeters per second, when slider A is 10 centimeters from point C? What will be the tension or compression in the connecting link? If this force at F is held constant, with what velocity will the sliders be moving when slider B is at point D? Neglect mass of sliders and self-inductance. (This strictly hypothetical problem is presented because practical problems involving the same principles contain so many other factors that they become too complicated for our use.)

Prob. 24–11. A pair of busbars, line and return, spaced 12 inches on centers, extend 15 feet vertically down a wall. At

the foot of the wall the busses extend out 15 feet perpendicularly to the wall. These horizontal sections are supported by insulators spaced 5 feet apart, the first pair being directly under the vertical parts of the busses. At both ends the busbars are attached to twin cables. Consider the busbars hinged at the bends and treat the horizontal parts as "simple beams." If the busbars carry 6000 amperes,

(*a*) What vertical stresses due to this current will be put upon the pair of insulators which are 5 feet from the wall?

(*b*) What horizontal stresses will be set up in these same insulators?

(*c*) What will be the total added stress on these insulators?

Prob. 25–11. If the vertical and the horizontal sections of the busses in Prob. 24–11 instead of being 15 feet long were so long that they could be considered infinite in length, what would be the answers to parts (*a*), (*b*) and (*c*) of that problem?

Prob. 26–11. If the busses of Prob. 24–11 were spaced 12 feet instead of 12 inches on centers, what would be the answers to parts (*a*), (*b*) and (*c*) of that problem?

Prob. 27–11. Three parallel conductors are strung at the corners of an 18-inch equilateral triangle. At a given instant conductor A is carrying 100 amperes in a given direction, and B and C are carrying 50 amperes each in the opposite direction. What is the amount and direction of the magnetic force exerted at this instant on 100 feet of each conductor?

Prob. 28–11. If the three conductors of Prob. 27–11 had been strung in one plane and the outside wires spaced 18 inches from the center wire, what would be the answers to that problem, if

(*a*) Conductor A were an outside wire?

(*b*) Conductor A were the middle wire?

CHAPTER XII

CONDUCTION THROUGH GASES

In the preceding chapters we have studied metallic conduction. This includes nearly all cases of the conduction of electricity where Ohm's law is obeyed. We have seen that in this case every element of a circuit has a certain definite resistance which can be expressed numerically. These resistances may be combined in series and in parallel in order to solve networks of circuits. Where several sources of electromotive force are involved, Kirchhoff's laws may be applied in order to arrive at the solution.

Metallic conduction, we saw, takes place by reason of the movement of free electrons through the body of a metal. These electrons in large numbers are present in the material and free to move through it, although the molecules of the material are fixed in position. Upon the application of an electromotive force, however small, the electrons wander towards the region of lower potential and thus constitute an electric current. They are confined in their motion entirely to the body of the wire. The attraction of the metallic molecules for the electrons is so great that the electrons are forced to remain in the material and cannot leave it. In their progress down the wire, they collide with the metallic molecules and impart some of their acquired energy to the latter. This results in a heating of the conductor and a friction to the motion of the electrons which we know as the resistance that the conductor offers to a flow of current. This resistance for a given conductor, we have seen, is a constant so long as the temperature remains fixed. It is true that the resistivity of the material depends upon the temperature at which it is used,

but temperature changes when they occur are comparatively slow, so that the flow of the current at any time can readily be computed by using the resistances of the several parts of the circuit at that particular instant.

Electric circuits are generally constructed of metal wires, so that the above type of conductor is most important from a practical standpoint. There are, however, a large number of devices of various sorts which utilize conduction of a different nature. Such conduction is not in accordance with Ohm's law. This law and the other laws of circuits cannot be applied in such exceptions without several modifications. It is therefore necessary that we make special study of apparatus to which Ohm's law does not apply.

107. Non-Metallic Conduction of Electricity. One form of non-metallic conduction we have studied under the heading of electrolytic conduction. This we have seen to consist of the movement of charged ions through the body of a liquid. Conduction through an electrolyte is accompanied by a movement of the molecules themselves, and hence is in distinction to conduction through a metal where the molecules of the material itself are fixed in position.

Another type of non-metallic conduction which is of great importance is the conduction through gases. The laws which govern conduction through gases are at the basis of the construction of mercury-arc rectifiers, arc lights, some types of lightning arresters, X-ray tubes and a multitude of other devices. They also govern the corona loss from transmission lines and much of the behavior of insulations.

In addition to gaseous conduction proper, there is a form of conduction depending upon thermionic emission. This is at the basis of the thermionic tube, which, not to mention its use in radio practice and other places, has an extensive use in telephony as a repeater. There are also in use several devices which do not obey Ohm's law, the action of which cannot be explained as simply electrolytic or gaseous in nature Among these phenomena may be

mentioned the conduction between a metal and a crystal surface, where the crystal is conducting. This is the crystal rectifier used in radio telegraphy. However, this chapter will be confined to thermionic conduction and conduction through gases.

108. Thermionic Conduction. In the early days of the electric light, it was discovered by Edison that if a cold metal plate was placed in an electric light bulb, a current could be passed between the plate and the heated filament if a potential was applied between them. In fact, he found that while a current could be made to flow through the vacuum in the direction from the plate to the filament, no current whatever would flow in the opposite direction, regardless of how great potential was applied, up to the breakdown point. This device, then, was a rectifier which would change alternating into direct current. Its current capacity, that is, the amount of current which could be passed through it, was very small, and hence it did not find a use as a power rectifier. Fleming, however, discovered that the rectifying properties of the device were still present at very high frequencies. He accordingly made use of this device as a thermionic valve to rectify the incoming high-frequency current of a radio-telegraph signal, and thereby to change it into direct current so that it could affect a receiving instrument. This device then became the most sensitive detector for radio telegraphy then available.

The current between the filament and the plate is evidently not conducted in any ordinary manner. Experiment shows that the conduction of the device is governed by unique laws. If the potential between filament and plate of a tube constructed and connected as shown in Fig. 194 is varied, the current, measured by an ammeter at A, will vary in accordance with a characteristic curve such as is shown in Fig. 195.

The device evidently does not obey Ohm's law. If it did, its characteristic curve would be a straight line. It

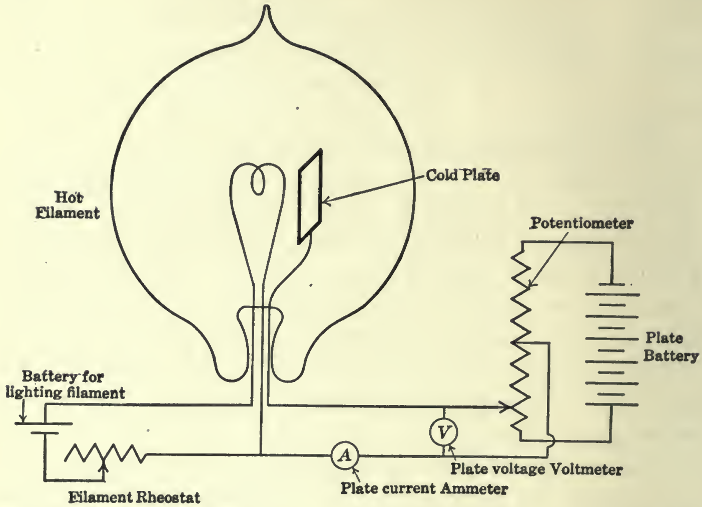

FIG. 194. When the filament is heated and an electric potential is applied between the filament and the cold plate, electrons pass from the filament to the plate.

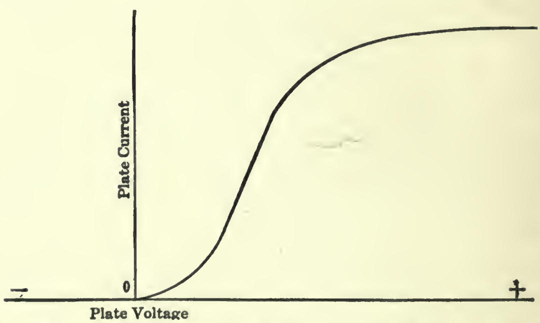

FIG. 195. The curve shows the relation existing between plate potential and plate current if the filament temperature of Fig. 194 is maintained constant.

does not have a constant resistance, but its resistance varies according to the current passing through the device. A study of this characteristic curve shows several things. In the first place, there is a current which flows when the plate is positive, but no current when it is negative. Second, the current rises slowly at first and then more rapidly as the voltage is increased, and finally reaches a maximum or saturation value which is not increased no matter how high the potential is made. The curve shown in Fig. 195 is for one given value of filament current. If the filament current is adjusted to several successive values, a series of curves can be obtained as shown in Fig. 196, each curve

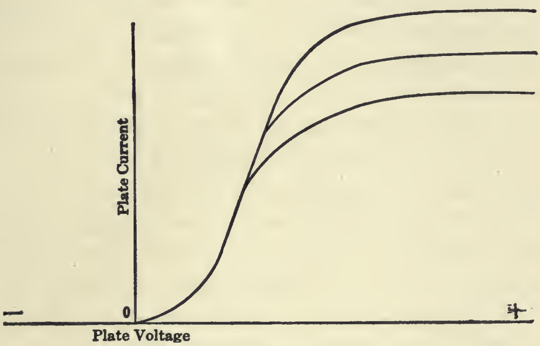

FIG. 196. The curves show the relations between plate current and plate voltage for different temperatures; the higher the temperature, the higher the saturation plate current.

being for a different value of filament temperature. It will be noted that the curves for different values of filament current coincide for low values of plate voltage, but that the saturation value of current reached in each case is different, and is higher the higher the filament current and hence the filament temperature.

109. Richardson's Law of Thermionic Emission. The behavior of the above device may be explained as follows.

As the metal of the filament is heated, the free electrons, which are in the body of the metal, move faster by reason of their heat velocity. In fact, if the temperature is sufficiently increased, some of the electrons will move so rapidly that they will escape from the body of the metal and fly out into space. This effect is exactly analogous to the evaporation of a liquid. When a liquid is heated, some of the molecules by reason of their heat motion acquire enough speed to be able to escape from the surface. In exactly the same mannei, we may evaporate electrons from an incandescent filament.

If the filament is made negative with respect to the cold plate, these electrons are repelled by the filament and attracted by the plate. Some of them accordingly move across to the plate. This constitutes an electric current which is measured by the ammeter. The direction of this current we take in accordance with convention to be in the opposite direction to that in which the electrons actually move. This, as we have seen, is because of the fact that due to a nomenclature which was adopted before the electron theory was known, the charge on an electron is found to be negative. Of course it makes no difference in which direction we draw the arrow indicating the direction of current provided we are consistent. The motion of the electron, however, is always from the hot filament to the cold plate, and never in the other direction; for the cold plate does not give out any electrons by which a current could be set up in the opposite direction.

When the plate voltage is small, only a few of the electrons evaporated from the filament move over to the plate, the remainder returning to the filament. As the voltage is increased, however, larger and larger numbers of the electrons move over to the plate; that is, the current increases. If the potential between filament and plate is sufficiently increased, all of the electrons will pass over to the plate as fast as they are evaporated from the filament.

Then no matter how much the potential increases beyond this, the current cannot increase, for saturation has been reached. This accounts for the saturation value of current which is shown on the curve of Figure 195, and which is the current which cannot be exceeded in the device at the given value of filament current. If the filament current is increased, the temperature is, of course, increased and hence a larger number of electrons per second is evaporated. Then the maximum current which can be passed through the device is also increased, or the saturation current becomes greater. This gives rise to the series of curves shown in Figure 196.

The evaporation of electrons described above is known as **thermionic emission,** and a device which utilizes such an effect is a **thermionic valve.** It is **not gaseous** conduction, for there is no gas which plays a part in the device. In fact, the more completely the bulb is exhausted of air, the better the device works. Thermionic tubes are accordingly freed as far as possible from gas; that is, they operate in a hard vacuum. The conduction is hence across an evacuated space by reason of electrons which are evaporated from the filament and which fly from the filament to the plate through empty space.

The law which governs the number of electrons evaporated from a filament was discovered by Richardson. It is of exactly the same form as the law that governs evaporation from a liquid and is therefore exponential. The number of electrons evaporated can be measured by determining the saturation current of the device. In this form, Richardson's law may be expressed thus:

$$i_s = a\, T^{\frac{1}{2}} \epsilon^{-\frac{K}{T}}, \tag{1}$$

where

i_s is the saturation current in amperes,

T is the temperature of the filament in centigrade degrees absolute,

a is a constant depending upon the length, diameter and material of the filament,

K is a constant depending upon the material of the filament only.

For drawn tungsten wire, the value of K is almost exactly 52,500. For tungsten also we find that a has the value approximately 23.6×10^6 times the surface of the filament in square centimeters. In utilizing the formula, we must be careful to note that it applies only to the portion of the filament which is actually at the temperature T, and that the ends which are cooled by the support will emit only to a lesser degree.

Example 1. As an example, if we have a filament 5 centimeters long and of 0.05 millimeter diameter, of tungsten, which is at a temperature of 2200 degrees absolute, the saturation current, that is, the maximum current which can be passed through a valve with the filament at this temperature, will be

$$i_s = 23.6 \times 10^6 \times 5 \times \pi \times 0.005 \sqrt{2200}\epsilon^{-52500/2200}$$
$$= 0.004 \text{ ampere.}$$

According to Richardson's formula, the filament will emit somewhat at any temperature. At ordinary temperatures, however, the emission is very slight. This can be checked by computing the current from the above filament at a temperature of say 100 degrees centigrade, that is, 373 degrees absolute. The current will be

$$i_s = 23.6 \times 10^6 \times 5 \times \pi \times 0.005 \sqrt{373}\,\epsilon^{-52500/373}$$
$$= 4 \times 10^{-53} \text{ampere}$$

which, it will be seen, is an exceedingly small current and quite negligible.

The electrons are speeded up by the action of the plate voltage and impinge upon the plate at high velocity. This naturally heats the plate, and if a sufficient current passes through the device at a high voltage drop, the plate will become very hot. Of course, it cannot be allowed to become so hot that it will also emit electrons to any great extent, for

in that case the device will cease to rectify. The loss in the plate circuit is, of course, equal to the current multiplied by the drop of potential in the tube. This loss in watts all appears as heat in the plate. In addition there is an expenditure of energy required to keep the filament hot.

The thermionic rectifier is beginning to be of importance in other fields than that of radio communication. In the next decade it will undoubtedly be used to considerable extent even in power work. Its current-carrying capacity is somewhat strictly limited, but the voltages it can handle are high. Even a current of 1 ampere at a potential of 100,000 volts represents a power of 100 kilowatts, and a tube which can rectify this amount is not a toy. In connection with controls by means of grids or magnetic fields it will be used for circuit interruption and similar service.*

Prob. 1–12. A thermionic valve is constructed with a filament 2 inches long and 0.005 inch in diameter. At a filament current of 1.1 amperes this filament is at a temperature of 2100 degrees absolute. A length of $\frac{1}{8}$ inch on each end of the filament is cooled by the support and may be considered not to emit. A voltage of 1000 volts applied between the filament and the plate causes the saturation current to flow. Compute the value of this saturation current and the watts transformed into heat in the plate.

Prob. 2–12. After the device in Prob. 1–12 has been operated steadily for a time, the plate is observed to come to a temperature of 400 degrees centigrade. Assuming that the heat dissipation from the plate is proportional to the fourth power of the absolute temperature, compute the temperature to which the plate will come if the filament temperature is increased to 2200 degrees absolute, the voltage across the tube remaining unchanged and being sufficient to produce saturation.

Prob. 3-12. If the mean temperature coefficient of tungsten is taken as 0.006, what filament potential will be necessary to produce 1.2 amperes in the filament of Prob. 1–12? Assume 800° C. as the temperature of $\frac{1}{8}$ in. at each end of filament.

* See prediction in paper by A. W. Hull, Journal A. I. E. E., 1921.

Prob. 4–12. Will the current be the same at all points along the filament of Prob. 1–12? If not, by what percentage will it vary from a mean value, assuming the filament to emit uniformly? What effect will this have on the behavior of the valve?

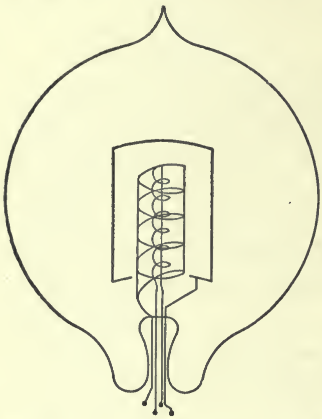

Fig. 197. A thermionic tube with a grid introduced between the filament and the cold plate.

110. Thermionic Amplifiers and Oscillators. De Forest placed a third electrode in a thermionic tube and obtained remarkable results. This third electrode is in the form of a grid located between the filament and the plate as shown in Fig. 197. The development of this idea by a large number of investigators has made possible present-day long-distance telephony. The repeaters used on the long lines of the American Telephone and Telegraph Company are now nearly all of this type. One of the tubes is shown in Fig. 198.

When a voltage is established between filament and plate in a three-electrode tube, the electrons in their flight leave the hot electrode, pass between the meshes of the grid and finally arrive at the plate. A small change of potential between the grid and filament will then greatly affect the electrons' flow. This is shown in the curves of Fig. 199, which give grid characteristics for a common type of amplifier tube plotted for different values of plate voltage. It will be noted that a small change in grid voltage may be made to cause a large change in plate current. If the grid is negative, as may be made the case by connecting in a small grid battery as shown in Fig. 200, there will be no grid current whatever, for the grid will at all times repel the electrons.

The input to the grid circuit can hence be made very small. We can thus control a large amount of output energy by an

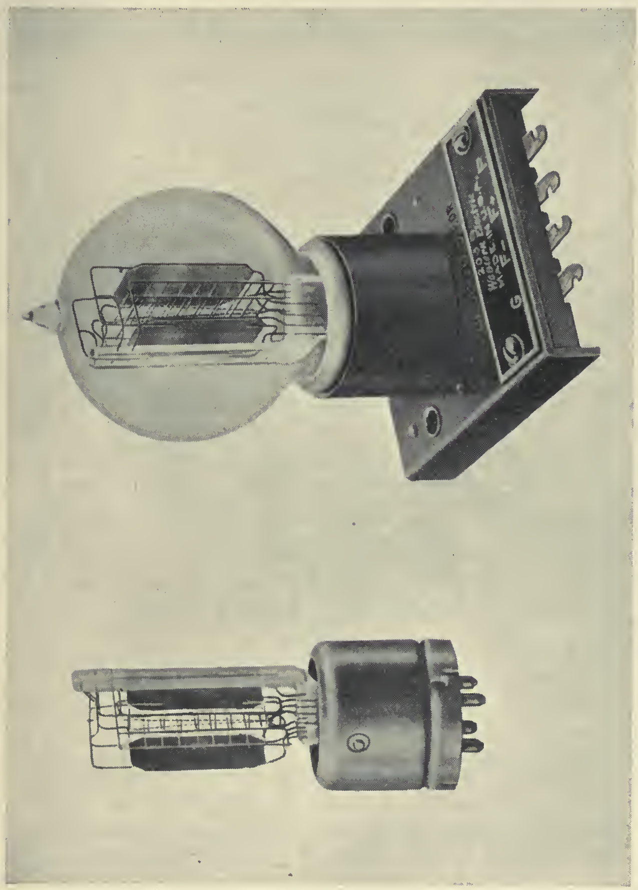

FIG. 198. A thermionic tube used as a telephone repeater on long-distance lines. *Western Electric Co.*

expenditure of practically no energy at all. Any device which in this way enables a small amount of input power to vary a large amount of output power is an **amplifier.**

It will be noted in Fig. 199 that for a certain range of grid potential, the characteristic is really a straight line.

If we operate the tube over this range only, the plate current will vary in linear relation to the grid potential. This is called distortionless amplification.

Amplifiers are used for very many purposes. Principal

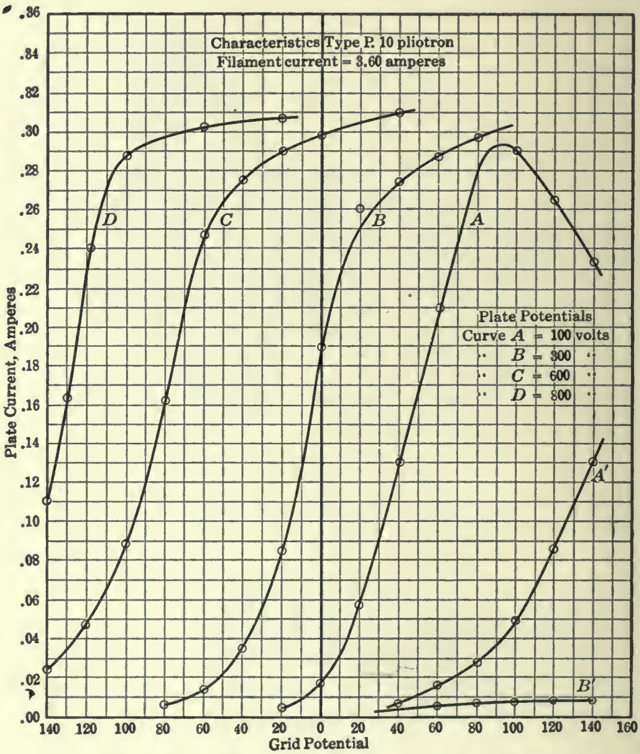

FIG. 199. The relation between grid potential and plate current. *From Morecroft's, "Principles of Radio Communication".*

among these is to amplify telephone currents. A telephone current is usually a current which is varying in accordance with the speech of the user. It is an alternating current of rapidly varying frequency and wave form. Cause this varying current to impress an exactly similar varying volt-

age upon the grid of an amplifying tube adjusted to the straight portion of its characteristic. The result is that the plate current will vary in just the way that the input does but its variation will be of much greater amplitude. A weak current may thus be amplified almost indefinitely.

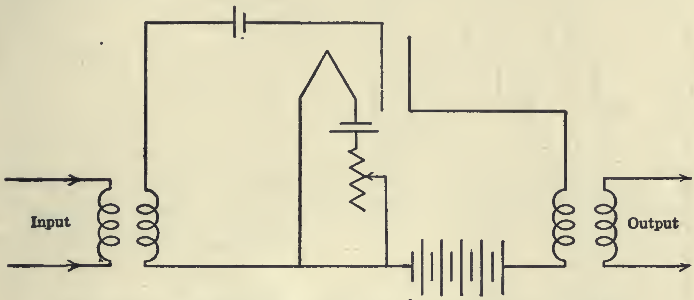

FIG. 200. The thermionic tube connected as an amplifier.

The action of the oscillator follows from the above. In fact, any amplifier of energy may be thus used as an oscillator, by deflecting part of its output back on its input circuit. Suppose, for example, that we have a thermionic tube with an input of 0.01 watt at 1000 cycles and an output of 10 watts. We may take a small part, about 0.01 watt, of this large output, and convert it by means of transformers so that it becomes input to the tube. We may then dispense with any other input, and the tube will continue to operate or oscillate. In order that the frequency may remain at 1000 cycles, it will be necessary to supply an oscillating circuit consisting of an inductance and a capacity, the natural frequency of which is 1000 cycles. One circuit by which this may be accomplished is shown in Fig. 201.

111. X-Ray Tubes. An electron is a negative charge of electricity. When situated between two plates which are of different polarities, it is attracted by one and repelled by the other. Accordingly it is set in motion; that is, it receives acceleration. The acceleration which it receives is pro-

portional to the force on the electron, and this is equal to the charge of an electron multiplied by the potential gradient at the point in question. By potential gradient we mean the volts per centimeter which exist at the point in question. It is measured in abvolts per centimeter, if we are working in the c.g.s. system. If the electric field is uniform, the potential gradient is equal to the voltage between the plates divided by the distance between them. That is, if 1000 volts exist between plates which are three centimeters apart, we have a potential gradient between them of 333 volts per centimeter. If the field is not uniform, the gradient varies from point to point.

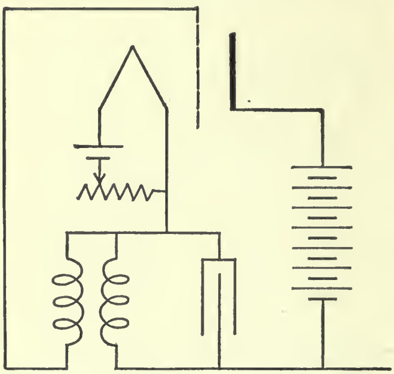

FIG. 201. The thermionic tube as an oscillator.

When we have, in an evacuated device, a heated filament and a cold plate between which electrons pass, the speed which the electrons attain may be computed in the following manner. In passing between filament and plate, their change of potential energy is, of course, the integral of force times the distance between the plate and the filament. This will be equal to the kinetic energy which they acquire. This gives rise to the equation

$$\int f dx = \frac{mv^2}{2}. \qquad (2)$$

If X is the potential gradient and q is the charge on the electron, then

$$f = Xq \qquad (3)$$

and

$$\int f dx = \int qX dx.$$

But $\int X dx$ is simply the voltage between filament and plate. Accordingly

$$Eq = \frac{mv^2}{2} \tag{4}$$

or

$$v = \sqrt{2E\frac{q}{m}}. \tag{5}$$

This equation gives us a means of finding the velocity which an electron will acquire in falling through a given difference of potential. The quantity

$$\frac{q}{m}, \tag{6}$$

which occurs in this equation, is the ratio of the charge to the mass of an electron. This constant is of fundamental importance and has been determined by a large number of different methods of measurement. Its value when q is in electromagnetic units and m in grams is approximately

$$1.77 \times 10^7.$$

Let us compute an example. Suppose that there is a potential of 1000 volts between filament and plate; that is,

$$E = 10^{11} \text{ abvolts.}$$

Substituting in our equation above, we obtain for the velocity

$$\begin{aligned} v &= \sqrt{2 \times 10^{11} \times 1.77 \times 10^7} \\ &= 1.9 \times 10^9 \text{ centimeters per second.} \end{aligned}$$

This is indeed a very high velocity. It is more striking if we convert it into the value

$$v = 12{,}000 \text{ miles per second.}$$

It is naturally to be expected that an electron moving with this velocity will produce a peculiar effect when it suddenly

strikes the hard surface of the metal. This is in fact the case, and it has been found that when electrons of very high velocity, corresponding to a potential drop of 10,000 to 50,000 volts, impinge upon the metal plate, the plate gives out a form of light of very short wave length which penetrates even very dense, opaque objects. These rays are known as X-rays and are of very great service in medical practice and in various other connections.

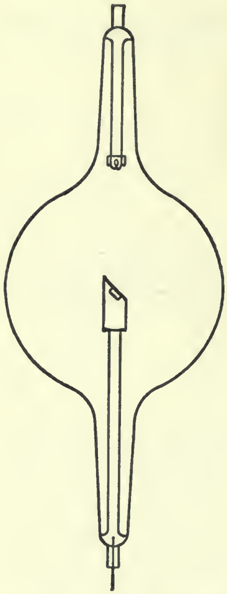

Fig. 202. A Coolidge X-ray tube.

The form of X-ray tube employing a heated filament and a cold plate or target is known as the Coolidge X-ray tube. It is arranged as shown in Fig. 202.

In order to intensify the effect of such a tube, an arrangement is used whereby the electrons are focused on a small spot on the target. This is accomplished by means of a shield placed around the filament and connected to the filament. The shield as well as the filament repels the electrons when they are first emitted and when they are moving at low velocity. By properly shaping this shield, they are so directed during the initial stages of their flight that they are aimed toward a certain spot on the target. After they have acquired a high speed, it is, of course, considerably more difficult to deflect them. The direction which they take is therefore determined almost entirely during the first part of their flight.

A Coolidge X-ray tube may employ potentials as high as 100,000 volts. Either an alternating or a direct potential may be used. When an alternating potential is used, the tube itself does the necessary rectifying.

The electrons in their motion are, of course, invisible. So, for that matter, are the X-rays. However, when electrons or ions impinge upon the glass of the tube, they cause it to fluoresce.* When an X-ray tube is in operation, a hemisphere of the bulb is accordingly fluorescent with a greenish light. This hemisphere is roughly bounded by the plane of the target extended. The X-rays, of course, are emitted from the target in all directions, and not focused as is the stream of electrons.

A Coolidge X-ray tube operates in a hard vacuum; that is, all of the gas, so far as possible, is exhausted from the bulb. The presence of even a very minute amount of gas will prevent the proper operation of the device.

A molecule consists, as we know, of a positive nucleus surrounded by a number of electrons more or less securely attached to the molecule. When an electron at high velocity encounters such a system, it sometimes knocks off one or more electrons. This process, we have seen in Chapter V, is called ionization. The electrons thus produced, together with the original electrons, proceed together to the positive plate. The remainder of the molecule, now lacking an electron, and therefore positively charged, is propelled in the opposite direction and hits the filament. The movement of these extra electrons and positive ions is also a movement of electricity and constitutes an electric current. Hence one effect produced when there is ionization in a thermionic device is an increase in current through the tube.

There is another important effect, however. The positive ions are large and comparatively heavy. When they impinge upon a metal surface, it is possible to cause them to emit electrons by the process of knocking them bodily out of the surface of the metal. It is thus possible to get conduction through a tube without utilizing a heated filament.

* A body is said to fluoresce when it throws out light waves of wave lengths different from any which it is receiving.

In fact, it was in this manner that X-ray tubes were first constructed. A small amount of gas was allowed to remain in the device. In any gas there are always a number of free electrons. The number may be as high as 1,000 per cubic centimeter for air in a normal state. This is, of course, very small when compared with the total number of molecules present. When a potential is applied between two cold electrodes in a tube containing a small amount of gas, these free electrons are speeded up and fly toward the positive electrode. In their path they encounter molecules and ionize them. The positive ions thus produced impinge upon the negative electrode and cause it to emit electrons. These in their turn proceed on their way and produce more ions. Thus the process is continuous.

The electrons leaving the surface of the negative electrode or cathode fly off in a direction almost perpendicular to the surface. By making the cathode a section of a spherical surface with the target at the center of the sphere, the electrons may be concentrated, and such of them as do not encounter molecules on their way will impinge upon the target within a comparatively small area. This is the older type of X-ray tube.

An X-ray tube of this sort is very sensitive to gas pressure. If there is too little gas present, no current at all can be passed, for in order that it shall be possible to pass a current, the process of ionization must be cumulative; that is, more ions must be formed by collision than are lost by absorption into the electrode. On the other hand, if the gas pressure is too high, it will be possible to have a current but nearly all of the electrons which leave the cathode will hit molecules before they arrive at the target. There are simply too many molecules in the way. Another way of saying the same thing is that the mean free path of the electrons in the gas must be of about the same order of magnitude as the distance between the cathode and the target. By the mean free path is meant the average distance which

the electrons will fly in a straight line before encountering molecules. If the gas pressure is high and there is a large number of molecules present, the mean free path will be short. In such a case almost all the electrons leaving the cathode will encounter molecules before they have proceeded far. Very few of them indeed will fly straight from cathode to anode and impinge upon the target. It is this latter action which we have seen to be necessary for the proper production of X-rays. The pressure in a gaseous X-ray tube must hence be very carefully adjusted to its proper value. This critical adjustment is very difficult to make correctly, and this fact has rendered the gaseous type of X-ray tube almost obsolete.

Prob. 5–12. The mass of an electron is approximately 9×10^{-28} gram. If we have an X-ray tube operating with a potential of 10,000 volts between filament and plate, what energy will be possessed by each electron when it arrives at the plate? How fast will it be moving, if we assume the mass to be constant? If a current of 100 milliamperes is being passed through the tube, how much heat is being produced at the target? Assuming the target to be of copper and weighing 250 grams, how fast will its temperature rise when the tube is first started?

Prob. 6–12. How many electrons per second are passing across between filament and plate in the tube of Prob. 5–12?

112. The Gaseous Discharge Tube. In a gaseous X-ray tube, the mean free path of an electron in the residual gas is of about the order of magnitude of the dimensions of the tube. This means that of the electrons which pass between filament and target, only a fair percentage collide with molecules on the way and ionize. The potential across the tube necessary to pass even a very small current is accordingly correspondingly high.

If we let more gas into a discharge tube of this sort, the mean free path of the electrons is decreased; that is, an electron will travel much less distance on the average before encountering a molecule and ionizing it. It is therefore much easier

to pass a current through the tube, because the potential drop in the tube for a given current will be less. In fact, if a sufficient amount of gas is present, considerable current may be passed through the tube with a drop of only two or three hundred volts. The current which can be passed is therefore determined only by the capacity of the tube, that is, by the amount of loss that it can stand without becoming too hot. When the pressure is about one millimeter of mercury, the tube then containing gas of an amount approximately one-seven-hundred-and-sixtieth of the amount which would be present at atmospheric pressure, conduction takes place very freely. There are so many molecules present that an electron does not proceed far on its way before it encounters one of them and ionizes it. The mean free path under these conditions for an electron in hydrogen is about 0.7 millimeter.

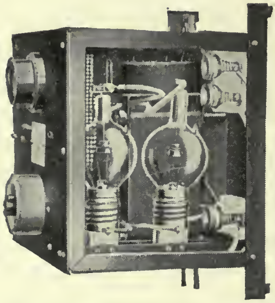

Fig. 202*a*. The tungar apparatus for rectifying small alternating currents. *General Electric Co.*

Fig. 202*b*. The tungar, a tube which operates as a rectifier. *General Electric Co.*

When a small amount of gas is admitted to a thermionic tube, the current that it can carry is greatly increased. Such a tube may be used as a rectifier for charging storage batteries from an alternating potential source. The "tungar," a tube of this type, shown in Fig. 202*a* and 202*b*, can carry several amperes with a drop of only a few volts. It cannot, however, rectify high potentials.

Gaseous conduction is accompanied by a brilliant glow in the gas. Every time a molecule is ionized, it gives out a spurt of light. This light is of a certain definite color dependent upon the nature of the molecule. If the glow between electrodes in such a device is examined with a spectroscope, it will be found to consist entirely of certain definite colors which appear as lines in the spectroscope. In this way gases can easily be identified and studied.

The discharge through the tube is often very beautiful and characteristic in its nature. It is divided into several different parts, and the phenomena which take place within the tube are quite complicated when examined completely.

Near the cathode is a dark space, called the cathode dark space. The reason for the existence of a region close to the cathode where no ionization occurs is readily apparent. The electrons, as they leave the cathode, must travel a certain distance before they acquire sufficient velocity to enable them to ionize. The velocity which they must attain depends somewhat upon the gas which is present, but in general they must fall through a potential difference of about twenty volts in order to acquire the necessary speed. Also, after they have acquired this speed, they will proceed a certain further distance (on the average) before they hit a molecule. There is a region near the cathode, therefore, where the electrons are coming up to speed and in which no glow is visible. Directly on the surface of the cathode there is a velvety light which is probably caused by the impact of the positive ions as they knock electrons out of the surface of the metal. At the end of the cathode dark space, there is a long beam of light which extends fully or almost to the anode. This is called the positive column. By varying the pressure and the current through the device, the form of the discharge may be varied within quite wide limits.

Discharge tubes of this sort are being used somewhat for lighting purposes at the present time. Such a tube is the so-called Moore light. There are other forms which are

just coming on the market, and which are made small enough to be screwed into a lamp socket. Such a light at the present time is not highly efficient, but it may possibly be developed considerably further and become a very important factor in lighting. It has the advantage that the color of the light given off may be adjusted almost at will by properly choosing the gas which is utilized in the tube and by varying the electrical conditions under which the tube operates. Another advantage is that there is no filament in the light to be burned out and thus shorten its life. At the present time it is not particularly efficient in terms of candlepower per watt. The radiation given off from the ionized molecules can be almost entirely in the visible spectrum; that is, it can be light accompanied by little heat. However, there is of necessity a certain amount of energy used in heating the electrodes, and this energy is lost. If all of the energy could be put into the glow itself, we should have a very efficient light indeed. Even all of the energy put into the glow does not appear as light, for some of it is used simply in heating the gas in the tube.

It has been noted that as the pressure is increased in the tube, the drop in the tube decreases. With a hard vacuum, we can, of course, pass no current through the tube unless a heated filament is present, for there are no electrons to conduct the current. When a little gas is admitted, such as in an X-ray tube, a small amount of current may be passed by a very high potential indeed. Admitting more and more gas lowers the potential drop by making it easier for the discharge to become cumulative. What is meant by this expression will be explained somewhat in detail, for it is very important.

Suppose that there is a discharge tube consisting of two electrodes in a gas, with a certain potential between the electrodes and a certain current flowing between them. Suppose that an electron starts from the cathode. If it produces the ionization of a molecule on the way, the two electrons will proceed together toward the anode, the positive

ion produced will impinge upon the cathode, and possibly collide with several molecules on its way to the cathode, and upon striking the metal surface probably knock out other electrons. The process is then continued. If each electron on its way produces on the average sufficient ions to start another fresh electron from the cathode, then the process can proceed. Ions are, however, lost in many ways. Some of them recombine by meeting free electrons, and hence becoming neutral molecules. Some of them, by reason of much jolting as they pass through the gas, lose so much energy that they cannot knock electrons off the cathode when they reach it. When the discharge is cumulative, we mean that more and more ionization tends to be produced in the body of the gas. Under these conditions, the potential across the device will fall if the current is held constant, until only sufficient ionization is produced to carry the current. On the other hand, if an insufficient amount of ionization is produced to carry the current, the potential will necessarily rise until the electrons are speeded up and sufficient ionization in the body of the gas is maintained.

In order to produce cumulative ionization we must have, first, sufficient potential to bring electrons up to ionizing speed, and second, ample opportunity for these high-speed electrons to collide with neutral molecules. In fact, if electrodes are sufficiently closely spaced, we may have the first of these conditions satisfied without the other. Thus a short gap in a gas may be made to be much harder to break down than a long one, provided all the discharge paths of the former are short. The recognition of this principle has recently enabled C. G. Smith to produce a gas-filled device which can rectify, oscillate and perform other similar functions.*

As the pressure of the gas is increased and the number of molecules between the electrodes goes up, it is easier to produce cumulative ionization, and accordingly the potential drop across the device for a given current will go down.

* See Bush and Smith, "A New Rectifier," Proc. I. R. E., Feb., 1922.

There is, however, a limit to this effect. If the pressure of the gas is sufficiently increased, the collisions of electrons with molecules will be so frequent that they will not have time to come up to ionizing velocity before impact. In other words, an electron as it flies between cathode and anode will be continually jostled by the molecules and will lose its energy. Between impacts with molecules it will speed up, but seldom come to so high a velocity that upon striking a molecule it can knock it apart. As the pressure is increased beyond a certain critical value, the conditions for ionization are therefore not so favorable, and the potential across the device will go up. The gas pressure which will give a minimum drop in the tube in most gases is one or two millimeters with ordinary electrode spacing. At higher gas pressures, the potential is determined by somewhat different conditions than it is at pressures below this value. We have seen that below this critical value of pressure, ionization occurs smoothly through the body of the gas, and the potential across the tube is regulated automatically to a constant value dependent upon the current. To state it another way, the device is stable. Above the critical pressure, however, we have no longer stability. No current at all will be passed until sufficient potential is reached so that somewhere in the body of the gas electrons are speeded up sufficiently between impact to produce ionization. That is, we must produce sufficient potential gradient at some point in the gas so that the electrons in falling through a distance equal to their mean free path will acquire an ionizing velocity. For gas at atmospheric pressure, this means that a potential gradient of about 30,000 volts per centimeter must be produced in order to start ionization.

When ionization is started, however, it is very rapidly cumulative indeed. There is plenty of gas present to ionize, and hence the potential across the space rapidly goes down and the current goes up. The discharge is therefore sudden and may be explosive. This is what we know as a spark.

It is usually accompanied by other effects which we will study in succeeding sections.

Prob. 7–12. If it requires a potential gradient of 30,000 volts per centimeter to break down air at atmospheric pressure, and if we assume that an electron speed of 2×10^8 centimeters per second is necessary in order to ionize, about what is the mean free path of an electron in air at atmospheric pressure?

Prob. 8–12. Why is it, under the conditions which are outlined in Prob. 7–12, that a spark discharge in air at atmospheric pressure is in a narrow path between electrodes, whereas a glow discharge at low pressure fills the entire body of the tube?

113. The Spark. The spark, we have seen, is a disruptive discharge. It has a steeply falling negative characteristic. This means that the greater the current, the less is the voltage drop in the spark. In fact, in air at atmospheric pressure it requires a very high potential to produce any appreciable current between separated electrodes, but when the air breaks down, by sufficient potential being impressed to give ionizing velocity to the electron, the potential drop is lowered to a comparatively small value. The passage of a spark is thus a sudden affair, and soon over, for the large current which can flow rapidly reaches the limit of the source. If there is much power behind the spark, as for instance when it is over the insulator of a high voltage transmission line, the spark may be followed by a destructive power arc as we shall see.

In order to produce a spark we must stress the air at some point to 30 kilovolts per centimeter. More exactly, this value varies directly with the density of the air, and is 30 kilovolts per centimeter under standard conditions of temperature and pressure, 25 degrees centigrade and 76 centimeters barometer height. The density δ at any other temperature t and barometer height b in centimeters is given by the formula

$$\delta = \frac{3.92\, b}{273 + t}.$$

If the potential is applied between parallel plates where the edge is protected in some manner, the critical potential will be reached when the voltage applied is 30,000 times the separation in centimeters. When this potential is reached, the air will break down suddenly all the way between the plates and a spark will pass.

With other shaped electrodes this simple rule will not, however, apply, for the potential gradient will not be uniform, and it is simply necessary to bring the air at **one point** up to the critical gradient.

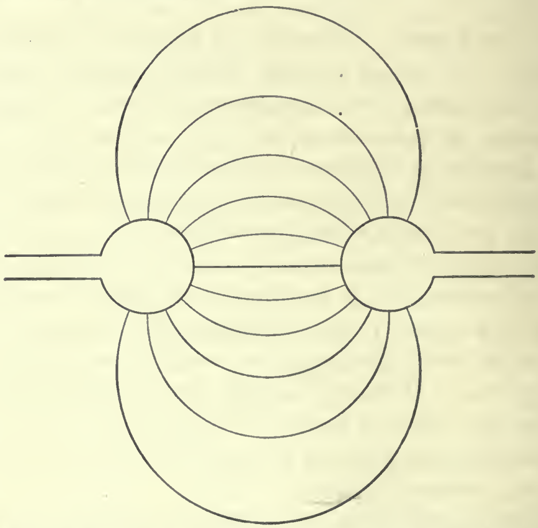

FIG. 203. The electrostatic field between two spherical electrodes.

An estimate of the factor introduced in this manner may be made by mapping the electrostatic field between the electrodes, that is, by drawing the lines of electrostatic stress. In Fig. 203 this field is shown for a pair of spherical electrodes, forming a sphere gap. We do not need to study the method of drawing these fields accurately. Their approximate form may readily be obtained as follows. The laws governing the distribution of electrostatic stress between electrodes are the same as those governing the distri-

bution of magnetic flux in an air gap. If we considered the spherical electrodes of Fig. 203 as the iron portions of a magnetic circuit and mapped the magnetic field in the gap, we should obtain just the same picture.

The density of the lines is proportional to the potential gradient. The average potential gradient is the total voltage divided by the separation. The ratio of the average to the maximum density of the lines gives a factor to apply to the gap in obtaining its critical potential. Thus if this ratio is 0.6 and the gap is two centimeters long, it will require

$$0.6 \times 2 \times 30 = 36 \text{ kilovolts}$$

to break the gap down. When this voltage across the gap is reached, the air close to the electrodes will ionize or break down. This layer of air close to the electrodes then becomes conducting so that the full voltage is applied across the remainder of the space. This raises the gradient on the next layer of air, which in turn breaks down, and so on. Thus as soon as the gradient in one part of the gap is reached, a spark passes completely across.

The more the lines of electrostatic stress are crowded together, the easier will a gap break down. Small balls with a given separation break down at a smaller potential than large balls.

The Standardization Rules of the American Institute of Electrical Engineers give the spark gap settings, for example, for 110,000 volts break-down at 25° C. and 76 centimeters barometer

for 125 mm. spheres,	spacing		110 mm;	
" 250	"	"	90	";
" 500	"	"	83	".

A calibrated spark gap may be used for measuring very high voltages by noting the separation when the gap breaks down. For a given separation the easiest gap to break down is the needle gap, for the stress is highly concentrated at the point of the needle.

All of these matters pertain to a gradually applied voltage. Where a gap breaks down close to the electrode, and then spreads across as the stress is redistributed, it takes time for conduction to take place in the ionized region, and hence for the discharge to spread. For very suddenly applied potentials, therefore, the concentration of stress is relatively ineffective. For a sudden surge of voltage of very short duration, all gaps of the same separation will break down at the same voltage. This has much application in lightning-arrester design.

It should be noted that the distribution of electrostatic stress in a gap can be simply mapped only when no current is passing. For instance, the field between filament and plate of a conducting thermionic tube cannot be simply drawn. The presence of the electrons in the space between, that is, the **space-charge effect**,* causes the gradient to be modified and concentrates the stress near the positive electrode.

Similarly in a glow tube which is ionized, we may have a space charge. When a molecule is ionized, the electron and the positive ion both tend to move out of the field. If the gas is hydrogen, however, the ion weighs about 1700 times as much as the electron. The charge and hence the force on each, are the same. The electrons thus move more quickly and are sooner swept out of the ionized space. The preponderance of positive ions in this space near the anode brings a concentration of gradient nearer to the cathode. In a glow tube the greater part of the voltage drop will therefore be found to be within a short distance of the cathode. This effect is sometimes modified by negative ions which are formed as follows. An electron in its flight often attaches itself to a molecule and carries it along with it. This constitutes a negative ion, which is as heavy and moves as slowly as a positive ion, that is, as a molecule minus an electron.

Air on the surface of an insulator breaks down more easily

* See page 396 for further explanation of this effect.

than air in a space between separated electrodes. Greater creepage distance must therefore be allowed over surfaces in order to sustain a given voltage. This is probably due

FIG. 203*a*. A flash-over on a wet bushing during a test. A potential of 305,000 volts was being applied when this picture was taken. *The General Electric Co.*

in part to a surface layer of moisture on even a surface which appears dry. The appearance of such a flash-over on a wet bushing is shown in Fig. 203*a*

Prob. 9–12. A 125-millimeter diameter sphere is situated 55 millimeters above the floor, which is of metal. What potential will be necessary between sphere and floor to cause a spark to pass? Assume standard conditions.

Prob. 10–12. What potential will break down a ball gap of spheres of 25-centimeter diameter, spaced 9 centimeters, when the temperature is 15° C. and the barometer 70 centimeters?

Prob. 11–12. A transmission line carries power at 110 kilovolts from a station in the mountains to a city in the valley. If the same insulators are used throughout, where are they more likely to flash over? If the barometer in the mountains is 60 centimeters when it is 76 centimeters in the valley, how great a factor is thus introduced? If it is also colder in the mountains, how does this affect flash-overs?

114. Corona. It does not always follow that break down of air at one point between electrodes will result in a complete break down. When the layer of gas adjacent to the electrode is ionized, the full stress is brought to bear on the remaining space. Whether the stress on the next layer of gas of this remaining space is thus brought to the critical value depends upon the shape of the electrodes and their separation. With small electrodes far apart, it will often happen that in the new distribution of stress when the surface layer of gas is ionized, there will be no new point further out from the wire where the gradient is sufficiently raised to continue the spread of the discharge.

In such cases we have a brush discharge or a corona. Such a case occurs particularly on the wires of a high-tension transmission line. For the voltage at which many of these are used, the gradient at the surface of the wire is above the critical gradient, and yet the configuration is such that break down from wire to wire does not occur. In use each wire is surrounded by a luminous envelope of ionized air. The production of ions and the discharging on the wire constitute a leakage current from the wire. This occasions a loss in the transmission line. Due, however, to the great advantages of the use of high voltages, a small amount of corona loss can often be tolerated.

Prob. 12–12. The leakage current from each wire of a transmission line due to corona is one ampere, the line being 100 miles long. Assuming two wires only with 110,000 volts

applied, how much loss does this involve? The line current at full load is 100 amperes. The line wires have a resistance of 0.4 ohm per mile. Dropping the voltage to 88,000 volts would stop corona entirely. From the standpoint of efficiency of transmission, would it be advisable to make this change?

Prob. 13–12. With the corona of Prob. 12–12, how many ions per second are being discharged on each centimeter length of line wire, assuming each ion singly charged?

Prob. 14–12. How could the corona in Prob. 12–12 be avoided without decreasing the line voltage?

115. Arcs. It requires an enormous potential gradient to break down air at atmospheric pressure, but once the air is disrupted and a spark is passed, even a fairly small voltage can maintain a current across the gap.

The sudden explosive flow of current when air is broken down by a high potential gradient is called a spark. The steady current which follows at low potential drop is called an arc.

The arc has certain distinctive properties which cause it to differ from discharge through gas at low pressure. It is confined to a relatively narrow path between the electrodes instead of being uniformly spread out. It therefore produces an intense heat at a small point on the electrode. If the arc is long continued, this point is brought to incandescence and melted or even vaporized. When this occurs, the molecules of vapor are charged when they leave the electrode and assist in carrying the current. The positive ions when they bombard the incandescent metal cause it to emit electrons copiously. There is also a thermionic emission by which electrons are evaporated from the cathode surface. In the air space through which the arc passes, there is intense heat which renders the air molecules easily ionized. For all these reasons, an arc which has continued long enough to heat the electrodes is easily maintained and large amounts of current can pass at low voltage. Fifty volts will maintain an intense arc between carbon electrodes in air.

An arc has a negative characteristic. The higher the current passed, the lower is the drop between electrodes. The reason for this is easily seen. The heavier the current carried, the fatter the arc becomes. The resistance is therefore much less for a heavy current. In fact, the conductivity of the arc path increases at a greater rate than simple proportionality to the current.

There are many kinds of arcs. Arcs between carbon electrodes are much used for lighting. In such a case, most of the light comes from the incandescent electrodes themselves. The arc stream is relatively much less luminous. In certain arc lights, however, known as flaming arcs, the light from the arc stream itself is increased by impregnating the carbon electrodes with certain salts.

An arc light must be operated on a constant-current circuit or with a resistance in series for stabilizing. Due to its negative characteristic, an arc connected to a constant-potential circuit will be very unstable. The more current it takes, the less is its drop, and it hence acts like a complete short circuit. A resistance in series will give a rising characteristic for lamp and resistance and hence stabilize it. For a street-lighting circuit, arcs are connected in series and supplied by a transformer which automatically maintains a constant current, thus avoiding possible instability.

The mercury-arc lamp is also used for lighting. In this case the arc passes between a solid metal electrode and a pool of liquid mercury, or between two mercury pools. The gas present is simply the mercury vapor. To start the lamp, it is tilted until a stream of mercury passes over and touches the opposite electrode. When this stream breaks, the arc follows. Since the mercury vapor is at low pressure, the arc diffuses throughout the tube. It concentrates, however, at a small spot on the pool of mercury. The intense heat at this point, and the charged vapor and electrons thus produced, allow the passage of current at a low voltage.

In the mercury arc, there is an incandescent spot on the pool of mercury, but none on the other electrode. The electrons evaporated from this spot are the principal means by which the arc is sustained. Current can therefore pass in only one direction through a mercury arc between the mercury and a solid electrode. It may thus be used as a recti-

Fig. 204. Welding the ends of hot-rolled steel rings. *Westinghouse Electric and Mfg. Co.*

fier. A mercury-arc rectifier, for changing alternating into direct current, is quite similar in operation to the mercury arc used for lighting. It is designed, however, with a different purpose in view.

Arcs are also much used for welding metals. In the process which is most used for welding iron and steel as shown

in Fig. 204, one electrode is the material to be welded and the other is an iron rod held in the hand. The rod is touched to the work to start the arc, and then withdrawn slightly to form an arc about one-fourth of an inch long. Only 30 to 50 volts are required to maintain the arc. A current of 50 to 150 amperes is used, depending upon the nature of the work. The arc raises the material to be welded to fusion temperature. Small globules of the "pencil" or electrode held in the hand are dropped in molten condition into the weld. Considerable skill is required in order to produce uniform work.

Besides the useful applications of arcs, there are many places where they must be dealt with by the electrical engineer in connection with switching and line protection. Their nature should therefore be thoroughly understood.

It has been mentioned that an arc has a negative characteristic. In Fig. 205, page 430, is shown the characteristic for an arc in air between metal electrodes. It will be noted that as the current is decreased, a point is reached where the voltage rises very rapidly indeed. It requires a certain current to maintain an arc. If the current falls below this value, the arc will suddenly go out. The current necessary to maintain the arc depends upon its length, the electrode material, the electrode temperature and several other factors.

When a switch is opened in a power circuit, an arc follows the separation of the blades. If the potential of the source is high or if there is much inductance in the circuit, the arc may be drawn out very long, in fact, several feet on high-voltage power circuits. If the source is capable of maintaining the arc at its maximum length, it will continue until interrupted in some manner, or until it has fused and destroyed the switch. If the arc is drawn out, however, until the current passing becomes too small to maintain it, the arc will suddenly go out. It is this sudden extinguishing of the arc which produces a rapid change of current and a high potential when an inductive circuit is opened. Thus

when the field circuit of a large machine is suddenly opened by a switch, it is not the instant when the contacts separate that is dangerous, for a hundred volts or so will start and maintain the arc, but it is the instant when the arc breaks.

An arc stream which is situated in a magnetic field is forced sideways in exactly the same way that a wire carrying current would be acted upon. The cause of the action is the same in both cases, namely the tendency of a current-carrying conductor in a magnetic field to be deflected sideways. The direction in which the arc will move can be determined by the left-hand rule, just as in the case of a motor.

This principle is made use of in magnetic blowouts as applied to trolley-car controllers, contactors, etc. When the contact points separate, there is a magnetic field in the space between them which forces the arc out. This field is usually produced by a coil connected in series with the circuit to be opened. Then the heavier the current to be opened, the stronger will be the field acting to blow the arc. The movement of the arc in the field accomplishes two things. First, the arc is lengthened and moved out to where it will do no harm. A longer arc, we have seen, will break more easily. Second, it is moved over to a cool surface. It has been shown that one condition necessary for the maintenance of an arc at low voltage is an incandescent spot on the surface of the electrode. If the arc is moved off this spot, it will have a powerful tendency to go out. This second point is not always fully appreciated.

Prob. 15–12. Using the curve of Fig. 205, draw the characteristic for the combination of this arc in series with 10 ohms resistance. Will the combination be stable at 6 amperes? What is the apparent resistance of the combination at this current value? At 8 amperes?

Prob. 16–12. Make a sketch showing a switch with magnetic blowout, indicating the coil connections and the direction in which the arc will be blown. Will the action be correct if the current to be broken is reversed in direction?

SUMMARY OF CHAPTER XII

THE FLOW OF ELECTRICITY THROUGH GASES is not governed simply by Ohm's law.

BY THERMIONIC CONDUCTION is meant the conduction of electricity by electrons which are evaporated from a hot conductor. The hot conductor, together with a cold plate, is placed in a vacuum bulb and a voltage applied between the hot conductor and the cold plate.

RICHARDSON'S LAW. The amount of current which passes in a thermionic tube with the hot filament at a given temperature increases as the voltage between filament and plate increases up to a certain point, where saturation occurs. The saturation point depends upon the temperature of the hot filament. The saturation current is given by the equation

$$i_s = a\,T^{\frac{1}{2}}\,\epsilon^{-\frac{K}{T}}\ .$$

AN AMPLIFIER OR REPEATER is made by placing a grid between the hot filament and cold plate. The amount of current passed between the hot filament and cold plate is caused to vary through wide ranges by means of slight changes in the potential on the grid. Thus the small telephone currents may be made to cause the grid potential to vary and in this way control the flow of much larger currents in the circuit of the plate. The output current will thus vary in the same manner as the input current. This is the action of the telephone repeater.

X-RAYS ARE SHORT WAVES set up at the target of a bulb when electrons at very high velocity impinge upon the metal surface of the target. These X-rays can penetrate substances impervious to ordinary light waves.

THE VELOCITY OF ELECTRONS can be found from the following equation

$$v = \sqrt{2\,E\,\frac{q}{m}}\,.$$

This equation does not hold for velocities approaching that of light unless we consider the value of m to increase with the velocity.

CONDUCTION MAY TAKE PLACE IN A TUBE CONTAINING GAS at low pressure. Free electrons in the gas are set in motion by the electric force. These electrons hit molecules and ionize them by knocking electrons away from them. During ionization a molecule emits a spurt of light the color of which depends upon the nature of the molecules. The ions upon hitting the cathode knock other electrons from it, keeping up the supply of free electrons.

TO PRODUCE A CUMULATIVE DISCHARGE the electric potential must be sufficiently high to give the electrons enough acceleration and the **MEAN PATH** of the electron must be long enough to allow the electron to acquire sufficient velocity to break up a molecule with which it may collide and thus ionize it.

SPARKS ARE PRODUCED when the gas pressure is so great as to require a comparatively high potential to give ionizing velocity to the electrons. When the potential is reached at one spot of the gap, the air throughout the gap becomes suddenly ionized and a spark passes.

THE SPARKING DISTANCE in air depends upon the atmospheric pressure and the temperature as well as upon the shape of the terminals, pointed terminals requiring less potential.

CORONA IS PRODUCED when the ionization of one section of gas between electrodes does not result in raising the potential gradient of any other section to the ionizing point. This is often the condition on high-voltage transmission lines.

ARCS ARE CHARACTERIZED BY THE VAPORIZATION of electrodes and ionization of the vapor. Arcs differ from sparks in being confined to the comparatively narrow path of vapor flow between electrodes which may be solid, as in carbon-arc lamps, or liquid as in the mercury-arc lamp.

PROBLEMS ON CHAPTER XII

Prob. 17–12. In the Western Electric standard repeater, the filament has a coating of the oxides of barium and strontium. The area of the coating is approximately 95 square millimeters. The value of a in Richardson's equation is 2.0×10^5. At a temperature of 750° C. the saturation current is 0.04 ampere. What is the value of K in Richardson's equation for this substance?

Prob. 18–12. If the filament of the repeater in Prob. 17–12 is heated to 850° C., what will the saturation current become? One of the good features of the oxides composing the coating of the filament in these repeaters is the high current of saturation at comparatively low temperatures.

Prob. 19–12. Determine the path followed by a charged particle, moving with a velocity u in a magnetic field of strength B. The charge on the particle is q and the mass is m. The angle between the initial direction and that of the magnetic flux is 90°. Neglect the effect of gravity.

Prob. 20–12. The copper target of an X-ray tube weighs 200 grams. When the tube is first started the temperature of the target rises at the rate of 8° C. per second. If 10 milliamperes are being passed through the tube, how fast are the electrons moving when they arrive at the target? Assume the mass of the electron to be constant.

Prob. 21–12. A thermionic valve has been tested for its saturation current at two filament currents. The results obtained are as follows:

Filament current	Saturation current
0.90	0.0105
1.05	0.0455

The value of a in Richardson's equation for this tube is 6.0×10^4. Assuming that the heat radiated from the filament varies as the fourth power of the absolute temperature, and that the resistance of the filament is not appreciably changed by the change in temperature, determine the value of k in the saturation-current formula, and also determine the saturation current at a filament current of 1.1 amperes.

Prob. 22–12. Determine the absolute temperature of the filament in each of the cases above. Plot a curve showing the relation between filament current and saturation current for the region including the above two points. Plot another curve showing the relation between saturation current and absolute temperature for the same region. If necessary, compute a few other points, in order to get a good curve.

Prob. 23–12. If the temperature coefficient of resistance of a filament is taken as c, and the heat radiated from it is taken as k times the fourth power of the absolute temperature, derive equations for the following:

(a) relation between the filament current and the absolute temperature;

(b) relation between power input and resistance of filament.

(c) Plot a curve for each of the above cases on the assumption that $c = 0.009$ and $k = 5 \times 10^{-11}$.

Prob. 24–12. The saturation current of a tube maintains the plate at a constant temperature of 400° C., with a fall in potential between filament and plate of 180 volts. To what must the plate potential be increased if the temperature is to be raised to 650° C., the filament current remaining constant?

Prob. 25–12. The plate in the tube in Prob. 21–12 is made of the same material as the filament, and its area (on the filament side) is 100 times the filament area. The plate current bombardment maintains it at a temperature of 350° C. If the direction of the plate potential is now reversed, find the current which will flow from plate to filament. What temperature must the plate assume if this current is to be one-hundredth of the saturation current from plate to filament, when the filament current is 1.0 ampere?

Prob. 26–12. Using the mean free path of an electron in air at normal atmospheric pressure found in Prob. 7–12, determine the potential which must exist on the surface of a sphere to start ionization at its surface. Assume an ionization velocity of 2×10^8 centimeters per second.

Prob. 27–12. A transmission line is 50 miles long and has a resistance of 0.403 ohm per thousand feet. The maximum allowable carrying capacity of this wire is 50 amperes. At 90,000 volts at the receiving end, the corona loss on this line is 5 kw. per mile, and at 110,000 volts the loss is 35 kw. per mile. The corona loss can be assumed to vary according to the law

$$p = c(V - V_0)^2,$$

where V is the line voltage, and c is a constant, dependent on spacing, diameter of wire and atmospheric conditions. V_0 is the voltage at which the corona first appears. If power costs $2\frac{1}{2}$ cents per kilowatt-hour to generate and sells for 4 cents per kilowatt-hour, compute the theoretical voltage at which this line will operate when bringing the greatest income. Neglect the effect of line drop upon corona loss. (In practice the operating voltage would be determined by many other additional considerations.)

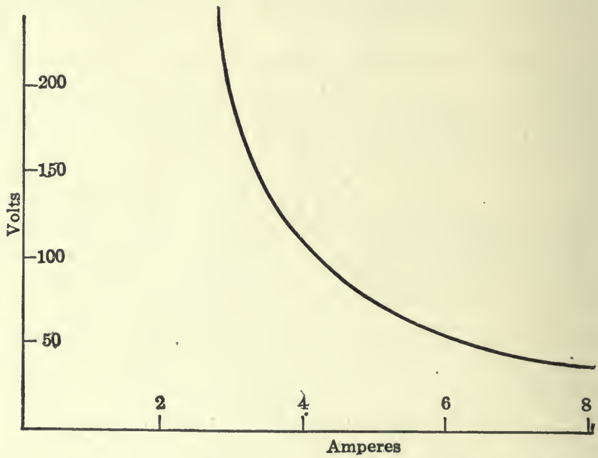

FIG. 205. Relation between current and voltage of an arc between metal electrodes.

Prob. 28–12. An arc between metal electrodes has the characteristic shown in Fig. 205.

(a) Plot a curve using as abscissas the values of current, and as ordinates, the values of the minimum resistance that must be placed in series with the arc to give stable operation.

(b) Plot a curve of voltage across both arc and resistance for the conditions in (a), using the same abscissas.

CHAPTER XIII

DIELECTRICS

An electric current flows whenever electrons are caused to move. When a voltage is applied to the ends of a wire, the free electrons in the conductor wander along the wire and there is metallic conduction. If the wire is broken and the two ends placed in an electrolyte, a current continues to flow. Electrons from the wire become attached to ions in the solution and move with them through the electrolyte. Even if there is an air gap in the wire, current can still flow through a spark or an arc if the potential is high enough, conduction in this case being due to electrons, gaseous ions and charged particles of metal which move from one electrode to the other. Thus in order to have a current, we must in all cases have electric charges free to move.

All substances are constructed of atoms, and all atoms consist of electrons attached in some mysterious manner to a charged nucleus. If the substance is a metal, the electrons are continually interchanging between atom and atom, and even a small force impressed upon them will cause a movement along the wire. If the substance is an insulator such as glass, however, there are about as many electrons present, but they are securely attached to the atoms and cannot get away under the action of moderate electric forces. The application of an electromotive force of small value to an insulator hence produces practically no steady current.

116. Dielectric Strength. A substance which will conduct practically no steady current of electricity at all under a moderate applied electromotive force is called an insulator or a dielectric. The principal insulators used in electrical work are: first, mica and the vitreous materials such as glass,

porcelain and so on; second, rubber and its derivatives; third, paper and moulded composition; and fourth, the oils and varnishes.

In a dielectric, we have said that the electrons are securely attached to the atoms and hence cannot flow along the material when a moderate electromotive force is applied. However, if the electromotive force is raised to a sufficient value, the electrons can possibly be torn loose and the material thus broken down or disrupted in exactly the same way that air is broken down. When an insulator is thus punctured, the electrons are torn forcibly loose from the atoms. The potential gradient necessary to accomplish this is very high and depends upon the material. The potential gradient necessary to disrupt an insulation is called its **dielectric strength.**

As in the case of air, it is not necessary that a high potential gradient be produced at all points in the body of the material in order to break down the insulator. It is sufficient that at one point the potential gradient shall exceed the dielectric strength. In such a case, this point will break down, thus bringing the whole potential to bear on the remaining insulation and so breaking it down in turn. Thus a given slab of insulating material may stand a certain voltage satisfactorily when it is applied between flat electrodes held against each surface, whereas it will soon break down if the same potential is applied between points on each side of the slab.

Glass is one of the best dielectrics from an electrical standpoint. It has a dielectric strength of about 100,000 volts per centimeter. Mechanically, it is, of course, open to considerable objection. Porcelain has nearly the same dielectric strength and is much stronger mechanically. For places where an insulator of small thickness is desired, mica is much used. Its dielectric strength is about one-half that of glass.

The dielectric strength of a material varies with a great

many factors. For instance, any such material as oil tends to absorb moisture, and the percentage of moisture content greatly affects the dielectric strength. This is particularly true of transformer oil. Even one-tenth of one percent of moisture dissolved in the oil will reduce the dielectric strength to less than ten percent of full value. The temperature also has a large effect. In general, the higher the temperature of the material, the less will be its dielectric strength. Finally, it should be mentioned that the length of time during which the voltage is applied to the specimen is a great factor, — certain substances that will stand up for one minute under a voltage of 125,000 volts per centimeter will break down under 80,000 volts per centimeter applied for a half hour.

In practice, in constructing insulators great care must be used that no air pockets are left in the material. If there is a bubble of air in a dielectric which is subjected to high stress, the air in the bubble may be broken down although the material itself is far below puncture voltage. In such a case, ionization produced in the bubble results in a great deal of heating. This heats the insulator locally and often causes it to break down where it would otherwise have been fully strong enough for the conditions at hand.

The proper design of an insulator brings in many factors. For instance, in designing an insulator for a transmission line, we must first construct it so that there is sufficient dielectric strength of the material to avoid puncture under the maximum voltage gradient which will occur. Second, it must have sufficient surface so that it will not break down between wire and ground over the surface of the material. This is the reason that insulators are constructed with a deeply corrugated surface or with petticoats, as shown in Fig. 206. Also these petticoats must be of such shape that when it rains, there will be enough surface of the insulator left dry to stand the applied potential. In addition, of course, the insulator must be strong enough to stand the mechanical stresses which are brought to bear

upon it, and these are often high when a heavy transmission wire is being supported. Finally, there must be no point close to the insulator at which the air is highly ionized, and no bubbles of air in the material of which it is constructed.

FIG. 206. An insulator having petticoats to increase the length of dry leakage path. *Westinghouse Electric & Manufacturing Co.*

Prob. 1–13. Bakelite prepared for the electrical trade may have a dielectric strength of 15,000 volts per millimeter. In a certain machine it was desired to replace some mica insulating sheets 1 millimeter thick with bakelite sheets. How thick should the bakelite sheets be? Assume that the same voltage will be applied in each case between the surfaces of the sheets.

Prob. 2–13. For rubber insulation the relative dielectric strengths for various times of electrification are given in the following table.

Time of electrification in minutes	Relative dielectric strength
1	180
3	110
5	100
10	90
15	85
30	80

The 5-minute factory test of No. 00 rubber-insulated cables having $\frac{7}{64}$ inch insulation is 11.5 kilovolts. What voltage will these cables safely stand for 1 minute?

117. Condenser Action. We now come to a very important matter, — the subject of electric condensers.

We have seen that the electrons in a dielectric are fixed in position by being attached to specific atoms. However, the **bonds** which are holding the electrons to the atoms are more or less flexible. It is true that with only moderate electromotive forces it is impossible to break these bonds and cause the electrons to move through the material. However, any electromotive force whatever will somewhat stretch the bonds and cause the electrons to move slightly in position.

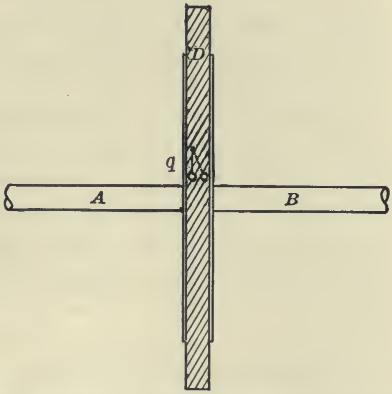

FIG. 207. A simple condenser, consisting of a dielectric D held between two electrodes A and B.

Thus in Fig. 207, suppose that D is a slab of dielectric material, and that A and B are metal electrodes placed against its surface. The material D is made up of an enormous number of atoms with attached electrons. Let us represent one of these electrons by q, and represent the bond by which it is attached to the atom by the short straight line. Suppose that a potential is applied between A and B, and that A is made negative. The electron will now be repelled by A and attracted by B. It will therefore move into some such position as is shown by the dotted line. While it is thus moving, we have an electric current through the dielectric. Note, however, that this current continues for only an instant. If we suddenly apply the voltage between A and B, there will be a small instantaneous current as the electrons move slightly in their positions in the atom, but this current will soon die out, and then after a time there will be zero current as long as the applied voltage is steady.

Moreover, if the voltage between A and B is removed, the electrons by reason of the elasticity of their bonds will return to their previous positions. That is, we get a current on applying a voltage, and a current in the opposite direction upon removing the voltage. In fact, the effects produced in these two cases are exactly equal and opposite.

In general, whenever the voltage impressed upon a dielectric is changed, there will be a current in the dielectric which we call a displacement current, and which is due to the slight movement of the electrons in their positions in the atoms. When the applied voltage is steady, there is no current. When the voltage is varying, a current is produced which is proportional to the rate of change of the voltage.

It has been checked experimentally that the current produced in a dielectric is proportional to the rate of change of voltage. Thus, if in Fig. 207 a voltage is gradually applied between A and B, and in such a way that it increases from zero to 100 volts in one second, a certain current will be produced which we will call I_1. If now the rate of change of voltage is increased, the current will be increased. The current depends simply upon the rate of change of voltage. For example, if the voltage between A and B is increased to 100 volts in one-half a second, the current produced in this interval will be on the average $2I_1$, and so on. This proportionality may be expressed by means of the equation

$$i = C\frac{de}{dt}. \tag{1}$$

The proportionality factor C is called the **capacitance**, and an arrangement consisting of a dielectric between two electrodes is called a **condenser.**

In general, then, the current produced in a condenser is equal to the product of its capacitance times the rate of change of the voltage. This may be compared with an inductance coil, where we saw that the voltage produced was

equal to the inductance of the coil times the rate of change of the current.

The unit of capacitance in practical units is the **farad.** A condenser has a capacitance of one farad when a current of one ampere will flow through it upon the application of a voltage changing at the rate of one volt per second. It has been found, however, that this unit of capacitance is usually much too large for engineering use. In practical computations it is therefore much more convenient to use one-millionth of this unit, called a **microfarad.** A microfarad is the capacitance of a condenser in which a microampere will be produced by a voltage changing at the rate of one volt per second.

We may also write equation (1) in c.g.s. units. The c.g.s. unit of capacitance in the electromagnetic system is called the abfarad. The capacitance of a condenser in abfarads is equal to the abamperes of current produced divided by the rate of change of potential in abvolts per second.

Prob. 3–13. A condenser with a capacitance of 1 microfarad is connected as shown in Fig. 208. If the slide wire is 120 inches long and of a total resistance of 5 ohms and if the slide is being moved to the right at the rate of 5 feet per second, what current will flow in the condenser?

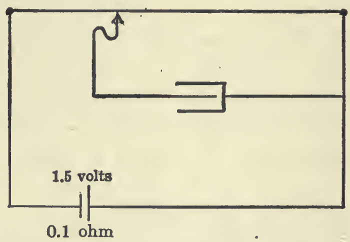

Fig. 208. As the slider moves to the right, the voltage across the condenser drops.

Prob. 4–13. How much would you modify the result in Prob. 3–13 if the slide wire were replaced by another with a resistance of one ohm?

Prob. 5–13. What is the capacitance of a condenser in which a current of 0.012 ampere flows when the voltage across its terminals rises at a uniform rate from 100 to 110 volts in 0.08 second?

Prob. 6–13. What maximum current will flow in a condenser of 2.2 microfarads when an alternating pressure of 150 volts

(maximum value) is applied to its terminals? The frequency of the voltage is 60 cycles per second.

118. Dielectric Constant. The capacitance of such an arrangement as is shown in Fig. 207 has been found to depend upon the material of the dielectric used. If B is a sheet of glass, the capacitance will be found to be about twice what it is when B is composed of mica.

The **dielectric constant** of a material is a measure of its effectiveness when used in a condenser, just as the permeability of a material is a measure of the effectiveness of the material for use in a magnetic circuit. It will be remembered from the study of magnetic circuits that the permeability of iron is a number of thousands, and of other materials much lower, but that the permeability of even air is unity. In the same way, when we study condensers we find that while the dielectric constant of most insulating materials is high compared with that of air, yet air or even a vacuum has a dielectric constant; that is, both will give condenser effects when between parallel plates. When the dielectric of a condenser is stressed, we can visualize the movement of electrons which constitutes the displacement current. It has never been satisfactorily explained, however, why there is still a current when the dielectric is removed and replaced by a vacuum. Such, however, has been found to be the case experimentally. There is by no means as much difference in the dielectric constant of various materials, however, as there is in the permeability.

The dielectric constant of air is so nearly the same as that of a vacuum that it is not usually necessary to distinguish between them. We may therefore define the dielectric constant of a dielectric material as the capacitance of a condenser constructed with this material as dielectric divided by the capacitance of exactly the same condenser when air replaces the dielectric material.

Glass has a dielectric constant ranging between five and ten, depending upon the grade of the glass. Mica has about

half of this. Some liquids have high dielectric constants, that of glycerine being about fifty-six and of picric acid about eighty. Very pure water is an insulator and has a dielectric constant which can be measured. It is in the neighborhood of eighty. Table VI in the appendix gives the dielectric constants for materials commonly used in condensers.

Prob. 7–13. A certain condenser using glycerine as the dielectric has a capacitance of 1.84 microfarads. The pressure across it is varies from 8 volts to 24 volts in 0.003 second. What average current will flow in this condenser under this condition, if the glycerine is replaced by acetone?

119. Parallel-Plate Condensers. A parallel-plate condenser may be constructed as shown in Fig. 207 or, in order to increase the amount of capacitance, as shown in Fig. 209.

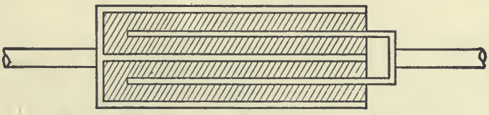

Fig. 209. A parallel-plate condenser.

Such condensers may be built up of sheets of metal and plates of glass. They may be constructed of paraffined paper and tinfoil. The construction depends, of course, upon the voltage to which they are to be subjected.

The capacitance of such a unit is its current for unit rate of change of voltage. Since this current depends upon the number of electrons in a cross-section which can move in position, it is to be expected that this capacitance will be proportional to the cross-sectional area of the sheet of dielectric. Also, since the extent to which the electrons will move depends upon the force to which they are subjected, it is to be expected that the capacitance of such a condenser will be inversely proportional to the thickness of the dielectric, and hence proportional to the voltage gradient applied to the electrons. Such is found experimentally to be the case. The capacitance is also proportional to the dielectric con-

stant, K, of the material used. We may thus write for the capacity of a parallel-plate condenser

$$C = a\frac{KA}{S}, \tag{2}$$

where a is the proportionality factor. If we take for convenience the dielectric constant of air to be unity, then the value of a comes out, in c.g.s. units,

$$a = \frac{1}{4\pi \times 9 \times 10^{20}}. \tag{3}$$

This peculiar factor comes in by reason of the way in which capacitance was initially defined. We may therefore write, in c.g.s. units,

$$C = \frac{1}{36\pi\ 10^{20}}\ \frac{KA}{S}\ \text{abfarads}, \tag{4}$$

where C is the capacitance in abfarads, A is the cross-section of the dielectric in square centimeters, S is the thickness in centimeters and K is the dielectric constant. In practical units, we have

$$C = 0.08842 \times 10^{-6}\frac{KA}{S}\ \text{microfarads}, \tag{5}$$

where C is in microfarads, the other quantities being as before.

Prob. 8–13. A condenser is constructed of 2000 sheets each of paraffined paper and tinfoil. The paper sheets are 0.008 cm. thick and the area of each sheet actually between the tinfoil sheets is 16×20 cm. At what rate must the voltage be varied across this condenser to produce a current of 2.43 amperes?

Prob. 9–13. If mica sheets 0.003 cm. thick are substituted for the paraffined paper in the condenser in Prob. 8–13, at what rate must the voltage be varied to produce 2.43 amperes?

120. Charge on a Condenser. Electrons repel one another so powerfully that they become evenly distributed throughout the body of a metal. They are also crowded

out to the surface, but cannot leave, except when the temperature is high, on account of the attraction of the metal for them. They act as a practically incompressible fluid in the body of the metal. However, if a large voltage is applied, a few more can be crowded into the surface.

This is especially true if a second metal surface is close by which is oppositely charged, that is, which has a deficiency from its normal supply of electrons, for the surface electrons are attracted by the metal of the second plate. This partially accounts for the fact that two plates separated by a vacuum can act as a condenser, although it does not explain the experimental fact that a current also passes in the space between the plates.

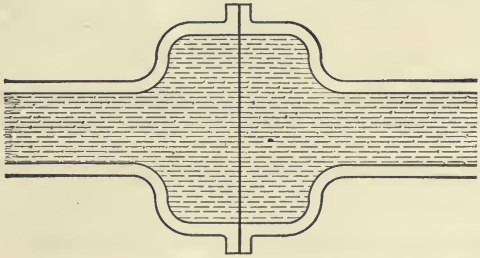

FIG. 210. A hydraulic analogy to a condenser. It consists of a pipe with a rubber diaphragm stretched across it.

When a dielectric is placed between the plates, more electrons can be crowded into the metal surface because of the adjacent molecules of insulating material in which the electrons have moved over in position. These neighboring molecules then attract the electrons of the metal.

An electric condenser has an excellent hydraulic analogy in a rubber diaphragm stretched across a pipe as shown in Fig. 210.

If this pipe is connected to a centrifugal pump, as in Fig. 211, and the pump is started running, the diaphragm will be stretched as shown, and will stay stretched as long as

the pump operates. During the period when the pump is accelerating, there will be a flow of water in the pipe which will soon cease. If the pump is stopped, there will be a flow of

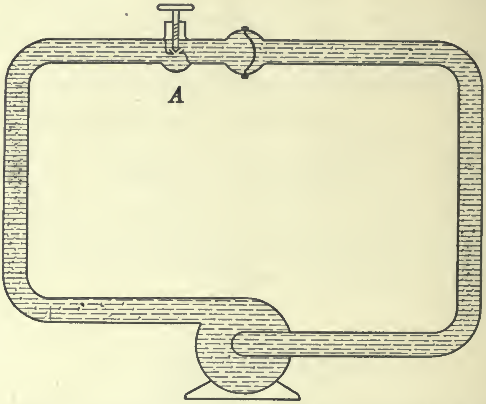

FIG. 211. When the pump is started a current flows for a short period until the diaphragm is stretched to its maximum.

water back, and the diaphragm will spring back to its mid-position.

Similarly, if an electric condenser is connected in series with a generator, as shown in Fig. 212, and the generator started, there will be a brief flow of current around the circuit for a moment after the generator is started, but the flow will soon stop if the generator speed and hence the voltage remain constant. If the generator is stopped, and the voltage thus removed, there will be a flow back through the circuit for a short period of time.

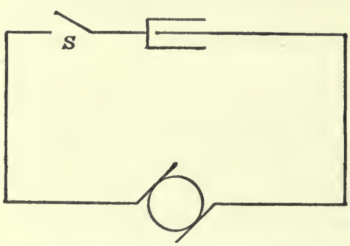

FIG. 212. If the switch S is closed and the generator started, a current will flow for a moment as in Fig. 211.

Suppose that instead of stopping the generator we first open a switch at S. This is equivalent to closing a valve at A in Fig. 211. In such a case the current cannot flow back even although the generator is stopped. We now say that the condenser is charged. The electrons tend to spring back in the dielectric but they cannot do so because the electrons in the plate act as if they were incompressible. There is a voltage between the ends of the circuit which are open which is equal to the generator voltage at the instant the switch was opened. We say that the condenser is charged to this voltage.

The current in the condenser we have seen to be

$$i = C\frac{de}{dt}. \tag{6}$$

Integrating this expression, we have

$$E = \frac{1}{C}\int i\,dt. \tag{7}$$

The current i is the number of electrons, measured in coulombs, flowing per second along the circuit, or it is the quantity of electricity per second. That is,

$$i = \frac{dQ}{dt}, \tag{8}$$

where Q is the quantity of electricity. Insert this in the above, and we have

$$E = \frac{1}{C}\int\frac{dQ}{dt}dt = \frac{1}{C}\int dQ, \tag{9}$$

or

$$E = \frac{Q}{C}. \tag{10}$$

This states that the voltage across a condenser is equal to the quantity or charge which has been forced into it, divided by its capacitance.

Expressed in the form

$$C = \frac{Q}{E}, \tag{11}$$

it leads to another definition of capacitance, namely: the capacitance of a condenser is equal to its charge per applied volt. Thus a condenser has a capacitance of a microfarad when an applied potential of one volt will force into it a charge of a microcoulomb. There are hence two equivalent definitions for capacitance, just as there are for inductance.

Prob. 10–13. A condenser of 12 microfarads capacitance is connected to a 220-volt battery. How many ampere-seconds of charge will flow into the condenser?

Prob. 11–13. What capacitance must a condenser have if 110 volts are to put a charge of 0.016 coulomb into it?

121. Measurement of Capacitance. There are two ways of measuring the capacitance of a condenser, depending upon the two definitions.

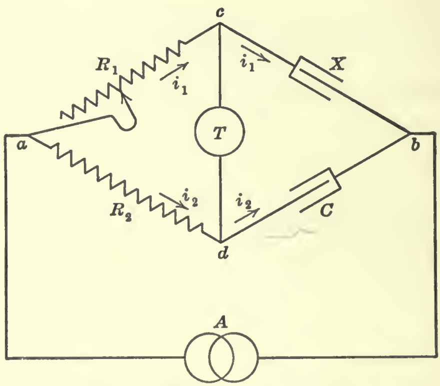

Fig. 213. A bridge arrangement for measuring capacitance.

The first method consists of applying a varying voltage to the condenser and measuring the current flowing, or comparing this current with that flowing in a standard condenser under the same conditions. This can be done very

simply with a bridge circuit such as that shown in Fig. 213. C is the capacitance of a standard condenser which is known. X is the capacitance to be measured. R_1 and R_2 are resistances. A is an alternating-current generator which supplies a variable voltage to the bridge. T is a telephone receiver.

The resistance R_1 is varied until no sound is heard in the telephone. When this is done, we know that points c and d are at the same potential at all instants. Under these conditions, currents flow in X and C which are at all instants proportional to their capacitances, for they are subjected to the same rate of change of voltage. Since there is no current in the telephone, these same currents must flow also in R_1 and R_2. But the drop from a to c is the same as that from a to d. Hence at any instant

$$R_1 i_1 = R_2 i_2. \tag{12}$$

We know, however, that

$$\frac{i_1}{i_2} = \frac{X}{C}. \tag{13}$$

Combining these equations gives

$$\frac{R_2}{R_1} = \frac{X}{C}, \tag{14}$$

or

$$X = \frac{R_2}{R_1} C, \tag{15}$$

so that when R_1 and R_2 are known, X can be computed.

This method assumes that there is no resistance in the leads to the condensers and no leakage through the dielectric. In the next section, we shall study the effect of a resistance in series with a condenser.

The above way of measuring a capacitance simply compares it with a capacitance of known value. The following method, which depends upon the second definition of

capacitance, allows us to measure a condenser without comparing it with another.

If a steady voltage is applied to a condenser, the charge produced in the condenser divided by the applied voltage gives the capacitance of the unit. The charge is equal to the total quantity of electricity passing through the leads during the process of charging.

We have seen, in measuring flux, that the deflection of a ballistic galvanometer is proportional to the quantity of electricity which is suddenly passed through it. Let us therefore connect a ballistic galvanometer in series with the capacitance to be measured, and suddenly close a switch to apply a known voltage E. The arrangement is shown in Fig. 214. Let d be the deflection of the galvanometer when the switch is closed. Let K be the galvanometer constant, that is. the deflection for one microcoulomb. Then, since a deflection $d = K$ when one volt is applied to a capacitance of one microfarad, we have in general

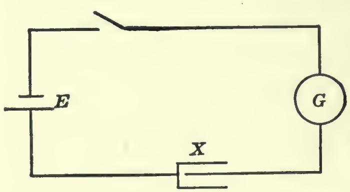

FIG. 214. A ballistic-galvanometer method of measuring capacitance.

$$X = \frac{d}{KE} \text{ microfarads.} \tag{16}$$

The constant K may be found in several ways. One is to use a standard condenser, measure the deflection, and use

$$K = \frac{d_1}{CE}, \tag{17}$$

where d_1 is the deflection obtained with voltage E and C is the known capacitance. This involves comparison, however, and we may have no standard capacitance available. Another method is to calibrate by using an inductance as outlined under flux measurement. A third way is to apply

a low voltage through a high non-inductive resistance to the galvanometer for a short instant by means of a contact-making device such as a commutator. An arrangement for doing this is shown in Fig. 215.

The contact device here shown is a pendulum which makes a contact for an instant at the lower part of its swing. From the period and amplitude of oscillation of the pendulum and the length of the contact arm, we can compute the short interval of time t_0 during which the pendulum causes contact to be made. The switch S is closed during one swing of the pendulum only, and the deflection of the galvanometer noted. During a time t_0 there is a current through the galvanometer of value

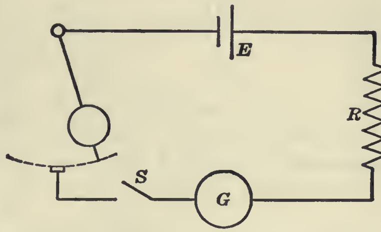

Fig. 215. The time of contact made by the pendulum can be determined. By means of this and the known values of E and R, the value of the constant of the galvanometer can be computed.

$$i = \frac{E}{R} \text{ amperes.} \tag{18}$$

The total quantity passing is hence

$$Q = it_0 = \frac{Et_0}{R} \text{ coulombs.} \tag{19}$$

If the galvanometer deflection due to this quantity is d'', the constant K of the deflection per microcoulomb will be

$$K = \frac{d''}{Q}\, 10^{-6}, \tag{20}$$

or

$$K = \frac{d''R}{Et_0} 10^{-6}. \tag{21}$$

This method of calibrating the galvanometer must be used with considerable care to obtain accurate results.

When a condenser is discharged, the same quantity flows out as flowed in during charge, if no electrons are lost by leakage. Accordingly, if we wish we may connect the circuit as shown in Fig. 216 and measure the quantity passing the galvanometer during discharge of the condenser, by throwing the two-way switch first to the battery side and then to the galvanometer. The equations are, of course, the same as before.

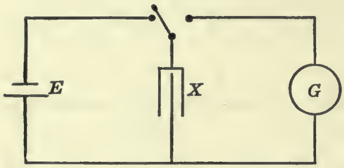

FIG. 216. The galvanometer will measure the quantity of electricity passed through it upon discharge of the condenser.

Prob. 12–13. In a bridge arranged as in Fig. 213, no sound is heard in the telephone receiver when $R_1 = 1.297$ ohms and $R_2 = 3.000$ ohms. What is the capacitance of X if the capacitance of C is 2.461 microfarads?

Prob. 13–13. In Fig. 213, suppose that A and T are interchanged in position. What will be the equation of the bridge for this condition?

Prob. 14–13. A pressure of 0.100 volt is applied for 0.010 second to a ballistic galvanometer having a total resistance of 50,000 ohms. The galvanometer deflects 2.00 cm. A condenser freshly charged to 20 volts is discharged through the ballistic galvanometer and it deflects 15.99 cm. What is the capacitance of the condenser?

Prob. 15–13. What capacitance must a condenser have if, when charged to 100 volts, it will deflect the galvanometer of Prob. 14–13, 20 cm. on being discharged through it?

122. Charging a Condenser Through a Resistance. Let us examine the transient which occurs in charging a condenser through a resistance. That is, let us study the manner in which the current varies in the circuit of Fig. 217 during the interval after the switch is closed.

Call the current in the circuit at any instant i. No matter how this current varies, the total quantity which

passes through the circuit up to any given time, t, can be written

$$q = \int_0^t i\,dt. \tag{22}$$

This quantity is the charge on the condenser at this instant. The voltage to which the condenser is charged is then

$$e = \frac{q}{C} = \frac{1}{C}\int_0^t i\,dt. \tag{23}$$

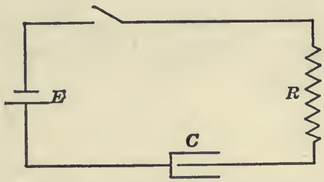

FIG. 217. A circuit containing resistance and capacitance in series.

Now there are two voltages in the circuit, the applied voltage of the battery and the back voltage to which the condenser is charged. The difference between these two acts to produce a current through the circuit. That is,

$$i = \frac{E - e}{R} = \frac{E - \frac{1}{C}\int_0^t i\,dt}{R}. \tag{24}$$

Rewrite this in the form

$$E - iR = \frac{1}{C}\int_0^t i\,dt, \tag{25}$$

and take the derivative with respect to t of both sides of the equation. Note that by definition

$$\frac{d}{dt}\int i\,dt = i. \tag{26}$$

The differential equation of the circuit is therefore

$$-R\frac{di}{dt} = \frac{i}{C}. \tag{27}$$

To solve, separate the variables

$$\frac{di}{i} = -\frac{dt}{RC}, \tag{28}$$

and integrate

$$\log_\epsilon i = -\frac{t}{RC} + A, \tag{29}$$

where A is a constant of integration. To determine A, note that when $t = 0$, that is, when the switch is first closed, the current is simply E/R, for at that time there is no charge on the condenser to oppose the flow of current. This relation may be obtained also from equation (19) by inserting $t = 0$. Since

$$t = 0$$

and

$$i = \frac{E}{R},$$

we have

$$A = \log_\epsilon \frac{E}{R}, \tag{30}$$

and inserting this in (29)

$$\log_\epsilon i = -\frac{t}{RC} + \log_\epsilon \frac{E}{R}, \tag{31}$$

or

$$\log_\epsilon \frac{i}{\frac{E}{R}} = -\frac{t}{RC}. \tag{32}$$

This equation can also be written in the form

$$\frac{i}{\frac{E}{R}} = \epsilon^{-\frac{t}{RC}}, \tag{33}$$

or

$$i = \frac{E}{R}\epsilon^{-\frac{t}{RC}}. \tag{34}$$

This is the equation for the charge of a condenser through a resistance. It is plotted in Fig. 218. We note that when the switch is closed, the current starts off at full value E/R and exponentially falls to zero. This curve is exactly similar to the transient by which the current dies out in an inductive

circuit. In the same way as there treated, we may note that when $t = RC$, the current will have fallen to $1/\epsilon$ of its initial value. The quantity RC is hence called the time constant of the condenser circuit.

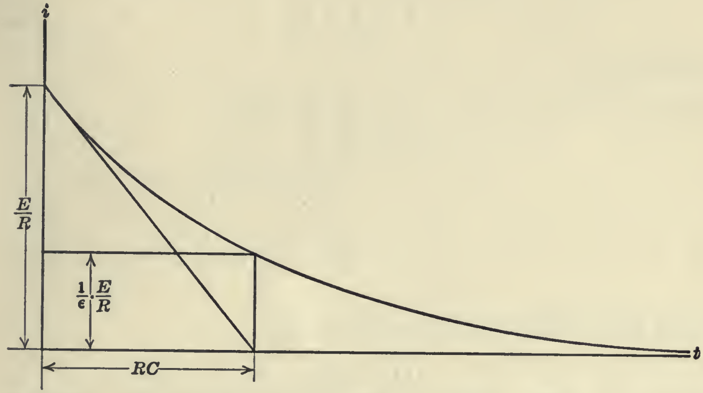

FIG. 218. The charging current of a condenser. The time represented by RC is called the time constant of the circuit.

The rate of decrease of the current at any instant is given by

$$\frac{di}{dt} = \frac{E}{R^2C}\epsilon^{-\frac{t}{RC}}, \tag{35}$$

obtained directly from (34). Inserting $t = 0$, we have

$$\left(\frac{di}{dt}\right)_{t=0} = \frac{E}{R^2C} \tag{36}$$

as the initial rate of decrease of the current. If the current had continued to decrease at this rate for a time RC, it would have fallen through the value

$$\frac{E}{R^2C} \times RC = \frac{E}{R}; \tag{37}$$

that is, it would have fallen to zero.

The time constant hence may also be expressed as the

time it would take the current to fall to zero if it continued to decrease at its initial rate.

The charge on the condenser at any instant is given by integrating the expression for current, or

$$q = \int_0^t i\, dt. \tag{38}$$

This gives

$$q = \int_0^t \frac{E}{R} \epsilon^{-\frac{t}{RC}} dt \tag{39}$$

$$= - EC\, \epsilon^{-\frac{t}{RC}} \Big]_0^t$$

$$= EC\,(1 - \epsilon^{-\frac{t}{RC}}). \tag{40}$$

The final charge on the condenser is

$$Q = EC,$$

so that we may write

$$q = Q\,(1 - \epsilon^{-\frac{t}{RC}}). \tag{41}$$

The charge therefore increases exponentially to its final value in accordance with the curve of Fig. 219. The same time constant as before applies to this curve also.

Prob. 16–13. The circuit of Fig. 217 has the following constants:

R = 50 ohms,
C = 5 microfarads,
E = 100 volts,
r = 2 ohms (internal resistance of the battery).

(*a*) At what rate will the current be decreasing 0.006 second after the switch is closed?

(*b*) What time will be necessary for the current to decrease to half its initial value?

(*c*) What is the initial rate of decrease?

(*d*) What is the value of the current when $t = CR$?

Prob. 17–13. In the circuit of Fig. 217:

(*a*) What will be the charge on the condenser 0.006 second after the switch is closed?

(*b*) What time will it take to put half the total charge on the condenser?

(*c*) What is the value of the charge when $t = CR$?

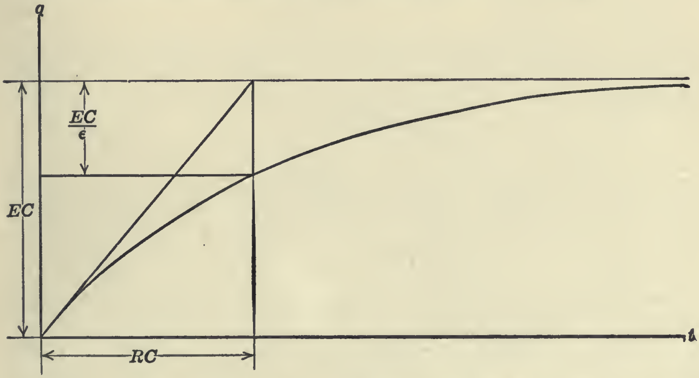

FIG. 219. The charge on a condenser increases in accordance with this curve. The time represented by *RC* is called the time constant of the circuit.

Prob. 18–13. After the condenser in the circuit of Fig. 217 has become fully charged, a second battery of negligible internal resistance and giving 75 volts terminal voltage is suddenly added to the circuit, in series with the one already acting.

(*a*) At what rate will the current be decreasing 0.002 second after the second battery has been added?

(*b*) What time must elapse before the current reaches half the value it has at the instant of adding the second battery?

(*c*) What time does it take to put in half the final added charge?

(*d*) What is the value of the added charge when the time since adding the second battery has the value CR?

Prob. 19–13. Determine from measurements made on the curve of the charging current in Fig. 220, the capacitance of the condenser used.

123. Discharge. The equation for discharge follows directly from that of charge. In the circuit of Fig. 221,

we will assume the condenser initially charged to a potential E, and then the switch closed at time $t = 0$.

The voltage of the condenser then is at any instant thereafter

$$E - \frac{1}{C}\int_0^t i\,dt, \tag{42}$$

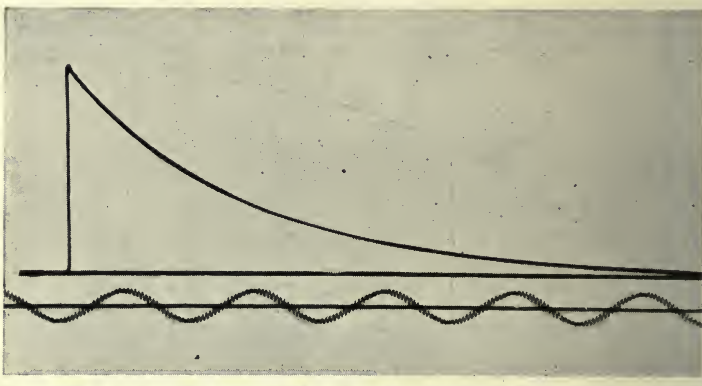

FIG. 220. An oscillogram of the charging current of a condenser with a 1000-ohm resistor in series when 115 volts are applied to the combination. The wave at the bottom is a 60-cycle wave from which the time may be obtained. *Taken by Prof. F. S. Dellenbaugh in the Research Laboratories of the Electrical Engineering Dept., Mass. Inst. of Technology.*

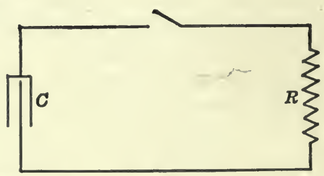

FIG. 221. A circuit containing capacitance and resistance.

and since this is the only voltage acting in the circuit, the current can be obtained by dividing by R

$$i = \frac{E - \frac{1}{C}\int_0^t i\,dt}{R}. \tag{43}$$

Rewriting this in the form

$$E = iR + \frac{1}{C}\int_0^t i\,dt, \tag{44}$$

it is seen to be exactly the same equation as (25) for charge, except that E now means an initial voltage instead of a battery voltage. The solution is accordingly exactly the same,

$$i = \frac{E}{R}\epsilon^{-\frac{t}{RC}}. \tag{45}$$

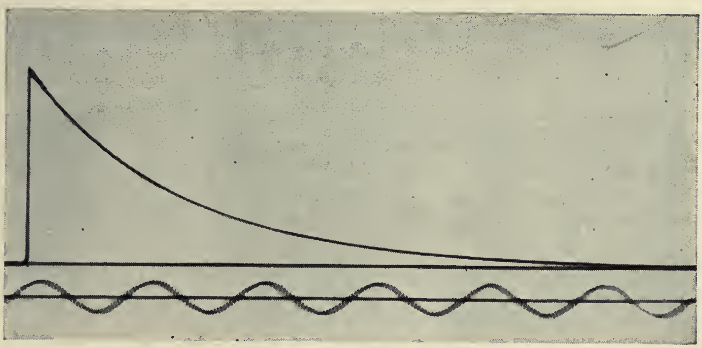

FIG. 222. Oscillogram of the discharge current of the condenser of which Fig. 220 is the charging current. *Prof. F. S. Dellenbaugh.*

The curves of current upon charge and discharge are hence identical. This fact is illustrated by Fig. 220 and 222 which are oscillograms of the charge and discharge currents of the same condenser.

The quantity on the condenser thus decreases during discharge at the same rate as it increased during charge. Hence the curve of q is that of Fig. 223, and the equation

$$q = Q\,\epsilon^{-\frac{t}{RC}}. \tag{46}$$

Prob. 20–13. The constants for the circuit of Fig. 221 are C = 6.28 mf. and R = 18 ohms. The condenser is charged to a potential of 150 volts and the switch is closed. Compute

(*a*) Current at instant of switch closing,
(*b*) Rate of decrease of current at instant of switch closing,
(*c*) Time in which charge on the condenser is reduced one-half,
(*d*) Charge left on condenser after 0.008 second.

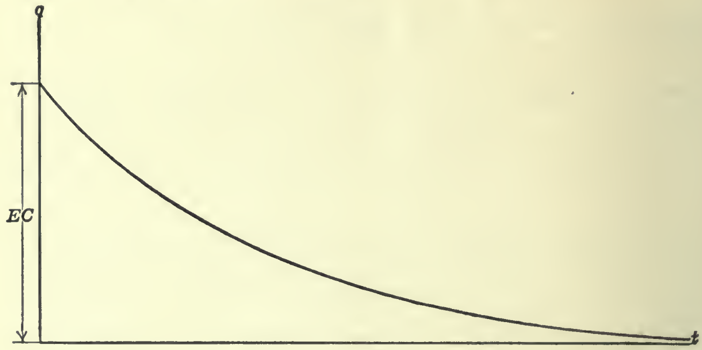

FIG. 223. The discharge curve showing the relation between the charge on a condenser and the time of discharge. The charge on the condenser decreases upon discharge through a resistance according to this curve.

124. Energy Relations. Referring to Fig. 217, the equation

$$E = iR + \frac{1}{C}\int i\,dt \tag{47}$$

applies to the current and voltage relations during **charge** and expresses the fact that at any instant the applied voltage E is partly used in overcoming the iR drop in the resistance, and partly in opposing the back voltage, $\frac{1}{C}\int i\,dt$, of the condenser. Multiply the equation by i, and we have

$$Ei = i^2R + \frac{i}{C}\int i\,dt. \tag{48}$$

This expresses the fact that of the power input Ei at any instant, a part i^2R is used in heating the resistance, and the remainder

$$\frac{i}{C}\int i\,dt$$

is stored in the condenser.

When the current is varying in accordance with

$$i = \frac{E}{R}\epsilon^{-\frac{t}{RC}}, \tag{49}$$

the rate of energy storage in the condenser becomes, upon substituting this value of i,

$$\frac{i}{C}\int i\,dt = \frac{E}{RC}\epsilon^{-\frac{t}{RC}}\int\frac{E}{R}\epsilon^{-\frac{t}{RC}}\,dt, \tag{50}$$

$$= \frac{E^2}{R}\epsilon^{-\frac{t}{RC}}\left(-\epsilon^{-\frac{t}{RC}}\right),$$

$$= -\frac{E^2}{R}\epsilon^{-\frac{2t}{RC}}. \tag{51}$$

The total energy stored is the integral of this expression over the total time used in charging the condenser: that is,

$$W_c = \int_0^\infty -\frac{E^2}{R}\epsilon^{-\frac{2t}{RC}}\,dt. \tag{52}$$

The integration gives

$$W_c = \frac{E^2C}{2}\epsilon^{-\frac{2t}{RC}}\Big]_0^\infty \tag{53}$$

or

$$W_c = \frac{E^2C}{2}\text{ joules.} \tag{54}$$

This is the expression for the energy stored in a condenser when it is charged to a potential E. It will be remembered that when a current I is flowing through an inductance L, the energy storage was

$$W_L = \frac{I^2L}{2}\text{ joules.} \tag{55}$$

The two expressions are seen to be very similar. The energy stored in a condenser is analogous to a potential energy, while that stored in an inductance is similar to a kinetic energy.

It is interesting to note the energy lost in the series

resistance during the process of charging the condenser. This is,

$$W_R = \int_0^\infty i^2 R dt. \tag{56}$$

Putting in

$$i = \frac{E}{R} \epsilon^{-\frac{t}{RC}}, \tag{57}$$

we have

$$W_R = \int_0^\infty \frac{E^2}{R} \epsilon^{-\frac{2t}{RC}} dt, \tag{58}$$

and integrating

$$W_R = -\frac{E^2 C}{2} \epsilon^{-\frac{2t}{RC}} \Big]_0^\infty \tag{59}$$

$$= \frac{E^2 C}{2}. \tag{60}$$

This gives us a curious fact. When a condenser is charged through a resistance from a constant potential source, exactly half of the energy put in is lost in the resistance, and the other half is stored in the condenser. This is true no matter whether the resistance is large or small. If the condenser is charged from a voltage source which is gradually increasing as the condenser charges, not nearly as much energy will be lost.

Prob. 21–13. A condenser of 1.00 mf. capacitance is charged to 500 volts. How much energy is stored in it? Express answer in ergs, joules and kilowatt-hours.

Prob. 22–13. How much energy is stored in a condenser of 0.005 mf. capacitance when charged to a pressure of 100,000 volts? Express answer in ergs, joules and kilowatt-hours.

Prob. 23–13. The condenser of Prob. 21–13 has a resistance of 25 ohms in series with it.

(*a*) At what rate is energy being stored in the condenser 0.08 second after the switch is thrown?

(*b*) At what rate is energy being consumed in the resistance at the same instant?

125. Mechanical Force on a Condenser. When two bodies are charged, there are always small mechanical forces acting between them, called electrostatic forces. These forces tend to pull the bodies together when the charges are opposite in sign, or to push them apart if the charges are similar. Electrostatic attraction is somewhat similar to magnetic attraction, but is not as important practically.

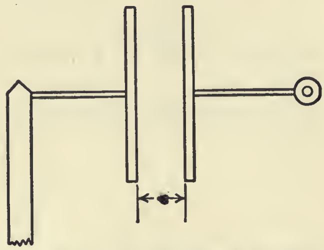

FIG. 224. A condenser the two plates of which are separated by a distance S.

The consideration of the last section will enable us to compute the force acting between the plates of a parallel - plate condenser when the condenser is charged to a difference of potential E.

Consider the condenser of Fig. 224. Assume that we have charged it to a potential E and then disconnected it from the circuit. The energy stored in the condenser is

$$W = \frac{CE^2}{2} \text{ joules.} \tag{60}$$

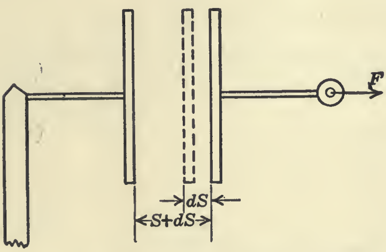

FIG. 225. The plates of the condenser in Fig. 224 have been pulled apart by a force F to a distance $S + dS$.

Consider one plate attached fast. Let us apply a force F to the other plate and pull it away an additional distance dS as shown in Fig. 225. In so doing we have expended work to the amount

$$dW = FdS, \tag{61}$$

which, assuming no friction or other losses, must appear as additional energy stored in the condenser. The energy now stored in the condenser is

$$W_1 = \frac{C_1 E_1^2}{2}, \tag{62}$$

where C_1 and E_1 are the new capacitance and voltage respectively.

For the new separation the capacity is decreased to the value

$$C_1 = \frac{S}{S + dS} C \tag{63}$$

since the capacity of a parallel-plate condenser is inversely proportional to the separation of the plates.

Also, since the number of electrons on the plates has remained unchanged during the process, and since the potential on a condenser is the charge divided by the capacity, we have

$$E_1 = \frac{S + dS}{S} E. \tag{64}$$

Inserting these two expressions, we then have

$$W_1 = \tfrac{1}{2} \left(\frac{S}{S + dS} C\right) \left(\frac{S + dS}{S} E\right)^2 \tag{65}$$

$$= \frac{S + dS}{S} W, \tag{66}$$

and hence

$$dW = W_1 - W \tag{67}$$

$$= \frac{dS}{S} W$$

$$= \frac{dS}{S} \cdot \frac{CE^2}{2}. \tag{68}$$

But we had above

$$dW = FdS. \tag{69}$$

So, equating these expressions,

$$f = \frac{CE^2}{2S} \times 10^7 \text{ dynes.} \tag{70}$$

This expression gives the force acting on the plates of a charged parallel-plate condenser. If all quantities on the right are in the c.g.s. system, the force will be given in dynes and the factor 10^7 will not be needed.

We can put the expression in a different form by inserting the expression for the capacitance,

$$C = \frac{1}{36\,\pi 10^{20}} \frac{KA}{S} \text{ abfarads,} \tag{71}$$

so that

$$f = \frac{1}{72\,\pi 10^{20}} \frac{KAE^2}{S^2} \text{ dynes.} \tag{72}$$

The potential gradient between the plates or the electromotive force per unit of length is uniform and of value

$$F = \frac{E}{S}. \tag{73}$$

The **electric potential gradient** F is exactly analogous to the **magnetic potential gradient** H, sometimes called the magnetizing force. It will be remembered that in a uniform magnetic field

$$H = \frac{\mathcal{F}}{l}, \tag{73a}$$

where

H = the magnetic potential gradient,
$\mathcal{F}$ = the magnetomotive force,
l = the length of the magnetic flux path.

Equation (73*a*) for a magnetic field is exactly analogous to equation (73) for an electrostatic field. The magnetomotive force $\mathcal{F}$ is ordinarily measured in ampere-turns or in gilberts and the length l of the field in centimeters. The magnetic

potential gradient is therefore in units of ampere-turns per centimeter, or in gilberts per centimeter. 1 ampere-turn $= 0.4\,\pi$ gilberts.

Similarly we measure the electromotive force E in volts or in statvolts* and the length of the field S in centimeters. 1 statvolt = 300 volts. Thus the electric potential gradient is in units of volts per centimeter or in statvolts per centimeter.

The analogy extends still further. The magnetic field is thought of as containing lines of magnetic force. Similarly the electric field is thought of as containing lines of electrostatic force. In the case of the magnetic field, we multiply the magnetic potential gradient H at a point in the field by the permeability μ of the medium at that point, and obtain the magnetic flux density B at that point. So in the case of an electrostatic field, we multiply the electric potential gradient F at a given point in the field by the permittivity K, (or dielectric constant) of the medium at that point, and obtain the electrostatic flux density D at that point. Expressed as equations,

$$B = \mu H, \tag{74}$$

$$D = KF. \tag{74a}$$

Of course, the proper units for D, K and F must be used to make this equation numerically correct. We will take up these units below.

If now the value of F as found in equation (73) is substituted in equation (74a), we obtain

$$D = \frac{KE}{S}. \tag{74b}$$

In terms of K, equation (72) becomes

$$f = \frac{1}{72\,\pi 10^{20}}\,\frac{AD^2}{K}\text{ dynes,} \tag{75}$$

which may be written

$$f = \frac{1}{9 \times 10^{20}} \times \frac{D^2A}{8\,\pi K}\text{ dynes.} \tag{75a}$$

* A statvolt per centimeter is the potential gradient which will produce unit electrostatic flux density in a medium of unit permittivity.

This equation is again analogous to the equation for the magnetic pull

$$f = \frac{B^2A}{8\,\pi\mu}\,\text{dynes}, \tag{75b}$$

the factor $\frac{1}{9 \times 10^{20}}$ coming into equation (75*a*) because of the arbitrary choice of unity for the dielectric constant of air.

The electrostatic pull is thus proportional to the cross-sectional area and to the square of the electrostatic flux density, just as the magnetic pull is proportional to the cross-sectional area and to the square of the magnetic flux density. In addition the electrostatic pull is inversely proportional to the dielectric constant of the medium. We always computed the magnetic pull in air, but in any other medium it would similarly have been found inversely proportional to the permeability.

The mechanical force due to the electrostatic field is usually small. For example, suppose we have two plates each 10 centimeters square, separated 1 centimeter and subjected to an electromotive force of 100 volts or 10^{10} abvolts. Suppose they are separated by air, of dielectric constant unity. Then inserting the values in the formula above,

$$f = \frac{1}{72\,\pi 10^{20}}\,\frac{10^2\,(10^{10})^2}{1^2} \tag{76}$$

$$= \frac{100}{72\pi} = \text{about } \tfrac{1}{2} \text{ dyne,}$$

$$= \text{about}\,\frac{1}{2000}\,\text{gram,} \tag{77}$$

a force so small that it could be detected with difficulty.

However, suppose the plates were in a hard vacuum so that the potential between them could be raised to 100,000 volts without flash-over. Since the force is as the square of the applied voltage, it is now $(1000)^2$ times as great as before, that is, about 500 grams, or a little more than a pound.

Electrostatic attraction can thus be made use of in meters for measuring high voltage, by causing the attraction to move a delicately suspended vane, as shown in Fig. 226 and 227. On the other hand, the force must be carefully balanced in a high voltage thermionic rectifier, for the very high potential gradient met with in such a vacuum device may pull over or even break the hot filament.

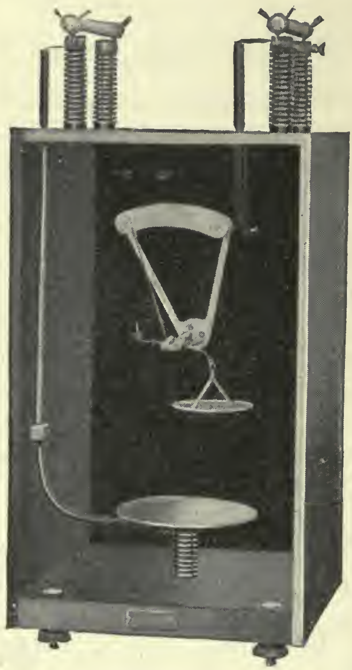

FIG. 226. An electrostatic voltmeter for measuring high voltages. The electrostatic force between the pans is a function of the voltage between them. *General Electric Co.*

Prob. 24–13. In an electrostatic voltmeter of the form shown in Fig. 226, the effective area of each plate is 80 square centimeters. The distance is 8 centimeters. What force acts on the plates when a potential difference of 25,000 volts exists between them?

Prob. 25–13. In designing an electrostatic voltmeter of the type shown in Fig. 226, for 80,000 volts, it is desired to produce an attractive force of $\frac{1}{10}$ gram. Determine the other dimensions of the voltmeter, using two plates of the same size and having a factor of safety of 4 against flash-over. Neglect fringing of the electrostatic lines of force.

Prob. 26–13. A certain condenser consists of two metal plates each having an area of 150 square centimeters and placed 0.60 centimeter apart. The dielectric is air. The plates are charged to a difference of potential of 500 volts and the source of the potential removed.

(a) How much work would be done in separating the plates until they were 1.40 centimeters apart?

(b) What would be the difference of potential between the plates if a mica sheet 0.4 centimeter thick were now thrust between them?

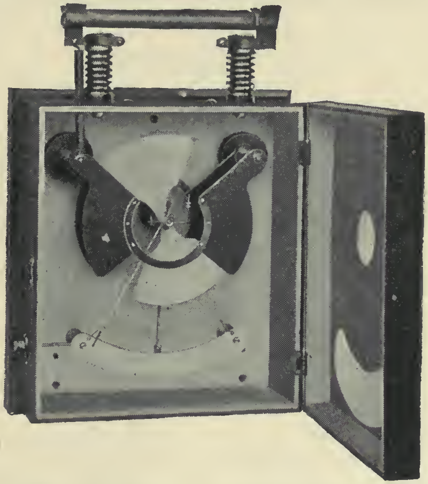

FIG. 227. Another type of electrostatic voltmeter. *General Electric Co.*

126. Electrostatic Fields. In the previous article we have met the idea of the electrostatic flux density D. We can, in fact, plot the electrostatic lines of force just as we have previously plotted magnetic lines. Thus in Fig. 228 is plotted the electrostatic field about a pair of parallel wires in air between which there exists an electromotive force. The density of the electrostatic flux lines D at any point in air is equal to the potential gradient at that point. If at any point this gradient is greater than the dielectric strength of the insulation (the air in this case) there will be break down, and corona or spark.

For a parallel plate condenser we had

$$D = \frac{KE}{3 \times 10^{10} S} \text{ lines per square centimeter,} \qquad (78)$$

but for such a condenser we had, in equation (71),

$$C = a\frac{KA}{S} \text{ abfarads} \tag{79}$$

where a is a proportionality factor equal to $\frac{1}{36\,\pi 10^{20}}$, and so

$$E = \frac{Q}{C} = \frac{QS}{aKA} \text{ abvolts.} \tag{80}$$

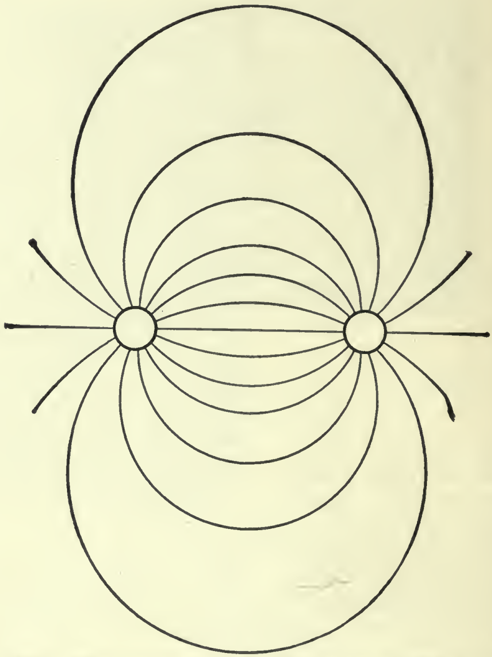

FIG. 228. The electrostatic field between a pair of charged parallel conductors of round cross-section.

Inserting this value in equation (78), we have

$$D = \frac{1}{a}\frac{Q}{A(3 \times 10^{10})} \text{ lines per square centimeter.} \tag{81}$$

This expression states that the electrostatic flux density is proportional to the charge per unit of area of the plates,

that is, to the number of excess electrons per square centimeter of plate.

In other words, in mapping the electrostatic field for a parallel-plate condenser, we should draw one electrostatic line to each charge consisting of a certain definite number of electrons.

Since in c.g.s. units

$$a = \frac{1}{4\,\pi(3 \times 10^{10})^2}, \tag{82}$$

it follows that we should draw one electrostatic line to each

$$\frac{3 \times 10^{10}}{4\,\pi(3 \times 10^{10})^2} = \frac{1}{4\,\pi(3 \times 10^{10})} \text{ abcoulomb of charge.} \tag{83}$$

That is, we should draw $4\,\pi(3 \times 10^{10})$ lines to each abcoulomb.

The quantity of electricity

$$\frac{1}{(3 \times 10^{10})} \text{ abcoulombs} \tag{84}$$

is called a statcoulomb. It is the quantity which situated at 1 centimeter distance from a similar quantity will repel it with a force of 1 dyne.

$4\,\pi$ electrostatic lines thus proceed from each statcoulomb of charge. It was the early choice of this value which led to the presence of π in the constants above. The quantity 3×10^{10} which appears in the relation between the units is equal, in centimeters per second, to the velocity of light. The numerical relations which exist among the various units of charge are as follows:

$$\begin{aligned}
&1 \text{ faraday} &&= 9{,}654 \text{ abcoulombs},\\
&1 \text{ abcoulomb} &&= 10 \text{ coulombs},\\
&1 \text{ coulomb} &&= 3 \times 10^9 \text{ statcoulombs},\\
&1 \text{ statcoulomb} &&= 2.1 \times 10^9 \text{ electrons}.
\end{aligned}$$

When we map any electrostatic field we should consider $4\,\pi$ lines to proceed from each statcoulomb of charge of one sign, and terminate on an equal charge of opposite sign.

The flux density at any point will then be equal to the voltage gradient in abvolts per centimeter. Of course in drawing an electrostatic field sectionally, it is usually convenient to actually plot in only a very small fraction of the lines as above obtained.

127. Capacitance of a Pair of Long Parallel Aerial Conductors. In developing the equation for the **inductance** of a pair of parallel aerial conductors, we first found the value of the magnetizing force or magnetic potential gradient H at any point in the space surrounding a long straight conductor. Similarly the **capacitance** of a pair of aerial conductors can be found by first finding the electric potential gradient F at any point in the air around such a conductor.

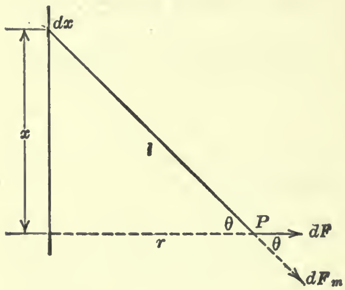

FIG. 229. The force dF is produced by the electric charge on the element dx of the conductor.

Fig. 229 is constructed similar to Fig. 85 which was used to find the inductance of a conductor. Let Q in statcoulombs represent the charge per centimeter of length on the long conductor. The charge on dx length will then be Qdx.

It will be remembered that the current flowing through an element dx of a straight conductor produces a magnetizing force or magnetic potential gradient H at a point P outside the conductor, the value of which is found by the equation

$$dH = \frac{Idx}{l^2}\cos\theta. \qquad (85)$$

Similarly the strength of the electrostatic force dF, Fig. 229, or electric potential gradient in this direction at the point P, due to a charge Qdx, is expressed by the equation

$$dF = \frac{Qdx}{l^2}\cos\theta, \qquad (86)$$

where

dF is the component of electrostatic force or potential gradient at P perpendicular to the conductor: it may be measured in dynes on a unit charge placed at the point or in statvolts per centimeter;

l is the distance in centimeters of point P from the element dx;

Q is the charge on the conductor in statcoulombs per centimeter;

θ is the angle between the electrostatic force at P due to dx and the component of this force perpendicular to the conductor.

The electrostatic force or potential gradient F at P due to the charge on a wire (of practically infinite length in comparison with r) is expressed by the equation

$$F = \int_{-\infty}^{+\infty} \frac{Q \cos\theta}{l^2}\, dx. \tag{87}$$

But from Fig. 229 it is seen that

$$x = r \tan \theta, \tag{88}$$

$$dx = r \sec^2\theta \, d\theta, \tag{89}$$

$$l = \frac{r}{\cos \theta}. \tag{90}$$

If x is indefinitely increased, θ will have $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$ as limits. Therefore substituting these limits and (88), (89) and (90) in (87) we have

$$F = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{Q \ \cos\theta \ \sec^2\theta}{\dfrac{r}{\cos^2 \theta}}\, d\theta \tag{91}$$

$$= \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{Q \cos \theta}{r}\, d\theta. \tag{92}$$

Integrating,

$$F = \frac{Q}{r}\Big[\sin \theta\Big]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} = \frac{2Q}{r}. \tag{93}$$

Note that the expression for electrostatic force F in statvolts per centimeter is again exactly analogous to the expression $\frac{2I}{r}$ in gilberts per centimeter for the magnetizing force H at a point outside of a conductor carrying a current.

To find the capacitance of a pair of long parallel aerial conductors, let A and B, Fig. 230, be a pair of such conductors spaced S centimeters apart and each having a radius of r centimeters and each charged with Q statcoulombs per centimeter length with a voltage of E statvolts between them. Consider the charge on A positive and that on B negative. These conductors must be far enough apart for the charge on each to be uniformly distributed around each, in order that the following development be valid. In other words, S must be large in comparison with r.

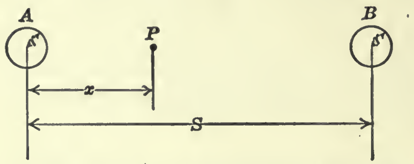

Fig. 230. A and B represent two long parallel conductors having unlike electric charges.

The value of F, the potential gradient or electrostatic force, at any point P a distance of x from A on a line joining the centers of the conductors A and B can be found as follows.

The electrostatic force F_A at P, due to the charge Q on A, is expressed by the equation

$$F_A = \frac{2Q}{x}. \tag{94}$$

The electrostatic force F_B at P, due to the negative charge $-Q$ on the conductor B, is in the same direction as D_A and is represented by the expression

$$F_B = \frac{2Q}{S - x}. \tag{95}$$

The total electrostatic force at P is expressed by

$$F = F_A + F_B = 2Q\left(\frac{1}{x} + \frac{1}{S - x}\right) \text{ statvolts per centimeter.} \tag{96}$$

Since the voltage between the wires must be the integral of the potential gradient between the wires,

$$E = \int_r^{S-r} F dx \text{ statvolts} \tag{97}$$

or

$$E = 2Q\int_r^{S-r}\left(\frac{1}{x} + \frac{1}{S - x}\right) dx.$$

Integrating this expression we obtain

$$E = 4Q \log_\epsilon \frac{S - r}{r} \text{ statvolts.} \tag{98}$$

But

$$C = \frac{Q}{E}. \tag{99}$$

Therefore substituting (98) in (99) we have

$$C = \frac{Q}{4Q \log_\epsilon \frac{S - r}{r}} = \frac{1}{4 \log_\epsilon \frac{S - r}{r}} \text{ statfarads} \tag{100}$$

per centimeter of line.

But

$$1 \text{ statfarad} = \frac{1}{9 \times 10^5} \text{ microfarad}$$

and

$$\log_\epsilon \frac{S - r}{r} = 2.3 \times \log \frac{S - r}{r}.$$

Therefore

$$C = \frac{2.54 \times 12 \times 5280}{2.3 \times 4 \times 9 \times 10^5 \log \frac{S - r}{r}}$$

$$= \frac{0.0194}{\log \frac{S - r}{r}} \text{ microfarads per mile of line.} \tag{101}$$

This value is an approximation only and can be used only when S is very large in comparison with r. For this reason $\dfrac{0.0194}{\log \frac{S}{r}}$ is usually about as close an approximation. In using this equation the effect of the ground is neglected, which may be done when the line is strung at considerable height above the ground. Even then trees along the line are likely to affect the capacitance of the conductor.

Prob. 27–13. The Sierra and San Francisco Power Co. transmits power from Stanislaus to San Francisco, a distance of 138 miles. The wires are No. 00 B & S and are spaced 96 inches apart. Compute the capacitance of two of these wires if used as line and return.

Prob. 28–13. What is the capacitance of a 40-mile two-wire transmission line, if the total inductance is 100 millihenries?

Prob. 29–13. What is the capacitance between two wires of the 154-mile Big Bend-Oakland transmission line? The wires, No. 000 B & S, are spaced 10 feet apart.

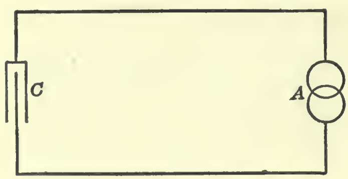

FIG. 231. *A* represents an alternator supplying a variable voltage to the condenser *C*.

128. Application of a Sinusoidal Voltage to a Condenser. In Fig. 231, A is an alternator which supplies a variable voltage to a condenser C. We will neglect resistance in the circuit, assuming it to be very small. The voltage of the alternator varies sinusoidally in accordance with the equation

$$e = E \sin \omega t, \tag{102}$$

and is shown in Fig. 232.

The values of t at which the voltage becomes zero are marked on the time axis.

Since there is no resistance in the circuit, the back e.m.f. of the condenser must at every instant equal the applied

voltage after conditions in the circuit have reached a steady condition; that is,

$$E \sin \omega t = \frac{1}{C}\int i\, dt. \tag{103}$$

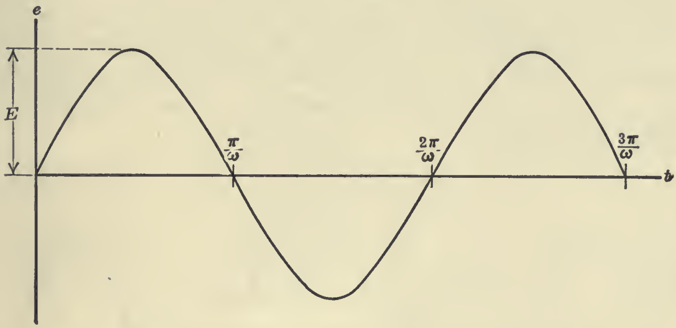

Fig. 232. The voltage of the alternator A in Fig. 231 varies according to this sine curve.

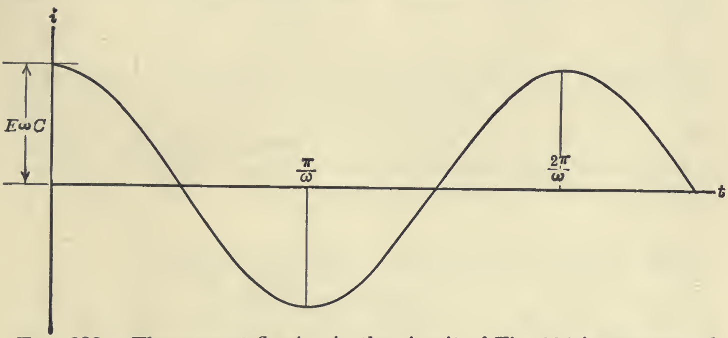

Fig. 233. The current flowing in the circuit of Fig. 231 is represented by this cosine curve. Note that the current has reached its maximum value at the instant when the voltage was zero. The current thus leads the voltage. Note that the time scale in this figure is different from that of Fig. 232.

Differentiate and we have

$$i = EC\,\omega \cos \omega t \tag{104}$$

as an expression for the current flowing. This curve is plotted in Fig. 233.

Several interesting things can be noted from this curve. First, the current also varies harmonically, but it comes to its maximum value sooner than the voltage. In fact, the current is a maximum when the voltage is zero. We state this by saying that the condenser draws a leading current. Also the magnitude of the current is proportional to the applied voltage, to the capacitance and to the frequency, for ω is equal to 2π times the frequency of the alternator in cycles per second.

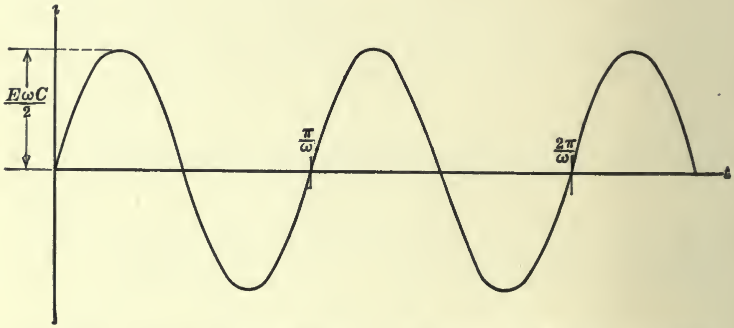

FIG. 234. The curve of power in the circuit of Fig. 231. This is also sinusoidal, but note that the frequency is double that of the voltage and the current.

The power in the condenser circuit is given by

$$p = ei = E^2C\,\omega \sin \omega t \cos \omega t \tag{105}$$

$$= \frac{CE^2}{2}\,\omega \sin 2\,\omega t. \tag{106}$$

This is plotted in Fig. 234.

The power therefore is also a sinusoidal function of the time, but it varies at double frequency.

The energy in the condenser at any instant is given by

$$W = \int p\,dt. \tag{107}$$

At time $t = \frac{\pi}{2\,\omega}$, we have for the voltage

$$e = E \sin \omega \frac{\pi}{2\,\omega} = E; \tag{108}$$

that is, at this instant the condenser is charged to maximum potential. The energy in the condenser at this instant is given by

$$\begin{aligned} W &= \int_0^{\frac{\pi}{2\omega}} p\,dt \qquad (109)\\ &= \int_0^{\frac{\pi}{2\omega}} \frac{CE^2}{2}\,\omega \sin 2\,\omega t\\ &= \frac{E^2C\omega}{4\,\omega}(-\cos 2\,\omega\,t)\Big]_0^{\frac{\pi}{2\omega}}, \end{aligned}$$

which gives

$$W = \frac{CE^2}{2}. \tag{110}$$

This is the expression we found by a different method for the energy stored in a charged condenser.

Note, however, that

$$\int_0^{\frac{\pi}{\omega}} p\,dt = 0, \tag{111}$$

as can be seen by inspection of Fig. 234, for the positive and negative loops are equal in area. That is, the energy stored in the condenser in the first half cycle is all returned to the generator during the next half cycle. This assumes, of course, no resistance and no losses in the condenser. Under these conditions, there will be no heating of the condenser, for there is no energy loss in it.

Prob. 30–13. A sinusoidal e.m.f. of 150 volts, maximum, at a frequency of 60 cycles per second is applied to a condenser of 5 mf. capacitance

(*a*) What maximum current flows?

(*b*) Plot voltage, current and power curves on same axes.

Prob. 31–13. A condenser is made up of 1200 sheets of mica 0.010 centimeter thick and leadfoil plates having an active area of 20 × 18 centimeters. If an alternating voltage with sinusoidal wave form of 150 volts (maximum) at a frequency of 60 cycles is applied to the terminals, what maximum current will flow?

Prob. 32–13. What will be the values of the current, voltage and power in the circuit of Prob. 31–13 when $t = 0.005$ second?

129. Condenser Losses, Dielectric Hysteresis. In the above treatment, we have assumed a perfect condenser, — that is, one which does not leak, has no resistance in its leads and no losses in its dielectric. Condensers constructed of glass or mica with metal plates can be made nearly perfect. No losses are apparent in such a condenser unless the frequency is high. Paper condensers, or condensers formed of sections of cables, are much less perfect. There are many places where condensers are used, or where there is a condenser action, in which there are very considerable losses. These are important not only on account of the cost of the lost energy, but also because of local heating. We have seen that temperature greatly affects the properties of insulators, a rise in temperature always lessening the resistance and lowering the flash-over point.

A cable acts as a condenser. In an ocean cable, the conducting wire is one plate of the condenser, the insulating material is the dielectric, and the sheath of the cable or the sea water forms the other plate. Also cables are often used for power transmission where the voltage used is 30,000 volts or less, particularly in cities. The transmission wires are assembled together with impregnated paper insulation between them, the whole being covered with a lead sheath to keep out moisture. In such an arrangement any local heating is very serious, as it is likely to lead to break down of the cable.

There are three sorts of losses in condensers: first, that due to the resistance of the leads, which is usually

small; second, the loss due to leakage, which in the case of a good insulator is also small; third, the loss in the dielectric itself. This last is most important.

We meet here a phenomenon known as **dielectric hysteresis** and also an effect due to absorption or " soaking into " the dielectric. When a steady voltage is applied to a paper condenser, careful measurement will show that the charge will soak into the condenser for a considerable length of time. Similarly it will gradually soak out on discharge. If a charged paper condenser is discharged, and then left for a short interval, a further small voltage will appear on it, and

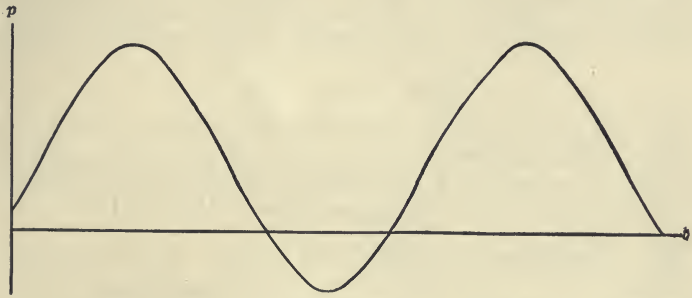

FIG. 235. The power curve of a condenser having losses. Note that the upper and lower loops are not equal as they are in Fig. 234.

it may be discharged again. Dielectric losses may be explained in either of two ways. The first is to assume that the electrons of the dielectric experience a certain viscosity in their motion. This is true dielectric hysteresis. The second is that the dielectric consists of a large number of layers with high resistances between layers. Probably both of these effects are present in many cases. The former exerts greatest influence at high frequencies. The latter is the cause of the principle loss in cable insulation.

If a condenser with losses is connected to an alternator and the curve of power plotted, it will appear as in Fig. 235. The positive and negative loops are no longer equal. Of

the energy supplied to the condenser, not all is returned during the next quarter cycle. Some of it is lost in heat. The expression

$$\int_0^{\frac{2\pi}{\omega}} p\,dt \tag{112}$$

is no longer equal to zero, but is twice the difference in area between a positive and a negative loop of the power curve. The value of this integral is equal to the energy loss in joules per cycle in the condenser.

Prob. 33–13. A sinusoidal voltage of 100 volts maximum is impressed across the terminals of a condenser having a capacity of 1.7 m.f. At 60 cycles frequency due to losses the current flowing is represented by the equation

$$i = E_{\max} C\omega \cos(\omega t - 5°).$$

Plot the current, voltage and power curves on the same axes.

Prob. 34–13. Either by using a planimeter, by counting squares or by integrating, determine the average power used up in overcoming the losses in the condenser of Prob. 33–13.

130. Condensers in Parallel and Series. If two condensers, of capacitances C_1 and C_2, are connected in parallel as shown in Fig. 236, they act simply as a larger condenser of capacitance

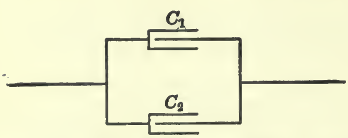

FIG. 236. Two condensers connected in parallel.

$$C_P = C_1 + C_2. \tag{113}$$

This is easily shown in the following manner. If e is the voltage across the pair, the currents in them are respectively

$$C_1\frac{de}{dt} \qquad \text{and} \qquad C_2\frac{de}{dt}.$$

The total current is

$$(C_1 + C_2)\frac{de}{dt},$$

but this is the same current as would be taken by a single condenser of capacitance

$$C_P = C_1 + C_2.$$

When the condensers are connected in series as in Fig. 237, the condition is different.

FIG. 237. Two condensers connected in series.

If a charge Q is passed into the lead to the condensers, the same charge Q appears on each. The voltages to which they are charged will be respectively

$$\frac{Q}{C_1} \quad \text{and} \quad \frac{Q}{C_2}.$$

The voltage across the combination will then be

$$Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right).$$

This same voltage would be produced if the pair of condensers were replaced by a single condenser of capacitance C_S, so chosen that

$$\frac{Q}{C_S} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right); \tag{114}$$

that is,

$$C_S = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}. \tag{115}$$

This expression hence gives the equivalent capacity of the two condensers in series.

The laws for combining condensers are thus the same as for combining conductances. To obtain the capacitance of condensers in parallel, add the individual capacitances. To obtain the capacitance of condensers in series, take the

reciprocal of the sum of the reciprocals of individual capacitances.

When condensers are connected in series, the smaller condenser is subjected to the higher stress. If a voltage E is applied to the two condensers of Fig. 237, a charge will be produced on each of value

$$Q = C_S E = \frac{C_1 C_2 E}{C_1 + C_2} \text{ coulombs.} \tag{116}$$

The voltage across C_1 will then be

$$E_1 = \frac{Q}{C_1} = \frac{C_2 E}{C_1 + C_2}, \tag{117}$$

and across C_2

$$E_2 = \frac{Q}{C_2} = \frac{C_1 E}{C_1 + C_2}. \tag{118}$$

Dividing these two expressions,

$$\frac{E_1}{E_2} = \frac{C_2}{C_1}. \tag{119}$$

The total voltage is therefore distributed in inverse ratio to the capacitances.

Prob. 35–13. Three condensers have respectively capacitances of 2, 4 and 5 microfarads.

(*a*) What is the capacitance of a parallel arrangement of these condensers?

(*b*) Of the 2-microfarad condenser put in series with a parallel arrangement of the 4-microfarad and 5-microfarad condensers?

Prob. 36–13. If a pressure of 110 volts is impressed across the terminals of the second arrangement in Prob. 35–13,

(*a*) What is the voltage across each condenser?

(*b*) What is the charge in each condenser?

(*c*) What is the total charge on the combination?

Prob. 37–13. It is desired to build up a capacitance of 2 microfarads. There are available three condensers of 4, 1 and 3 microfarads respectively.

(*a*) Show by a diagram the arrangement of these condensers

which will produce a capacitance most closely approximating 2 microfarads.

(*b*) Compute the capacitance of your arrangement.

131. Distribution of Stress in Insulation. The preceding section has considerable bearing on the design of insulation.

Suppose, for example, that we have two plates as in Fig. 238 separated one inch and subjected to an applied potential of 60,000 volts. This voltage is about all such a gap will stand when the plates are separated by air, even with the corners well rounded to avoid local high-voltage gradients. Suppose we insert a plate of glass $\frac{1}{2}$ inch thick of dielectric constant 5 as shown, in order to protect against possible breakdown. Examine the new conditions.

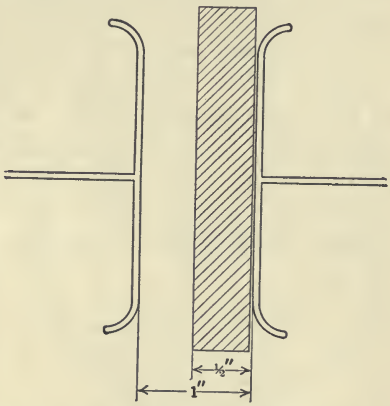

FIG. 238. Two plates separated by a layer of air and glass.

We now have in effect two condensers in series. The one formed by the glass has a capacitance five times as great as the one formed by the remaining half inch of air, because of the high dielectric constant of the glass. Accordingly if E_g and E_a are the voltages across glass and air respectively,

$$\frac{E_g}{E_a} = \frac{1}{5},$$

and since

$$E_g + E_a = 60{,}000,$$

we obtain

$$E_g = 10{,}000,$$
$$E_a = 50{,}000.$$

The introduction of a sheet of insulating material of high dielectric constant thus causes most of the stress to be brought to bear on the remaining material of low dielectric constant.

The above discussion assumes no leakage. However, a potential of 50,000 volts brought to bear upon one-half inch of air will surely break it down. A spark cannot pass clear across on account of the glass, but the air will be ionized and a corona will form. When the air thus becomes conducting, nearly all of the 60,000 volts stress will be brought to bear on the glass. A piece of glass one-half inch thick will surely be able to stand this voltage. There is much doubt, however, if it can stand the heating due to the continued formation of corona if the potential applied is alternating so that the air is continually being broken down. Such an arrangement as above, while it would be safe for a continually applied potential, would therefore be likely to lead to difficulty if the supply were alternating and of the same maximum value.

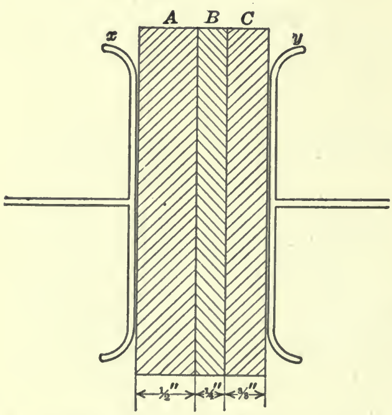

FIG. 239. The plates x and y are separated by three dielectric sheets A, B and C.

Prob. 38–13. Two plate electrodes similar to those in Fig. 238 are placed 2 inches apart and a potential of 50,000 volts applied. If a sheet dielectric $\frac{1}{2}$ inch thick and having a dielectric constant of 3 is inserted between the electrodes, what will be the new distribution of potential?

Prob. 39–13. The three dielectric sheets shown in Fig. 239 have the following dielectric constants: A, 2.5; B, 3; C, 4.5. What is the potential distribution across the different sheets when 60,000 volts are maintained between the plates x and y?

Prob. 40–13. Assuming the same data as in Prob. 39–13, compute the voltage distribution in Fig. 239:

(*a*) With A removed and B and C as in the figure,
(*b*) With B removed and A and C as in the figure,
(*c*) With C removed and A and B as in the figure.

Prob. 41–13. What dielectric sheet or sheets of arrangement in Prob. 40–13 must be removed to produce a condition most likely to cause a break down of the air? Assume the dielectric sheets to have much greater dielectric strength than air.

132. Cylindrical Condenser. A conductor, surrounded by insulation and a sheath, as shown in Fig. 240, forms a cylindrical condenser. This is the arrangement of a single-conductor cable used in power transmission or submarine telegraphy. The capacity of such a condenser may be found as follows.

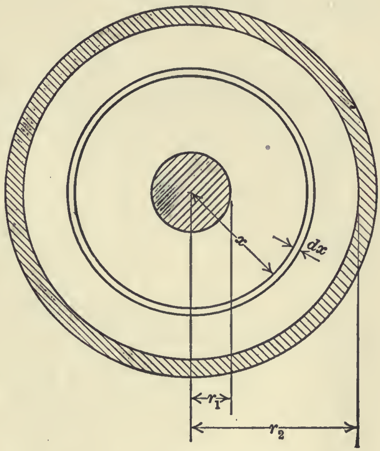

Fig. 240. A conductor surrounded by an insulation and a sheath forms a condenser.

Consider an elementary cylinder of length one centimeter perpendicular to the paper, thickness dx, at a radius x. The capacity of such a cylinder can be found from the formula for a flat-plate condenser,

$$C_{dx} = a\frac{KA}{dx}\text{ abfarads,} \tag{120}$$

where

K is the dielectric constant of the material,

a is the constant $\dfrac{1}{36\,\pi\,10^{20}}$.

The dimensions are in centimeters; A is the cross-sectional area of the dielectric. Since we are considering unit length of the cable,

$$A = 2\pi x.$$

Now the total condenser can be considered to be made up of these elementary condensers in series. In the case of condensers in series we have seen that to obtain the reciprocal of the total capacitance, we must find the sum of the reciprocals of the individual capacitances. Inserting the value of A, we have

$$\frac{1}{C_{dx}} = \frac{dx}{2\pi x a K}.$$

To obtain the total capacity we must integrate this reciprocal to cover all the elementary condensers.

Hence

$$\frac{1}{C} = \int_{r_1}^{r_2} \frac{1}{2\pi a K x} dx; \tag{121}$$

that is,

$$\frac{1}{C} = \frac{1}{2\pi aK} \log_\epsilon \frac{r_2}{r_1}, \tag{122}$$

or

$$C = \frac{2\pi aK}{\log_\epsilon \frac{r_2}{r_1}}. \tag{123}$$

Inserting the value of a, we have, if dimensions are in centimeters, the formula for the capacitance of the cable in abfarads per centimeter length

$$C = \frac{1}{18 \times 10^{20}} \frac{K}{\log_\epsilon \frac{r_2}{r_1}} \text{ abfarads per centimeter,} \tag{124}$$

or, since

$$\begin{aligned} 1 \text{ abfarad} &= 10^9 \text{ farads} \\ &= 10^{15} \text{ microfarads,} \end{aligned}$$

$$C = \frac{1}{18 \times 10^5} \frac{K}{\log_\epsilon \frac{r_2}{r_1}} \text{ microfarads per centimeter,} \tag{125}$$

or changing to common logarithms,

$$C = 0.0388 \frac{K}{\log_{10} \frac{r_2}{r_1}} \text{ microfarads per mile.} \tag{126}$$

The distribution of stress in the cable can be found as follows. The voltage across each elementary condenser is inversely proportional to its area since each receives the same charge. This gives

$$\frac{de}{dx} = \frac{b}{x}, \tag{127}$$

where b is a constant to be determined. Integrating,

$$E = \int_{r_1}^{r_2} b \frac{dx}{x} = b \log_\epsilon \frac{r_2}{r_1}; \tag{128}$$

or

$$b = \frac{E}{\log_\epsilon \frac{r_2}{r_1}} \tag{129}$$

where E is the voltage applied to the cable.

Hence for the potential gradient at any point, we have

$$\frac{de}{dx} = \frac{E}{x \log_\epsilon \frac{r_2}{r_1}} \text{ volts per centimeter.} \tag{130}$$

This distribution of stress is plotted in Fig. 241. We obtain the maximum stress when $x = r_1$.

$$\left(\frac{de}{dx}\right)_{\max} = \frac{E}{r_1 \log_\epsilon \frac{r_2}{r_1}} \text{ volts per centimeter.} \tag{131}$$

It is this value of stress which must be considered in cable design.

Prob. 42–13. What is the capacitance to sheath per mile of a rubber-covered single-conductor cable having a wire of No. 0000 (B & S) and insulation $\frac{7}{64}$ inch thick?

Prob. 43–13. If impregnated paper is substituted for the rubber in the cable of Prob. 42–13, what will be the capacitance per mile? The dielectric constant of impregnated paper may be taken as 2.5.

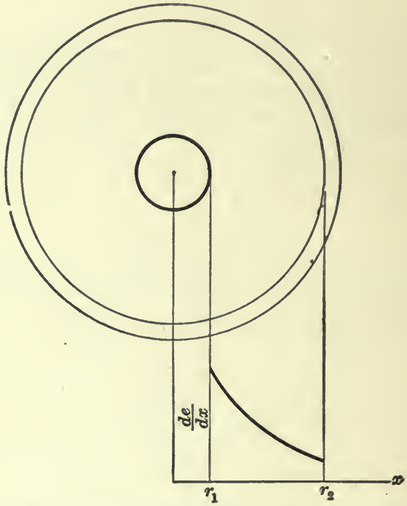

FIG. 241. The distribution of stress in the insulation of a cable with a metal sheath.

Prob. 44–13. Describe with diagrams how an ungrounded break in an insulated cable could be located provided the capacitance of the whole cable before the break was known.

Prob. 45–13. If a 1000-volt potential is applied to the cable of Prob. 42–13, plot the potential gradient throughout the insulation.

SUMMARY OF CHAPTER XIII

AN INSULATING MATERIAL is one which conducts no appreciable current when only moderate voltages are applied to it. Glass, porcelain, rubber, oil etc., are insulators under ordinary conditions.

THE DIELECTRIC STRENGTH of an insulating material is the potential gradient necessary to break it down and allow a current to flow. Temperature, freedom from moisture and duration of voltage are some of the factors affecting dielectric strength of a material.

CONDENSER ACTION is the name given to the phenomenon of the flow of current on to or off from a plate or conductor when its potential is raised or lowered. It is explained by assuming that while the electrons cannot leave the plate, they are moved a little in position and are under stress. The ratio of the current flowing to the rate of change of voltage is the **CAPACITANCE** of the condenser.

A CONDENSER HAS ONE MICROFARAD of capacitance when a voltage change of one volt per second causes a flow of a microampere, or when a change of one volt causes a change of one microcoulomb in the charge on the condenser.

THE DIELECTRIC CONSTANT of a material is the number which shows the ratio of the capacitance of a condenser using that material as a dielectric to a similar condenser using air.

THE CAPACITY OF A PARALLEL-PLATE CONDENSER can be found from the equation

$$C = 0.8842 \times 10^{-7} \frac{KA}{t} \text{ microfarads.}$$

THE CHARGE ON A CONDENSER is found by the equation

$$Q = CE.$$

CAPACITANCE MAY BE MEASURED by the bridge method or directly by a calibrated ballistic galvanometer.

THE CURRENT ENTERING OR LEAVING a condenser through a resistance when a voltage is suddenly applied or removed is represented by the following equation:

$$i = \frac{E}{R} e^{-\frac{t}{RC}}.$$

THE TIME CONSTANT of a circuit containing resistance and capacity is the time in which the current would reduce to zero if it continued to decrease at the initial rate, and equals RC.

THE CHARGE AT ANY TIME ON A CONDENSER which is being charged is

$$q = EC\left(1 - e^{-\frac{t}{RC}}\right)$$

and while being discharged is

$$q = EC\, e^{-\frac{t}{RC}}.$$

THE ENERGY STORED IN A CONDENSER is represented by the equation

$$W_c = \frac{CE^2}{2}.$$

THE ENERGY USED UP IN THE SERIES RESISTANCE in charging a condenser from a constant voltage source regardless of the amount of resistance also equals

$$W_R = \frac{CE^2}{2}.$$

THE MECHANICAL FORCE acting between the plates of a parallel-plate condenser is expressed by the equation

$$f = \frac{CE^2}{2S} \times 10^7 \text{ dynes}$$

or by

$$f = \frac{1}{9 \times 10^{20}} \frac{D^2 A}{8\pi K}$$

in which D is the electrostatic flux density and K the dielectric constant.

THE ELECTROSTATIC FORCE F MAY BE MEASURED in volts per centimeter and is analogous to the magnetization force H, measured in gilberts per centimeter. It is also called the **POTENTIAL GRADIENT**. The potential between two conductors of a condenser is the integral of the potential gradient between the conductors.

THE CAPACITANCE OF TWO LONG PARALLEL AERIAL CONDUCTORS is expressed by the equation

$$C = \frac{0.0194}{\log \frac{S}{r}} \text{ microfarads per mile of line.}$$

WHEN A SINUSOIDAL VOLTAGE IS APPLIED to a perfect condenser, a sinusoidal current flows which reaches its maximum value a quarter cycle before the voltage does. If

$$e = E_{max} \sin \omega T$$

then

$$i = CE_{max\omega} \cos \omega t.$$

NO POWER IS ABSORBED BY A PERFECT CONDENSER but if there is **LEAKAGE** or **HYSTERESIS** loss in the dielectric, then power is consumed and the temperature rises.

THE CAPACITANCE OF CONDENSERS IN PARALLEL equals

$$C_p = C_1 + C_2 + C_3 \cdots\cdots$$

THE CAPACITANCE OF CONDENSERS IN SERIES equals

$$C_s = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}$$

THE DISTRIBUTION OF STRESS IN INSULATION is **INVERSELY PROPORTIONAL** to the dielectric constant of the various series layers of material composing the insulator.

THE CAPACITY OF AN INSULATED CABLE may be found from the equation

$$C = 0.0388 \frac{K}{\log_{10} \frac{r_2}{r_1}} \text{ microfarads per mile.}$$

THE MAXIMUM STRESS IN A CABLE occurs when $x = r_1$ and can be found from the equation

$$\left(\frac{de}{dx}\right)_{max} = \frac{E}{r_1 \log_\epsilon \frac{r_2}{r_1}} \text{ volts per centimeter.}$$

THE FOLLOWING RELATIONS exist among the units of the various systems used in this chapter.

1 statvolt = 300 volts
1 volt = 10^8 abvolts
1 abcoulomb = 10 coulombs
1 coulomb = 3×10^9 statcoulombs
1 statcoulomb = 2.1×10^9 electrons
1 abfarad = 10^9 farads
1 farad = 10^6 microfarads
1 microfarad = 9×10^5 statfarads

PROBLEMS ON CHAPTER XIII

Prob. 46–13. A circular disk of glass $\frac{1}{4}$ inch thick and 6 inches in diameter has an electric potential applied between circular plates centrally located on opposite faces. Assume the surface resistivity to be 2×10^{10} ohm-centimeters. Assume also that the glass has a dielectric strength of 10^5 volts per centimeter but that 10,000 volts per centimeter surface gradient will flash it over. Neglect fringing of the lines of electrostatic force in the material.

(*a*) If the circular electrodes are 1 inch in diameter, at what potential will the disk break down and how?

(*b*) If the electrodes are 3 inches in diameter, at what potential will the disk break down and how?

Prob. 47–13. A sheet of tinfoil 2×24 inches and 0.001 inch thick is placed in the center of a sheet of paper 3×26 inches and 0.0015 inch thick, and having a dielectric constant of 4. The tinfoil is then covered with another sheet of paper of the same size as the bottom one, and another similar sheet of tinfoil is laid on the top sheet of paper. The whole is carefully wrapped about a mandrel of $\frac{3}{4}$ inch diameter, forming a cylindrical condenser 3 inches long and slightly over $\frac{3}{4}$ inch in diameter. Connections are now led to the two sheets of tinfoil. What is the capacitance of the condenser?

Prob. 48–13. What will be the capacitance found as in Prob. 47–13 if we initially pile up 12 sheets of tinfoil 2×240 inches and 0.001 inch thick, and 12 sheets of 3×242 inch paper of 0.0015 inch thickness?

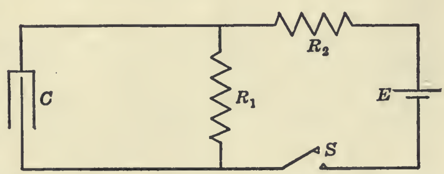

FIG. 242. A circuit containing resistance and capacitance.

Prob. 49–13. The capacitance of the condenser in Fig. 242 is 4 microfarads, the constant voltage of the battery E is 20 volts, R_1 is 30 ohms, R_2 is 45 ohms.

(*a*) Write the equation of the transient current delivered by the battery E upon closing the switch S.

(*b*) What current will be flowing 0.0005 second after closing the switch,

(1) in R_2?
(2) in R_1?
(3) in C?

Prob. 50–13. (*a*) What will be the charge in the condenser of Fig. 242, 0.08 second after the switch is closed?

(*b*) What will be the voltage across C, R_2 and R_1 at this instant?

Prob. 51–13. After the condenser of Prob. 49–13 has become fully charged, the switch S is opened. What current will be flowing through R_1, 0.0002 second after the switch is opened?

Prob. 52–13. What charge will there be on the condenser of Prob. 51–13, 0.0002 second after the switch is opened?

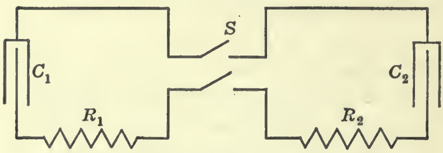

FIG. 243. The switch S connects the positively charged terminals of the condensers.

Prob. 53–13. In Fig. 243, C_1 has a capacitance of 2.4 mf. and is charged to a potential of 20 volts. $R_1 = 10$ ohms and $R_2 = 6$ ohms. C_2 has a capacitance of 4.6 mf. and is charged to a potential of 8 volts. When the switch is thrown it connects the positive sides of the condensers together. What current will be flowing 0.07 second after the switch is closed?

Prob. 54–13. If the initial polarity of one of the condensers in Prob. 53–13 is reversed, what current will be flowing 0.07 second after the switch is closed?

Prob. 55–13. It will be noted that the charge and discharge current curves of condensers through resistance are identical. The same is true even if the resistance is inductive. On the basis of this fact, prove that in charging condensers from a constant source of potential through an inductive resistance, just half the energy input is lost in heat.

Prob. 56–13. If the dielectric of a condenser has a constant high resistivity, and if the applied potential is maintained constant, will the maximum potential gradient in the dielectric become greater or less with time after application of constant potential?

Prob. 57–13. Explain the phenomenon of dielectric absorption on the basis of leakage in the dielectric.

Prob. 58–13. Four condensers each of 1 microfarad capacitance are connected in series. One of the end condensers has a conductance of 10^{-8} mhos, the others have zero conductance. Plot the potential across each condenser as a function of the time after applying a potential of 1000 volts to the series.

Prob. 59–13. Repeat Prob. 58–13 using conductances of 10^{-8}, 2×10^{-8}, 4×10^{-8} and 8×10^{-8} mhos respectively for the condensers in series.

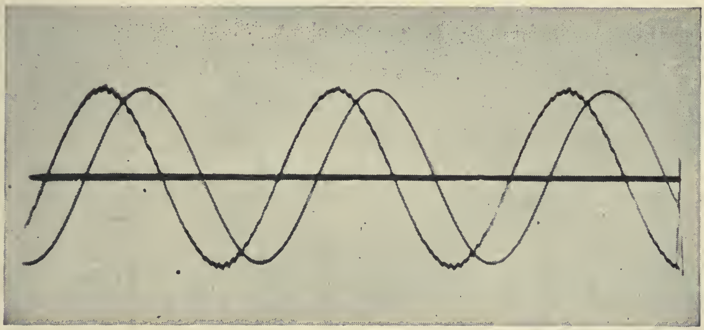

Fig. 244. The current and voltage curves of a leaky condenser. The curve with the small waves in it is the current curve. *Prof. F. S. Dellenbaugh.*

Prob. 60–13. Construct the power curve from the curves of current and voltage shown in Fig. 244. The frequency is 60 cycles per second; the voltage scale, 250 volts per inch; the current scale, 1.8 ampere per inch. What is the average power loss in the condenser under these conditions?

APPENDIX

TABLE I

Resistivity and Temperature Coefficient

Material	Resistivity in microhm-centimeters at 20° C.	Temperature coefficient per degree C. per ohm at 20° C.
Aluminum	2.828	0.0039
Antimony	36 to 46	0.0039
Bismuth	119	0.0045
Brass	8.7	0.0010
Constantin	50	0.000005
Copper		
Annealed Intnl. Standard	1.724	0.00393
Hard drawn	1.77	0.00382
Pure	1.68	0.0041
German Silver	30	0.00036
Gold	2.2 to 2.26	0.0037
Iron, Commercial	11 to 13.5	0.0055
Hard cast	98	
Lead	20.1 to 21.4	0.0042
Manganin	42 to 74	0.00003
Monel Metal	42.6	0.00198
Mercury	96	0.0009
Nickel	12 to 14	0.006
Platinum	9.7 to 16.5	0.0038
Platinium-iridium	24.6	0.0012
Silver	1.64 to 1.85	0.0040
Steel, Hard	47.2	0.0016
Soft	17.4	0.0042
Rail	13.8 to 21.6	
Tantalum	15.3	0.0027
Tin	10.4 to 12.5	0.0043
Tungsten	7.6	0.0039
Zinc	6.04 to 6.56	0.0040
Advance Metal	48.8	0.000018
IaIa	49.0	0.000005
Superior	86.4	0.00081
Nichrome	99.6	0.00044
Nichrome II	109.5	0.00016
Calorite	119.5	....

TABLE II
Resistance of International Standard Annealed Copper
American Wire Gauge (B. & S.)

B. & S. gauge, No.	Diameter in mils, d	Area in circular mils, d^2	Ohms per 1000 ft. at 20° C. or 68° F.	Pounds per 1000 ft.	B. & S. gauge, No.	Diameter in mils, d	Area in circular mils, d^2	Ohms per 1000 ft. at 20° C. or 68° F.	Pounds per 1000 ft.
0000	460.00	211,600	0.04901	640.5	21	28.462	810.10	12.80	2.452
000	409.64	167,810	0.06180	508.0	22	25.347	642.40	16.14	1.945
00	364.80	133,080	0.07793	402.8	23	22.571	509.45	20.36	1.542
0	324.86	105,530	0.09827	319.5	24	20.100	404.01	25.67	1.223
					25	17.900	320.40	32.37	0.9699
1	289.30	83,694	0.1239	253.3	26	15.940	254.10	40.81	0.7692
2	257.63	66,373	0.1563	200.9	27	14.195	201.50	51.47	0.6100
3	229.42	52,634	0.1970	159.3	28	12.641	159.79	64.90	0.4837
4	204.31	41,742	0.2485	126.4	29	11.257	126.72	81.83	0.3836
5	181.94	33,102	0.3133	100.2	30	10.025	100.50	103.2	0.3042
6	162.02	26,250	0.3951	79.46	31	8.928	79.70	130.1	0.2413
7	144.28	20,816	0.4982	63.02	32	7.950	63.21	164.1	0.1913
8	129.49	16,509	0.6282	49.98	33	7.080	50.13	206.9	0.1517
9	114.43	13,094	0.7921	39.63	34	6.305	39.75	260.9	0.1203
10	101.89	10,381	0.9989	31.43	35	5.615	31.52	329.0	0.0954
11	90.742	8,234.0	1.260	24.93	36	5.000	25.00	414.8	0.0757
12	80.808	6,529.9	1.588	19.77	37	4.453	19.82	523.1	0.0600
13	71.961	5,178.4	2.003	15.68	38	3.965	15.72	659.6	0.0476
14	64.084	4,106.8	2.525	12.43	39	3.531	12.47	831.8	0.0377
15	57.068	3,256.7	3.184	9.858	40	3.145	9.89	1049	0.0299
16	50.820	2,582.9	4.016	7.818					
17	45.257	2,048.2	5.064	6.200					
18	40.303	1,624.3	6.385	4.917					
19	35.890	1,288.1	8.051	3.899					
20	31.961	1,021.5	10.15	3.092					

TABLE III

Allowable Carrying Capacities of Wires

National Electrical Code, 1920

The following table is for copper wires of ninety-eight per cent conductivity, and must be followed in placing interior conductors.

For insulated aluminum wire, the safe carrying capacity is *eighty-four per cent* of that given for copper wire with the same kind of insulation.

B. & S. gauge	Diameter of solid wires in mils	Area in circular mils	Rubber insulation. Amperes	Varnished cloth insulation. Amperes	Other insulation. Amperes
18	40.3	1,624	3		5
16	50.8	2,583	6		10
14	64.1	4,107	15	18	20
12	80.8	6,530	20	25	25
10	101.9	10,380	25	30	30
8	128.5	16,510	35	40	50
6	162.0	26,250	50	60	70
5	181.9	33,100	55	65	80
4	204.3	41,740	70	85	90
3	229.4	52,630	80	95	100
2	257.6	66,370	90	110	125
1	289.3	83,690	100	120	150
0	325	105,500	125	150	200
00	364.8	133,100	150	180	225
000	409.6	167,800	175	210	275
		200,000	200	240	300
0000	460	211,600	225	270	325
		250,000	250	300	350
		300,000	275	330	400
		350,000	300	360	450
		400,000	325	390	500
		500,000	400	480	600
		600,000	450	540	680
		700,000	500	600	760
		800,000	550	660	840
		900,000	600	720	920
		1,000,000	650	780	1000
		1,100,000	690	830	1080
		1,200,000	730	880	1150
		1,300,000	770	920	1220
		1,400,000	810	970	1290
		1,500,000	850	1020	1360
		1,600,000	890	1070	1430
		1,700,000	930	1120	1490
		1,800,000	970	1160	1550
		1,900,000	1010	1210	1610
		2,000,000	1050	1260	1670

TABLE IV

Atomic Weights and Usual Valences of Some Elements

Compiled from "Handbook of Chemistry and Physics," The Chemical Rubber Company

Element	Atomic weight	Usual valence
Aluminum	27.1	3
Antimony	120.2	3 or 5
Bismuth	208.0	3
Bromine	79.92	1
Cadmium	112.4	2
Calcium	40.09	2
Carbon	12	2 or 4
Chlorine	35.46	1
Chromium	52	3 or 6
Cobalt	58.97	2 or 3
Copper	63.57	2 or 1
Gold	197.2	3
Hydrogen	1.008	1
Iodine	126.92	1
Iridium	193.1	3
Iron	55.85	2 or 3
Lead	207.1	2
Lithium	6.94	1
Magnesium	24.32	2
Manganese	54.93	2, 4 or 6
Mercury	200	1 or 2
Nickel	56.68	2 or 3
Nitrogen	14.01	3 or 5
Oxygen	16.00	2
Phosphorus	31.04	3 or 5
Platinum	195.2	2 or 4
Potassium	39.10	1
Radium	226.4	2
Silicon	28.3	4
Silver	107.88	1
Sodium	23.00	1
Sulphur	32.07	2, 4 or 6
Tantalum	181.0	5
Tin	119.0	2 or 4
Tungsten	184.0	6
Zinc	65.37	2

TABLE V

Specific Heat of Various Materials

Material	Specific heat in calories per gram	Material	Specific heat in calories per gram
Aluminum	0.0217	Sulphur	0.173
Antimony	0.0489	Tin	0.0551
Bismuth	0.0302	Tungsten	0.0336
Bromine	0.107	Zinc	0.0935
Cadmium	0.0560	Glass	
Calcium	0.1520	crown	0.161
Cobalt	0.103	flint	0.117
Copper	0.0931	Ice	0.530
Gold	0.0316	Rubber	0.481
Iron		Mica	0.206
cast	0.119	Paraffin	0.694
wrought	0.115	Porcelain	0.26
Lead	0.031	Brass	0.088
Manganese	0.107	German Silver	0.095
Mercury	0.0335	Alcohol	
Nickel	0.109	ethyl	0.648
Platinum	0.0323	methyl	0.601
Silver	0.0559	Petroleum	0.511

TABLE VI

Dielectric Constants (Average Values)

Material	Dielectric constant	Material	Dielectric constant
Ebonite	2.7	Acetone	26.6
Glass		Alcohol (0° C.)	
flint	9.9	amyl	17.4
hard crown	7.0	ethyl	28.4
lead	6.6	methyl	35
Guttapercha	4.1	Ammonia	22
Mica	5.8	Benzene	2.3
Paraffin	2.1	Glycerine	56.2
Shellac	3.1	Petroleum	2.1
Air	1	Water (pure)	81

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